A Simulation Based Comparative Study of Normalization Procedures in Multiattribute Decision Making

SUBRATA CHAKRABORTY
Student Member, IEEE
Clayton School of Information Technology,
Monash University, Clayton, Victoria 3800. AUSTRALIA.

CHUNG-HSING YEH
Senior Member, IEEE
Clayton School of Information Technology,
Monash University, Clayton, Victoria 3800. AUSTRALIA.

Abstract: – Normalization procedures are required in multiattribute decision making (MADM) to transform performance ratings with different data measurement units in a decision matrix into a compatible unit. MADM methods generally use one particular normalization procedure without considering the suitability of other available procedures. This study compares four commonly known normalization procedures in terms of their ranking consistency and overall preference value consistency when used with the most widely used simple additive weight method. To achieve this, new performance measure indices are introduced and new simulation settings are devised for dealing with various measurement settings. A wide range of MADM problems with various measurement scales are generated by simulation for the comparison study. The experiment result shows that vector normalization and linear scale transformation (the max method) outperforms other normalization procedures when used with SAW.

Key-Words: - MADM, SAW, Normalization, Decision making, Decision support systems, Simulation study, Method comparison, Decision consistency.

1 Introduction

Multiattribute decision making (MADM) problems involve ranking or evaluating a finite number of alternatives with multiple, often conflicting, attributes. Various MADM methods have been developed to solve different problem settings. MADM methods have shown their suitability to particular decision problems. The sheer complexity in MADM problems and the need for a timely solution prompts to choose any MADM method suitable to a problem on the basis of experience, knowledge and intuition. With quite a few available methods, it is extremely difficult to choose the best method that meets all the requirements. Several comparative studies on MADM methods have shown that certain methods are more suitable for specific decision settings as compared to other methods [2][7][12][13][15].

In MADM problems, each alternative has a performance rating for each attribute, which represents the characteristics of the alternative. It is common that performance ratings for different attribute are measured by different units. To transform performance ratings into a compatible measurement unit, normalization procedures are used. MADM methods often use one normalization procedure to achieve compatibility between different measurement units. For example, SAW uses linear scale transformation (max method) [1][3][5][6][13][14], TOPSIS uses vector normalization procedure [13][14][16], ELECTRE uses vector normalization [4][14] and AHP uses linear scale transformation (sum method) [8][9][10][14].

Enormous efforts have been made to comparative studies of MADM methods, but no significant study is conducted on the suitability of normalization procedures used in those MADM methods. This leaves the effectiveness of various MADM methods in doubt and certainly raises the necessity to examine the effects of various normalization procedures on decision outcome when used with given MADM methods.

It is thus the main purpose of this paper to find out the effect of four commonly used normalization procedures on decision outcomes of a given MADM method for the general MADM problem where attribute measurement units are different.

2 MADM & Normalization Procedures

An MADM problem usually involves a set of m alternatives A_i ($i = 1, 2, \ldots, m$), which are to be evaluated based on a set of n attributes (evaluation criteria) C_j ($j = 1, 2, \ldots, n$). Assessments are to be made to determine (a) the weighting vector $W = (w_1, w_2, \ldots, w_j, \ldots, w_n)$ and (b) the decision matrix $X = \{x_{ij}, i=1, 2, \ldots, m; j=1, 2, \ldots, n\}$. The weighting vector W represents the relative importance of n attributes C_j ($j=1, 2, \ldots, n$) for the problem. The decision matrix X represents the performance ratings x_{ij} of alternatives A_i ($i = 1, 2, \ldots, m$) with respect to attributes C_j ($j = 1, 2, \ldots, n$). Given the weighting vector W and decision matrix X, the objective is to rank or select the alternatives by giving each of them an overall preference value with respect to all attributes.

MADM methods generally require two processes to obtain the overall preference value for each alternative – (a) normalization and (b) aggregation. Normalization is first used to transform performance ratings to a compatible unit scale. An aggregation procedure is then used to combine normalized decision matrix and attributes weight W to achieve an overall preference value for each alternative, on which the overall ranking of alternatives is based.

To help present the comparative study, the four well known normalization procedures used in MADM are briefly described below, including: (a) vector normalization, (b) linear scale transformation (max-min method), (c) linear scale transformation (max method) and (d) linear scale transformation (sum method).

2.1 Vector Normalization (N1)

In this method, each performance rating of the decision matrix is divided by its norm. The normalized value r_{ij} is obtained by

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x^2_{ij}}} \qquad (1)$$

This method has the advantage of converting all attributes into dimensionless measurement unit, thus making inter-attribute comparison easier. But it has the drawback of having non-equal scale length leading to difficulties in straightforward comparison [6] [14].

2.2 Linear Scale Transformation, Max-Min Method (N2)

This method considers both the maximum and minimum performance ratings of attributes during calculation.

For benefit attributes, the normalized value r_{ij} is obtained by

$$r_{ij} = \frac{x_{ij} - x_j^{\min}}{x_j^{\max} - x_j^{\min}} \qquad (2)$$

For cost attributes, r_{ij} is computed as

$$r_{ij} = \frac{x_j^{\max} - x_{ij}}{x_j^{\max} - x_j^{\min}} \qquad (3)$$

where $x_j^{\max}$ is the maximum performance rating among alternatives for attribute C_j ($j = 1, 2, \ldots, n$) and $x_j^{\min}$ is the minimum performance rating among alternatives for attribute C_j ($j = 1, 2, \ldots, n$).

This method has the advantage that the scale measurement is precisely between 0 and 1 for each attribute. The drawback is that the scale transformation is not proportional to outcome [6].

2.3 Linear Scale Transformation, Max Method (N3)

This method divides the performance ratings of each attribute by the maximum performance rating for that attribute.

For benefit attributes, the normalized value r_{ij} is obtained by

$$r_{ij} = \frac{x_{ij}}{x_j^{\max}} \qquad (4)$$

For cost attributes, r_{ij} is computed as

$$r_{ij} = 1 - \frac{x_{ij}}{x_j^{\max}} \qquad (5)$$

where $x_j^{\max}$ is the maximum performance rating among alternatives for attribute C_j ($j = 1, 2, \ldots, n$).

Advantage of this method is that outcomes are transformed in a linear way [6][14].

2.4 Linear Scale Transformation, Sum Method (N4)

This method divides the performance ratings of each attribute by the sum of performance ratings for that attribute as follows

$$r_{ij} = \frac{x_{ij}}{\sum_{j=1}^{n} x_j} \quad (6)$$

where x_j is performance rating for each alternative for attribute C_j $(j = 1, 2, \ldots, n)$ [14].

In order to obtain the overall preference value, the normalized decision matrix generated by a normalization procedure needs to be aggregated by an MADM method. In this study, the simple additive weight (SAW) is used.

2.5 Simple Additive Weight (SAW)

The SAW method, also known as the weighted sum method, is probably the best known and most widely used MADM method [6]. The basic logic of the SAW method is to obtain a weighted sum of the performance ratings of each alternative over all attributes. The overall preference value of each alternative is obtained by

$$V_i = \sum_{j=1}^{n} w_j r_{ij} \quad ; i = 1, 2, \ldots, m. \quad (7)$$

Where $V(A_i)$ is the value function of alternative A_i, w_j is weight attribute C_j and r_{ij} are normalized performance ratings [6][14].

3 Experiment and Performance Validation

3.1 Simulation Study

Simulation based experiments were conducted to provide results applicable to the general MADM problem rather than a particular MADM problem. In the simulation study random decision matrices with alternatives A_i $(i = 1, 2, \ldots, 4)$ and attributes C_j $(j = 1, 2, \ldots, 4)$ were generated. Each decision matrix was normalized by using each of four normalization procedures. Then SAW was used to generate an overall preference value for each alternative. To simplify the process without losing generality, all the attributes were assigned equal weights.

To gain an unbiased result, the following settings were used in the experiments:

1) 10000 non-dominant decision matrices were generated randomly for each simulation run.
2) For each data range, the process was repeated 10 times and average was noted in final result table.
3) The data ranges used for four attributes (C_1, C_2, C_3, C_4) were 1 – 10, 1 – 100, 1 – 1000, 1 – 10000 respectively, each of which used 10 increment steps , given below:

Data range 1: [C_1 (1 – 10), C_2 (1 – 100), C_3 (1 – 1000), C_4 (1 – 10000)]
Data range 2: [C_1 (1 – 10), C_2 (10 – 100), C_3 (100 – 1000), C_4 (1000 – 10000)]
Data range 3: [C_1 (2 – 10), C_2 (20 – 100), C_3 (200 – 1000), C_4 (2000 – 10000)]
Data range 4: [C_1 (3 – 10), C_2 (30 – 100), C_3 (300 – 1000), C_4 (3000 – 10000)]
Data range 5: [C_1 (4 – 10), C_2 (40 – 100), C_3 (400 – 1000), C_4 (4000 – 10000)]
Data range 6: [C_1 (5 – 10), C_2 (50 – 100), C_3 (500 – 1000), C_4 (5000 – 10000)]
Data range 7: [C_1 (6 – 10), C_2 (60 – 100), C_3 (600 – 1000), C_4 (6000 – 10000)]
Data range 8: [C_1 (7 – 10), C_2 (70 – 100), C_3 (700 – 1000), C_4 (7000 – 10000)]
Data range 9: [C_1 (8 – 10), C_2 (80 – 100), C_3 (800 – 1000), C_4 (8000 – 10000)]
Data range 10: [C_1 (9 – 10), C_2 (90 – 100), C_3 (900 – 1000), C_4 (9000 – 10000)]

3.2 Performance Measures

3.2.1 Ranking Consistency

Ranking consistency is used to indicate how well a particular normalization procedure produces rankings similar to other procedures. To measure the ranking consistency index (RCI) of a particular normalization procedure, the total number of times the procedure showed similarities/ dissimilarities in various extents with other procedures applied is calculated, over 10,000 simulation runs and then its ratio with total number of simulation runs is calculated. The higher the RCI, the better the procedure performs.

In calculating RCI, a consistency weight (CW) is used as follows:

1) If a method is consistent with all 3 of other 3 methods, then $CW = 3/3 = 1$.
2) If a method is consistent with any 2 of other 3 methods, then $CW = 2/3$.

3) If a method is consistent with any 2 other methods, then $CW = 1/3$.
4) If a method is not consistent with any other methods, then $CW = 0/3 = 0$.

The consistency index of N1 is calculated as

$$RCI\,(N1) = [(T_{1234} * (CW=1) + T_{123} * (CW=2/3) + T_{124} * (CW=2/3) + T_{134} * (CW=2/3) + T_{12} * (CW=1/3) + T_{13} * (CW=1/3) + T_{14} * (CW=1/3) + TD_{1234} * (CW=0)) / TS] \quad (8)$$

The consistency index of N2 is calculated as

$$RCI\,(N2) = [(T_{1234} * (CW=1) + T_{123} * (CW=2/3) + T_{124} * (CW=2/3) + T_{234} * (CW=2/3) + T_{12} * (CW=1/3) + T_{23} * (CW=1/3) + T_{24} * (CW=1/3) + TD_{1234} * (CW=0)) / TS] \quad (9)$$

The consistency index of N3 is calculated as

$$RCI\,(N3) = [(T_{1234} * (CW=1) + T_{123} * (CW=2/3) + T_{134} * (CW=2/3) + T_{234} * (CW=2/3) + T_{13} * (CW=1/3) + T_{23} * (CW=1/3) + T_{34} * (CW=1/3) + TD_{1234} * (CW=0)) / TS] \quad (10)$$

The consistency of N4 is calculated as

$$RCI\,(N4) = [(T_{1234} * (CW=1) + T_{124} * (CW=2/3) + T_{134} * (CW=2/3) + T_{234} * (CW=2/3) + T_{14} * (CW=1/3) + T_{24} * (CW=1/3) + T_{34} * (CW=1/3) + TD_{1234} * (CW=0)) / TS] \quad (11)$$

where

$RCI\,(X)$ = Ranking consistency index for normalization procedure X.
TS = Total number of times the simulation was run (10,000 in this experiment).
T_{1234} = Total number of times N1, N2, N3 and N4 produced same rank.
T_{123} = Total number of times N1, N2 and N3 produced same rank.
T_{124} = Total number of times N1, N2 and N4 produced same rank.
T_{134} = Total number of times N1, N3 and N4 produced same rank.
T_{234} = Total number of times N2, N3 and N4 produced same rank.
T_{12} = Total number of times N1and N2 produced the same rank.
T_{13} = Total number of times N1and N3 produced the same rank.
T_{14} = Total number of times N1 and N produced the same rank.
T_{23} = Total number of times N2 and N3 produced the same rank.
T_{24} = Total number of times N2 and N4 produced the same rank.
T_{34} = Total number of times N3 and N4 produced the same rank.
TD_{1234} = Total number of times N1, N2, N3 and N4 produced different rank.

3.2.2 Overall Preference Value Consistency

An overall preference value for each alternative is generated by aggregating the normalized performance ratings; using an MADM method (e.g. SAW in this experiment). The overall preference value consistency is measured as the deviation in ratio between any pair of normalization procedures on the basis of their total positive distances in overall preference values produced by each procedure. Ideally the deviation is 0, so the method has a lower deviation is certainly a better one.

Fig. 1 shows the overall preference value matrix which is obtained by combining the overall preference value for each alternative for each normalization procedure.

	N_1	N_2	…	N_m
A_1	P_{11}	P_{12}	…	P_{1m}
A_2	P_{21}	P_{22}	…	P_{2m}
A_3	P_{31}	P_{32}	…	P_{3m}
…	…	…..	…	……..
A_n	P_{n1}	P_{n2}		P_{nm}

Fig. 1 Overall preference value matrix

In Fig. 1, P_{ij} are the overall preference values for each alternative A_i $(i= 1, 2, \ldots, n)$, for each normalization procedure N_j $(j= 1, 2, \ldots, m)$.

For simplicity and generality four alternatives (A1, A2, A3, A4) and four normalization procedures (N1, N2, N3, N4) has been used as an example in the experiments.

The total positive distance between overall preference values for each normalization procedure can be calculated as

$$TVD_1 = |P_{11} - P_{21}| + |P_{11} - P_{31}| + |P_{11} - P_{41}| + |P_{21} - P_{31}| + |P_{21} - P_{41}| + |P_{31} - P_{41}| \quad (12)$$

$$TVD2 = |P_{12} - P_{22}| + |P_{12} - P_{32}| + |P_{12} - P_{42}| + |P_{22} - P_{32}| + |P_{22} - P_{42}| + |P_{32} - P_{42}| \quad (13)$$

$$TVD3 = |P_{13} - P_{23}| + |P_{13} - P_{33}| + |P_{13} - P_{43}| + |P_{23} - P_{33}| + |P_{23} - P_{43}| + |P_{33} - P_{43}| \quad (14)$$

$$TVD4 = |P_{14} - P_{24}| + |P_{14} - P_{34}| + |P_{14} - P_{44}| + |P_{24} - P_{34}| + |P_{24} - P_{44}| + |P_{34} - P_{44}| \quad (15)$$

where
TVD_1 = Total overall preference value distance for N1
TVD_2 = Total overall preference value distance for N2
TVD_3 = Total overall preference value distance for N3
TVD_4 = Total overall preference value distance for N4

The pair wise distance ratios between normalization procedures are calculated as

$$RVD_{12} = TVD1 / TVD2 \quad (16)$$

$$RVD13 = TVD1 / TVD3 \quad (17)$$

$$RVD14 = TVD1 / TVD4 \quad (18)$$

$$RVD23 = TVD2 / TVD3 \quad (19)$$

$$RVD24 = TVD2 / TVD4 \quad (20)$$

$$RVD34 = TVD3 / TVD4 \quad (21)$$

where
RVD_{12} = Total Overall Preference Value Distance Ratio between N1 and N2
RVD_{13} = Total Overall Preference Value Distance Ratio between N1 and N3
RVD_{14} = Total Overall Preference Value Distance Ratio between N1 and N4
RVD_{23} = Total Overall Preference Value Distance Ratio between N2 and N3
RVD_{24} = Total Overall Preference Value Distance Ratio between N2 and N4
RVD_{34} = Total Overall Preference Value Distance Ratio between N3 and N4

The deviation in pair wise ratios are calculated by

$$DRVD_{12} = 1 - RVD_{12} \quad (22)$$

$$DRVD13 = 1 - RVD13 \quad (23)$$

$$DRVD14 = 1 - RVD14 \quad (24)$$

$$DRVD23 = 1 - RVD23 \quad (25)$$

$$DRVD24 = 1 - RVD24 \quad (26)$$

$$DRVD34 = 1 - RVD34 \quad (27)$$

where
$DRVD_{12}$ = Deviation in RVD between N1 and N2
$DRVD_{13}$ = Deviation in RVD between N1 and N3
$DRVD_{14}$ = Deviation in RVD between N1 and N4
$DRVD_{23}$ = Deviation in RVD between N2 and N3
$DRVD_{24}$ = Deviation in RVD between N2 and N4
$DRVD_{34}$ = Deviation in RVD between N3 and N4

The average deviation for each normalization procedure is calculated respectively as,

$$AD\ (N1) = (DRVD_{12} + DRVD_{13} + DRVD_{14}) / 3 \quad (28)$$

$$AD\ (N2) = (DRVD_{12} + DRVD_{23} + DRVD_{24}) / 3 \quad (29)$$

$$AD\ (N3) = (DRVD_{13} + DRVD_{23} + DRVD_{34}) / 3 \quad (30)$$

$$AD\ (N4) = (DRVD_{14} + DRVD_{24} + DRVD_{34}) / 3 \quad (31)$$

where
$AD\ (X)$ = Average Deviation for normalization procedure X

Equations (8) – (31) can be extended to accommodate any number of alternatives and any number of normalization procedures.

4 Experiment Results and Analysis

4.1 Results for Ranking Consistency

For each of the 10 data ranges, the simulation was run for 10,000 times (i.e. 10,000 performance matrices were generated) and the total number of times the normalization procedures performed with similar or different results as other procedures were recorded. Fig. 2 shows the result.

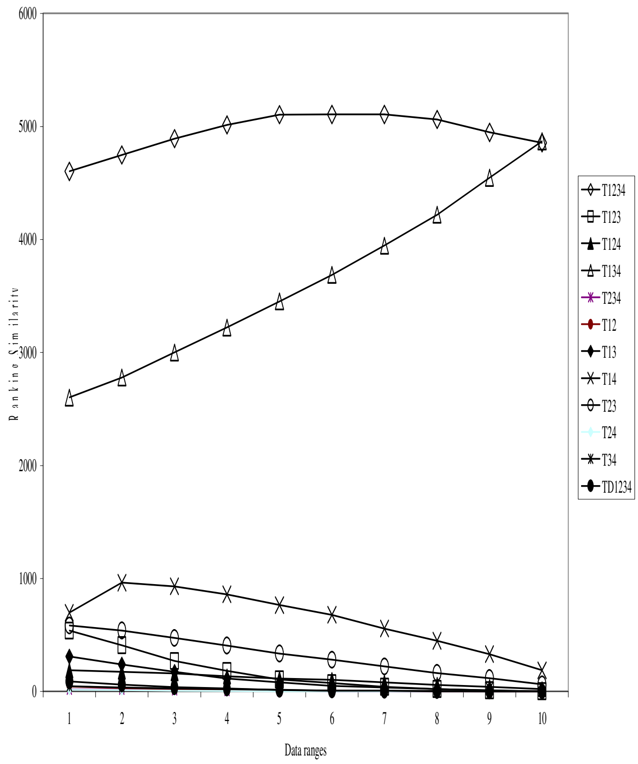

Fig. 2 Ranking similarity over various data ranges.

As shown in Fig. 2, the number of times N1, N2, N3 and N4 produce the same ranking is almost 50% of the total run over different data ranges. There is an initial increase with but is decreased slightly when the range becomes too narrow. The same rankings produced by N1, N3 and N4 increases consistently with narrow data ranges and almost reached 50% of total run with the narrowest data range. In addition, the number of times the same rankings produced by N1 and N4 has an initial increase but gradually decreases over narrow data ranges. It is also evident that the combination of N1, N2 and N3 and the combination of N2 and N3 have a significant number of similarities which dropped heavily with narrow data ranges. Other combinations produce same ranking few times, which are further decreased with narrow data ranges.

The result suggests that with wide data ranges, all four procedures produce similar outcomes for almost 50% of times. In particular N1, N3 and N4 produce similar results in most cases.

RCI for normalization procedures N1, N2, N3 and N4 can be calculated by applying Equations (8) – (11) on the ranking similarity result. Fig. 3 shows the results over various data ranges.

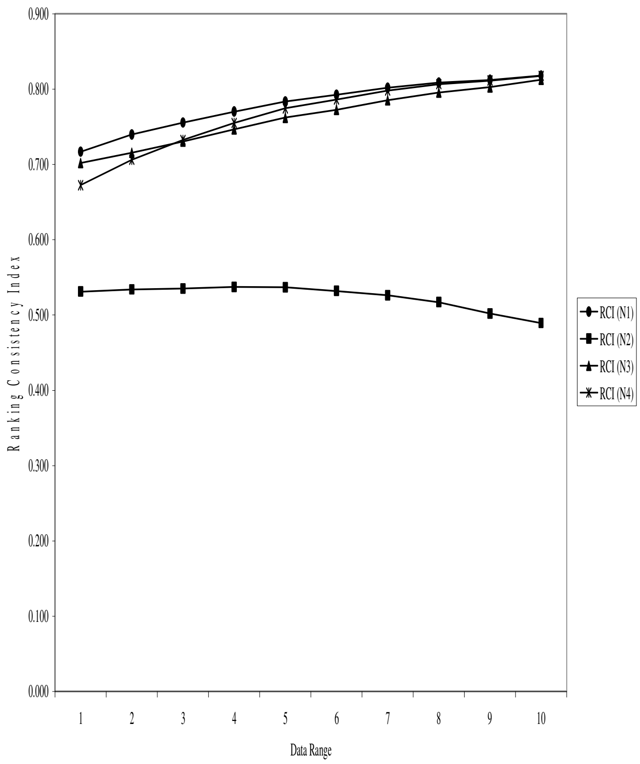

Fig. 3 Ranking consistency index for N1, N2, N3 and N4 over various data ranges.

As shown in Fig. 3, N1 has the highest RCI, closely followed by N3 and N4. All three procedures have an increase in RCI as data ranges narrow. With narrowing data ranges, N4 shows slightly better RCI over N3. N2 shows a poor level of RCI, as compared to other procedures. N2 shows a decreasing trend over narrow data ranges. On the basis of ranking consistency, N1 performs best, followed by N3 and N4.

B. Results for Overall Preference Value Consistency:

In order to calculate the pairwise ratio deviation, the overall preference value matrix, as shown in Fig. 1 was first generated using the results obtained from the ranking procedure. Equations (12) to (15) are then applied to obtain the total positive distance in preference values for each normalization procedure. Equations (16) to (21) are applied to calculate the pairwise ratio in the total distance for each pair of normalization procedures. Pairwise ratio deviations are obtained using equations (22) to (27). Fig. 4 shows the result.

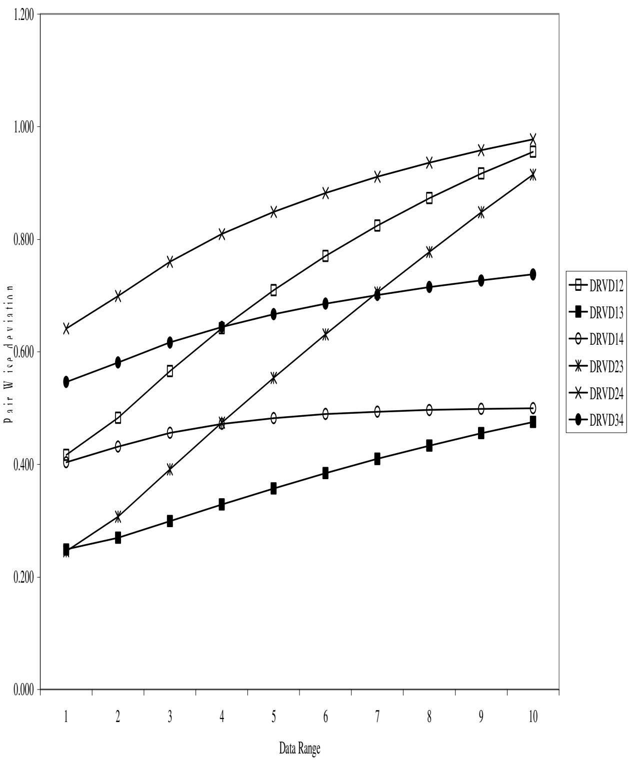

Fig. 4 Pair wise deviation in total overall preference value distance ratio.

As indicated in Fig. 4, pairwise deviation for the N1 and N3 pair has the lowest deviation over all data ranges with a slight increase for narrower ranges. The N1 and N4 pair shows moderate deviation with a minor increase over all data ranges. Deviation for the N2 and N3 pair shows a low deviation in wide data ranges but increases dramatically with narrow data ranges. The pair of N1 and N2 starts with a moderate deviation but increases heavily with narrower ranges. The N3 and N4 pair starts with a comparatively high deviation and has a gradual increment over narrow data ranges. Deviation for the N2 and N4 pair is highest over all data ranges with an increase over narrow ranges.

The average deviation for each of the normalization procedures N1, N2, N3 and N4 is calculated by using Equations (28) to (31) respectively. Fig. 5 shows the average deviation for each normalization procedure over various data ranges.

As shown in Fig. 5, the average deviation is less for N1 and N3 over all data ranges with an increasing trend over narrow data ranges. N3 performs slightly better, although very similar, with wide data ranges, while N1 performs better over narrow data ranges. Deviation for N4 shows less variation over different data ranges but shows more deviation than N1 and N3. N2 shows less deviation than N4 for wide data ranges but has a dramatic increase with narrow data ranges. In terms of overall preference value consistency, N1 performs best closely followed by N3.

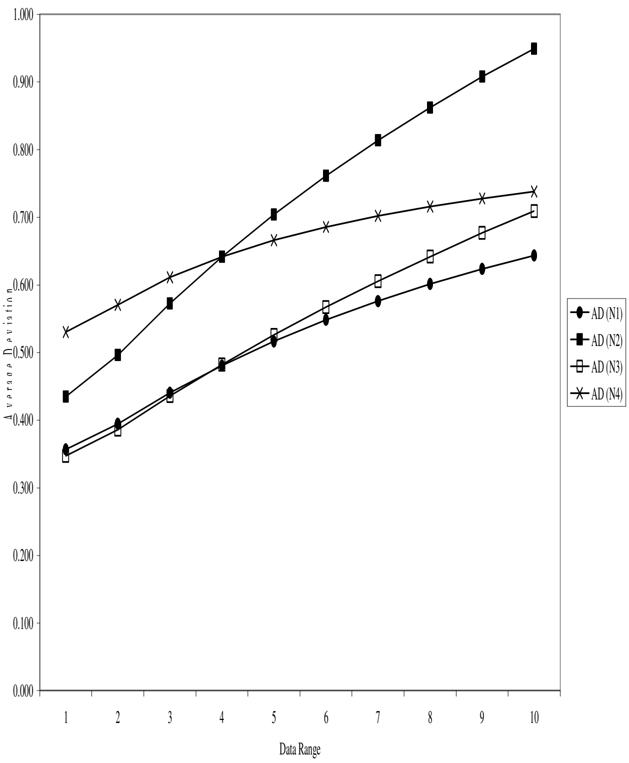

Fig. 5 Average deviation for N1, N2, N3 and N4

5 Conclusion

Multiattribute decision making generally requires using a normalization procedure to transform different measurement units of attributes to a comparable unit. The suitability study of a specific normalization procedure for a given MADM method is required. A simulation based study has been conducted to examine the effects of commonly used normalization procedures, when applied with SAW for the general MADM problem.

The result of the simulation study suggests that with ranking consistency and overall preference value consistency as the performance measures, vector normalization (N1) and linear scale transformation, max method (N3) are more suitable for SAW in decision settings where the attributes measurement units are diverse in range and there are a small number of alternatives to be ranked.

The new performance measures introduced and the new simulation process devised can be extended to other MADM methods and to determine the best combination of the normalization procedure and the MADM method.

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