A New Method for Grasp Stability Analysis

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Abstract: - This paper introduces the concept of dual space for grasp stability analysis. We transform friction cones in the robot work space into line segments in the dual space. Grasp stability is discussed from a novel point of view. We newly propose the Grasp Stability Index (GSI) by calculating intersection condition between line segments in the dual space. Its validity and effectiveness are investigated and verified by simulations for a quadrangular object.

Key-Words: – Friction Cone, Grasp Stability Index

1 Introduction

When an object is grasped, there are three constraints arising from the task, the grasped object and the hand. Within these constraints, Cutkosky [1] defined various analytical measures used to describe a grasp as stability, compliance, connectivity, isotropy, etc. Also he classified shapes of manufacturing grasps considering grasp geometry. When the grasp returned to its initial configuration after being disturbed by external forces or moments, he referred to that at low speeds the grasp is stable if the overall stiffness matrix is positively definite and at higher speeds dynamic stability must be considered.

When being disturbed by external forces and moments, grasp stability shows the degree it is able to maintain equilibrium state without sliding between object and robot fingers. The high stability means that it will be able to maintain equilibrium state well. Because the stability is the ability which is able to resist from disturbance, and gives many effects to the grasp relationship between robot and object, many researchers have interest and advanced researches regarding a stability with various methods [2]-[7].

Many Studies have analyzed grasp stability by using potential energy and stiffness matrix. Funahashi *et al.* [2] analyzed to consider the curvatures of both hand and object at contact points by using potential energy, and showed that the grasp using round fingers was more stable than using sharp fingers. Jenmalm *et al.* [3] verified grasp stability change with different surface curvatures by tests. Howard and Kumar [4] classified the categories of equilibrium grasps and established a general framework for the determination of the stability of grasps by using stiffness matrix. Yamada *et al.* [5] analyzed stability of 3D grasps by using potential energy of a three-dimensional spring model by a multifingered hand. Recently Yamada *et al.* [6] analyzed stability of simultaneous grasps of two objects in two dimensions by using potential energy method. However, potential energy and stiffness matrix methods requested experiences about the work and have a weak point of complex calculation because these methods have to know active force and moved displacement after grasp.

Recently, Sudsang and Phoka [7] proposed a method of testing whether three contact points form a three-fingered force-closure grasp in two dimensions. However, this study was limited to a three-fingered hand in two dimensions and studied only whether or not it grasps objects.

This paper is limited to a two-fingered robot which can be used to the industrial robot and to the case of grasping the object which is placed on the ground (two dimensions). To keep the analysis tractable, we assume rigid-body model with point contacts between the robot fingertips and the grasped object, no sliding or rolling of the fingertips, and quasi-static analysis (no inertial or viscous terms).

This paper has an advantage by using the dual space which is a new method. It can decide grasps possibility with only form information and friction coefficient of an object without relationship with magnitude of force which robot fingers inflict on an object. Also it can calculate the Grasp Stability Index of every grasp point.

2 Background

2.1 Mapping between Robot Work Space and Dual Space

Generally, when a robot grasps an object, achieving force-closure is judged by geometrical representation of the friction cones at the contact points. Sudsang and Phoka [7] studied testing force-closure by representing the friction cones as line segments in the dual plane in two dimensions. Mapping between a friction cone in the robot work space and a line segment in the dual space in two dimensions is shown in Fig. 1.

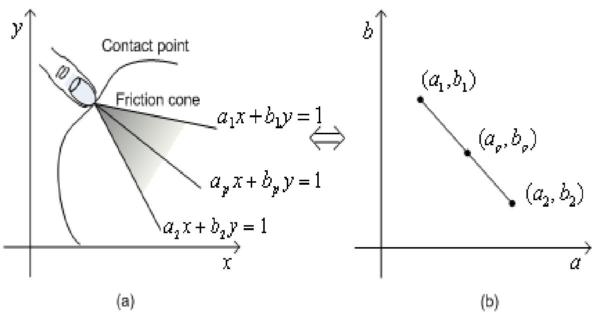

Fig. 1. Mapping between a Friction Cone in the Robot Work Space and a Line Segment in the Dual Space.

$$a_1x+b_1y=1 \quad (1)$$

$$a_2x+b_2y=1 \quad (2)$$

If we transform the two equations (1) and (2) into the points in the dual space, we can get two points (a_1,b_1) and (a_2,b_2). Using the same method, if we transform any equation ($a_px+b_py=1$) which is in the inner friction cone, the transformed point is on a line segment which is between (a_1,b_1) and (a_2,b_2). So we can transform the friction cone in the robot work space into a line segment in the dual space.

2.2 Constraint for Containing the Origin

To transform a friction cone in the robot work space into a line segment in the dual space, the friction cone should not contain the origin. If the linear equation in the friction cone is on the origin, it is impossible to transform the equation into the point in the dual space. Because the *y*-intercept (1/b) must be zero when equation ($a_px+b_py=1$) contains the origin, b is infinite. Thus, we can not transform a friction cone into a line segment in the dual space. Therefore, the friction cone should not include the origin in the robot work space.

2.3 Advantage for Dual Space

Because of method for representing a friction cone in two-dimensional space as a line segment in one- dimensional space, we have the advantage of reducing a dimension. Also, we can easily judge the possibility of grasping an object through geometrical relation of two line segments.

3 Condition of Grasping an Object

3.1 Robot Work Space

To judge the possibility of grasping an object in the robot work space, we can use the friction cone at the contact point.

When two fingers of a robot grasp an object, it is possible for it to grasp if base point and homologous point are in each opposite friction cone. That is to say, as shown in Fig. 2, if a homologous point is in the inner part of the friction cone of base point R which is between B (coordinate (2, 5.73)) and D (coordinate (2, 4.27)), it is possible for it to grasp an object.

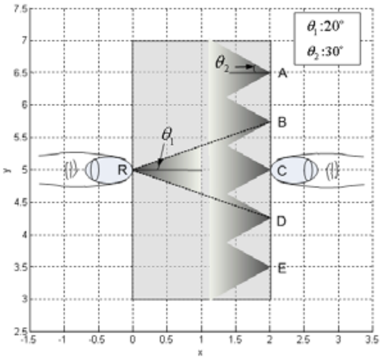

Fig. 2. Grasping an Object in the Robot Work Space. (R: Base Point, A~E: Homologous Points)

3.2 Dual Space

Figure 3 is the results of transforming friction cones in Fig. 2 as line segments in the dual space.

In the robot work space, when a summit of the friction cone is on the *y*-axis and the inner unit normal vector at contact point is normal to the *y*-axis, a transformed line segment in the dual space is parallel to the *a*-axis and symmetrical about the *b*-axis. Accordingly, the friction cone of grasping point R in Fig. 2 is transformed into line segment R' in the dual space that is parallel to the *a*-axis. Using the same method, we get the transformed line segments A', B', C', D' and E' for friction cones at the grasping points A, B, C, D and E. From this, we can infer the following fact from the mapping relation.

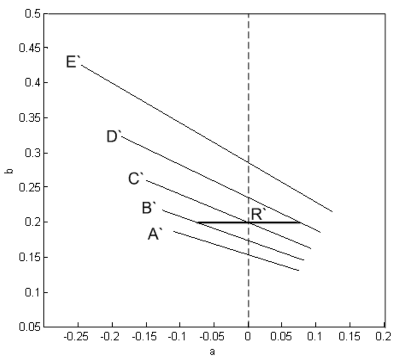

Fig. 3. Grasping an Object in the Dual Space.
(R': Line segment which transform friction cone of a base point R in the robot work space)
(A'~ E': Line segments which transform friction cone of homologous points A~E in the robot work space)

Fact 1: In the dual space, if two line segments intersect, it is possible for it to grasp an object.

As previously stated, in the robot work space base point R and homologous points B, C and D are able to grasp a quadratic object. Because these points are transformed into line segment R' and line segments B', C' and D' in the dual space as shown in Fig. 3, we know that these are possible to grasp the object. However, base point R and homologous points A and E are unable to grasp the object. Because these points are transformed into line segment R' and line segments A' and E' in the dual space as shown in Fig. 3, we know that these can not grasp the object.

Therefore, if two line segments intersect in the dual space, robot can grasp the object without slippage. It is easier to judge grasp of the object in the dual space than in the robot work space, because of visual representation (intersection of line segments).

3.3 Solution for the Problem of Origin Constraint

When the friction cone includes the origin in the robot work space, in accordance with constraint for containing the origin it is impossible to transform the friction cone into the line segment in the dual space. A method of solving the origin constraint is parallel translation of the object along the *y*-axis. After parallel translation, the length of the transformed line segment has to keep constant ratio a/b in Fig 4. The method of a parallel translation is given in the following contents.

1) Selecting a base point on the outline of an object.
2) Locating the object in order to be perpendicular between the inner unit normal vector at a base point and *y*-axis in the robot work space.
3) If the friction cone of a homologous point includes the origin, we should relocate the object. When the unit normal vector of the

homologous point indicates a positive y-axis, then we let parallel translation the object toward a negative y-axis. If the vector indicates a negative side, then we let parallel translation the object toward a positive side. Only a base point always isn't on the origin.

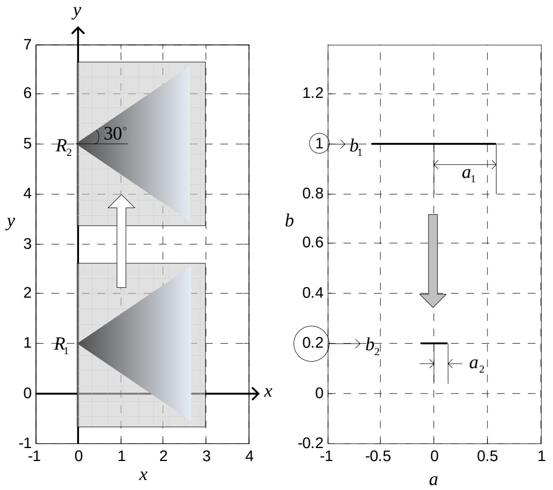

Fig. 4. Changing the Origin.

Figure 4 shows that the ratio of length between a and b is regular despite parallel translation.

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = h \tag{3}$$

As shown in Fig. 4(a), the object is translated along the y-axis with 4, and the values of a, b in the dual space are translated $a_1 = 0.5774$, $b_1 = 1$ into $a_2 = 0.1155$, $b_2 = 0.2$. But the values of h of (3) is regular as 0.5774.

4 Grasp Stability Index

In the former section, we found out that intersection of line segments which transform friction cones of a base point and homologous points means possibility of grasping the object. Although it is possible to grasp by intersection of line segments, grasp stability is quite different depending on the location of intersecting point. In this section, we define the Grasp Stability Index (GSI) in the dual space for quantitative expression of grasp stability.

Fact 2: Division ratio of line segments which transform friction cones of a base point and homologous points is an important factor for grasp stability.

In the robot work space of Fig. 2, a homologous point C is the most stable case to grasp the object. In the same manner, in the dual space of Fig. 3 which transforms the robot work space, line segment R' is bisected by line segment C'. Also, homologous points B and D are the boundary cases to grasp the object, and when we transform the points in the robot work space into a line segment in the dual space, line segments B' and D' are in a contact with each end of the line segment R'.

The friction cones of homologous points that can grasp the object between point B and point D are transformed into line segments between B' and D'. If the homologous point is shifted from point C, which has the most stable grasp, to the point B or point D, the hand of the robot grasps the object become unstable gradually. At this time, in the dual space the intersection point between line segment R' and the transformed line segments of a homologous point gradually shifts from bisected point to end point. Therefore, when the line segment for a base point is bisected, grasp stability is at its maximum. When each end of the line segment is contacted, grasp stability is at its minimum.

When we translate the object along the y-axis, the ratio of length for a base point is kept as shown in (3), but the ratio of length for homologous points is changed. In order to keep ratio of length, we propose as follows.

Proposition 1: The line segments for homologous points can be converted into the standardized line segment which is projected on the line segment for a base point.

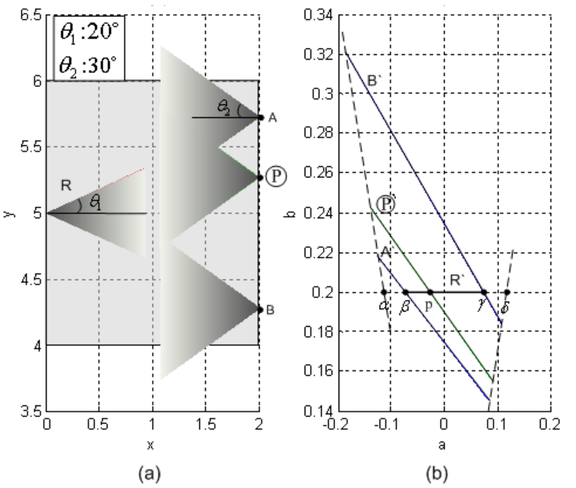

Fig. 5. Transforming Line Segment ⓟ' into Standardized Line Segment $\overline{\alpha\delta}$.

Figure 5(b) represents the method which standardize line segment ⓟ' for defining the Grasp Stability Index. We projects line segment ⓟ' on line segment R'. When we transform points A and B which is on a tangent line of point ⓟ into line segments A' and B' in the dual space, the trace equations which connect both end points of line segments A', ⓟ' and B' is represented by the form of straight lines. From crossing these straight lines and the extension line of line segment R', we can get crossing points α and δ . From this, line segment $\overline{\alpha\delta}$ is a standardized line segment for homologous points.

The angle θ of friction cone at a base point in the robot work space relates to value of length of the *a*-axis in the dual space, as follows.

$$a_0 = |\tan\theta| \qquad \text{(for b=1)} \qquad (4)$$

The greater the angle value of the friction cone becomes, the longer the length of the line segment for a base point enlarges. Therefore, we propose defining the Grasp Stability Index (GSI).

Proposition 2: Grasp Stability Index (GSI)

$$GSI \equiv \begin{pmatrix} \text{Overlapped length} \\ \text{of line segment for} \\ \text{a base point and} \\ \text{a homologous point} \end{pmatrix} \times \begin{pmatrix} \text{ratio of interior} \\ \text{division of line} \\ \text{segment for} \\ \text{a base point} \end{pmatrix} \times \begin{pmatrix} \text{ratio of interior} \\ \text{division of line} \\ \text{segment for} \\ \text{a homologous point} \end{pmatrix}$$

Figure 6 is enlarged section of $\alpha \sim \delta$ to calculate the GSI. We transform line segments for homologous points into standardized line segment L_2.

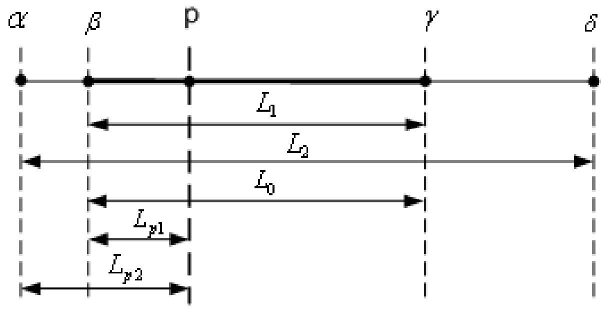

Fig. 6. Representation of Line Segments in the Dual Space.

We get the following form of the Grasp Stability Index (GSI) according to proposition 2.

$$GSI(R,P) = \left(\frac{L_0}{2b_0} \times \frac{L_{p1}}{(L_1/2)} \times \frac{L_{p2}}{(L_2/2)} \right) \qquad (5)$$

The definition of line segments is described in the following form.

$L_1 = \overline{\beta\gamma}$ (Length of the line segment for a base point)

$L_2 = \overline{\alpha\delta}$ (Length of the standardized line segment for homologous points)

$L_0 = \overline{\beta\gamma}$ (Overlapped length of L_1 and L_2)

$L_{p1} = \overline{\beta p}$ (Short length from each end of L_1 to ⓟ point)

$$L_{p1} = \begin{cases} L_{p1} & \text{for } L_{p1} \le \frac{L_1}{2} \\ L_1 - L_{p1} & \text{for } L_{p1} > \frac{L_1}{2} \end{cases} \qquad (6)$$

$L_{p2} = \overline{\alpha p}$ (Short length from each end of L_2 to ⓟ point)

$$L_{p2} = \begin{cases} L_{p2} & \text{for } L_{p2} \le \frac{L_2}{2} \\ L_2 - L_{p2} & \text{for } L_{p2} > \frac{L_2}{2} \end{cases} \qquad (7)$$

b_0 : Value of *b*-axis of line segment R' in the dual space

(In case of Fig. 5(b), b_0 = 0.2)

5 Simulation

In this section, we calculate the Grasp Stability Index (GSI) about a simple example, and determine the Optimal Grasp Point (OGP) from the viewpoint of grasp stability.

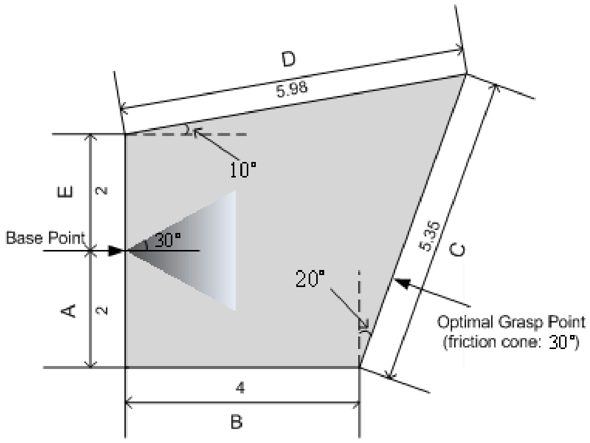

Fig. 7. Grasping a Quadrangular Object

Figure 8 is the result of calculating the GSI about a quadrangular object of Fig. 7. The horizontal axis is counterclockwise distance along the contour of the quadrangular object. The total distance is 19.33. When the angle of the friction cone has 30° at a base point and homologous points, the distance of the maximum GSI point, which is the Optimal Grasp Point (OGP), is 7.62 from a base point and the value of the GSI is 0.1354.

When the angle of the friction cone has 60°, it is possible to grasp the object at three sections. The Optimal Grasp Point (OGP) is a distance of 6.54 from the base point, and the value of the GSI is 0.3808. The GSI is discontinuous at the 6.00 and 11.35 due to the discontinuity of the object curvature.

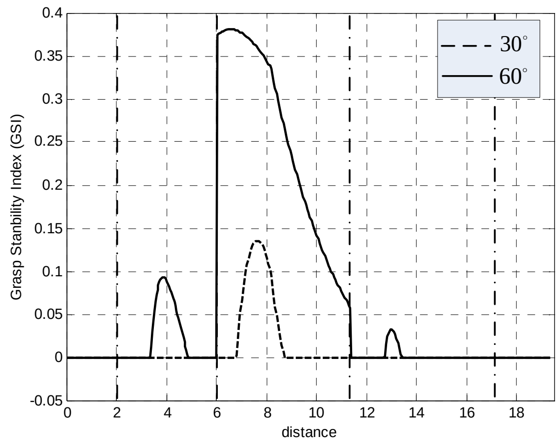

Fig. 8. GSI about the quadrangular object

6 Conclusion

We have presented throughout this paper a new method to calculate grasp stability by using the dual space. This method can judge the possibility of grasping by transforming friction cones in the robot work space into line segments in the dual space, and it needs only a friction coefficient and form information of an object without relationship to the magnitude of force which the robot fingers inflicts on an object.

Also, it can calculate the Grasp Stability Index (GSI) at every grasp point by considering the intersection condition between line segments in the dual space. As a result, we can confirm effectiveness and propriety of the GSI.

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