A Mixed VS-PA Model Reference Adaptive Controller

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Abstract: - A model reference adaptive controller with mixed variable structure-conventional adaptation law (VS-PA MRAS) using input–output measurement is proposed for single input single output systems. Adaptation law is mixed of variable structure type for numerator parameters and conventional type for denominator parameters of transfer function. Therefore only the bounds on numerator parameters of transfer function are required. Global exponential stability is proved based on Lyapunov criterion. Transient behavior is analyzed using sliding mode theory.

Key-Words: - adaptive control, model reference, variable structure.

1 Introduction

Model reference adaptive control using input-output measurement were mainly developed in 1980's [1, 2]. Adaptation law was based on parameter estimation using a pure integral action. In the subsequent years, it became evident that continuous adaptation laws had some problems such as 1) difficulty in the analysis of transient behaviour 2) guarantee of only global (but not asymptotic) stability 3) undesirable transient responses and tracking performance and 4) lack of robustness. Some researchers proposed some improvements on adaptation law [3,4,5,6,7]. All of these methods were categorized of parametric model reference adaptive control.

On the other hand, variable structure model reference adaptive control was proposed by some researchers [5]. The variable structure systems (VSS) have been studied in great detail in the literature [8]. The basic concept of the variable structure control is that of sliding mode control. Switching control functions are generally designed to generate sliding surfaces, or sliding modes, in the state space [9]. When this is attained, the switching functions keep the trajectory on the sliding surfaces and the closed loop system becomes insensitive, to a certain extent, to parameter variations and disturbances [9, 10 ,11].

2 MRAC with Mixed Adaptation Law

2.1 Problem definition and formulation

Consider a linear time-invariant plant as

$$\dot{\mathbf{X}} = \mathbf{AX(t)} + \mathbf{bu(t)} \ , \qquad \mathbf{y} = \mathbf{h^T X(t)} \tag{1}$$

where A is an $(n\times n)$ matrix and h and b are n-vectors. The transfer function of the plant is G(s) where

$$\mathbf{G(s)} = \mathbf{h^T (sI - A)^{-1} b} = \frac{\mathbf{KN(s)}}{\mathbf{D(s)}} \tag{2}$$

with G(s) strictly proper, D(s) is a monic polynomial of degree n, N(s) is a monic polynomial of degree m($\le$n-1), and K is a constant gain parameter. A reference model having output $\mathbf{y_m}$ characterized by

$$\dot{\mathbf{X}}_\mathbf{m} = \mathbf{A_m X_m} \ , \qquad \mathbf{y_m} = \mathbf{h^T X_m} \tag{3}$$

where A_m is an $(n\times n)$ matrix. The transfer function of the reference model is $G_m(s)$ where

$$\mathbf{G_m(s)} = \frac{\mathbf{K_m N_m(s)}}{\mathbf{D_m(s)}} \tag{4}$$

We further assume that: 1) the plant is completely observable and controllable,

2) N(s) is Hurwitz, i.e., G(s) is minimum phase,

3) the reference model has the same relative degree n*(= n – m) = 1 as the plant,

4) sgn(K) = sgn(K_m), positive for simplicity.

The purpose is to find a control law u(t) such that the output error

$$\mathbf{e}_0 = \mathbf{y} - \mathbf{y_m} \tag{5}$$

tends to zero asymptotically for arbitrary initial conditions.

2.2 Mixed adaptation law

The following input and output filters are used [1,5]

$$\dot{\mathbf{V}} = \Lambda\mathbf{V} + \lambda\,\mathbf{u} \tag{6}$$

$$\dot{\mathbf{W}} = \Lambda \mathbf{W} + \lambda\, \mathbf{y} \tag{7}$$

where Λ is an (n-1$\times$n-1) matrix and is chosen such that $\mathbf{N_m(s)} = \det(\mathbf{sI} - \Lambda)$. The regressor vector is defined as

$$\rho^T = [V^T \;\; y \;\; W^T r] \tag{8}$$

We choose structure of controller as

$$\mathbf{u(t)} = \beta^{\mathbf{T}}\mathbf{V} + \theta^{\mathbf{T}}\omega \tag{9}$$

where

$$\beta^{\mathbf{T}} = [\beta_1 \;\; \beta_2 \;\; \dots \;\; \beta_{\mathbf{n-1}}] \;,$$

$$\theta^T = [\sigma_0 \;\; \sigma_1^T k] \;, \qquad \omega^T = [y \;\; W^T r] \tag{10}$$

and the elements of β are adjusted using VS approach by designing switching functions $\beta_\mathbf{i}$ as described in the followings.

We know there exist constant control parameter vectors σ^* and $\theta^{*T} = [\sigma_0^* \;\; \sigma_1^{*T} k^*]$ such that if $\mathbf{u(t)} = \sigma^{*\mathbf{T}}\mathbf{V} + \theta^{*\mathbf{T}}\omega$ then the plant transfer function becomes similar to the reference model transfer function. In this situation we have

$$\sigma^{*\mathbf{T}}(\mathbf{sI} - \Lambda)^{-1}\lambda = \frac{\mathbf{N_m} - \mathbf{N}}{\mathbf{N_m}} \;,$$

$$\sigma_0^* + \sigma_1^{*\mathbf{T}}(\mathbf{sI} - \Lambda)\lambda = \frac{\mathbf{D} - \mathbf{D_m}}{\mathbf{KN_m}} \;, \qquad k^* = \frac{K_m}{K} \tag{11}$$

therefore $\sigma_0{}^*$, $\sigma_1{}^*$ and k^* would be known if $\mathbf{G(s)}$ was known. But we are encountered to the case that $\mathbf{G(s)}$ is not known, and the vectors β and θ must be adapted such that the error $\mathbf{e}_0\mathbf{(t)}$ tends to zero in finite time. Substituting control u from (9) to (1) and (6) we have

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{b}\beta^{\mathbf{T}}\mathbf{V} + \mathbf{b}\theta^{\mathbf{T}}\omega \tag{12}$$

$$\dot{\mathbf{V}} = \Lambda\mathbf{V} + \lambda\,\beta^{\mathrm{T}}\mathbf{V} + \lambda\theta^{\mathrm{T}}\omega \tag{13}$$

and substituting output y from (1) to (7) we have

$$\dot{\mathbf{W}} = \Lambda\mathbf{W} + \lambda\,\mathrm{h}^{\mathrm{T}}\mathbf{X} \tag{14}$$

adding terms $\pm b\beta^{*T} V \pm b\theta^{*T}\omega$ to the right side of (12) and terms $\pm \lambda\beta^{*T} V \pm \lambda\theta^{*T}\omega$ to the right side of (13), yields

$$\dot{X} = AX + b(\beta - \beta^*)^T V + b\phi^T\omega + b\beta^{*T} V + b\theta^{*T}\omega \tag{15}$$

$$\dot{V} = \Lambda V + \lambda(\beta - \beta^*)^T V + \lambda\phi^T\omega + \lambda\beta^{*T} V + \lambda\theta^{*T}\omega \tag{16}$$

where $\phi = \theta - \theta^*$. Introducing the state vector Z for the whole system composed of the plant and filters, ie, relations (1), (6) and (7), as

$$\mathbf{Z}^{\mathbf{T}} = [\mathbf{X}^{\mathbf{T}} \;\; \mathbf{V}^{\mathbf{T}} \;\; \mathbf{W}^{\mathbf{T}}] \tag{17}$$

we can write

$$\dot{Z} = \overline{A}\,Z + \overline{B}(\beta - \beta^*)^T V + \overline{B}\,\phi^T\omega \;, \qquad \mathbf{y} = \overline{h}^{\mathbf{T}}\mathbf{Z} \tag{18}$$

where

$$\overline{A} = \begin{bmatrix} A + b\sigma_0^{*T} h^T & b\beta^{*T} & b\sigma_1^{*T} \\ \lambda\sigma_0^* h^T & \Lambda + \lambda\beta^{*T} & \lambda\sigma_1^{*T} \\ \lambda\, h^T & 0 & \Lambda \end{bmatrix} \tag{19}$$

$$\overline{B}^{\mathbf{T}} = [\mathbf{b}^{\mathbf{T}} \;\; \lambda^{\mathbf{T}} \;\; 0 \; \dots \; 0] \;, \quad \overline{h}^{\mathbf{T}} = [\mathbf{h}^{\mathbf{T}} \;\; 0 \;\; 0 \; \dots \; 0] \tag{20}$$

Also, the system defined by $\{\overline{A}, \overline{B}, \overline{h}\}$ is a non minimal realization of $G_m(s)$. Thus the reference model can be represented by (3n-2)th order differential equations as

$$\dot{\mathbf{Z}}_{\mathbf{m}} = \overline{A}\,\mathbf{Z_m} \;, \qquad \mathbf{y_m} = \overline{h}^{\mathbf{T}}\mathbf{Z_m} \tag{21}$$

where $\quad \mathbf{Z_m^T} = [\mathbf{X}^{*\mathbf{T}} \;\; \mathbf{V}^{*\mathbf{T}} \;\; \mathbf{W}^{*\mathbf{T}}] \qquad (22)$

then we can write the error equation of the total system as

$$\dot{\mathbf{e}} = \overline{A}\,\mathbf{e} + \overline{B}(\beta - \beta^*)^{\mathbf{T}}\mathbf{V} + \overline{B}\,\phi^{\mathbf{T}}\omega \;, \quad \mathbf{e}_0 = \overline{h}^{\mathbf{T}}\mathbf{e} \tag{23}$$

where $\quad \mathbf{e}_0 = \mathbf{y} - \mathbf{y_m} \;, \; \phi = \theta - \theta^* \;, \; \mathbf{e} = \mathbf{Z} - \mathbf{Z_m} \qquad (24)$

because the transfer function $G_m(s)$ is a real positive definite transfer function, there exist matrices $\mathbf{P} = \mathbf{P^T} \rangle\, 0$ and $\mathbf{Q} = \mathbf{Q^T} \rangle\, 0$ such that

$$\overline{A}^{\mathbf{T}}\mathbf{P} + \mathbf{P}\overline{A} = -\mathbf{Q} \;, \qquad \mathbf{P}\overline{B}\,k = \overline{h} \tag{25}$$

A block diagram of the system is presented in Figure [1].

2.3 Stability

We introduce a Liapunov function as

$$\Gamma = \frac{1}{2}[\mathbf{e^T P e} + \frac{1}{|k|}\phi^{\mathbf{T}}\phi] \tag{26}$$

time differentiating it, with the first consider to equations (23), and then regarding equations (25) and (15), and finally consider to equation (18), yields

$$\dot{\Gamma} = -\frac{1}{2}\mathbf{e^T Q e} + \frac{1}{k}\mathbf{e}_0(\beta - \theta^*)^{\mathbf{T}}\mathbf{V} + [\frac{1}{k}\phi^{\mathbf{T}}\mathbf{e}_0\omega + \frac{1}{|k|}\phi^{\mathbf{T}}\dot{\phi}] \tag{27}$$

using the conventional adaptation law for adapting the parameters θ, as

$$\dot{\phi} = \dot{\theta} = -\mathrm{sgn}(k)\,\mathbf{e}_0\,\omega \tag{28}$$

we can write

$$\dot{\Gamma} = -\frac{1}{2}\mathbf{e^T Q e} + \frac{1}{k}\sum_{\mathbf{i=1}}^{\mathbf{n-1}}[(\beta_\mathbf{i} - \sigma_\mathbf{i}^*)^{\mathbf{T}}\mathbf{v_i}\,\mathbf{e}_0] \tag{29}$$

Now, we choose the switching functions $\beta_\mathbf{i}$, as

$$\beta_i = -\overline{\sigma}_i\,\mathrm{sgn}(k\,v_i\,e_0) \;, \text{ where } \overline{\sigma}_i \rangle \left|\sigma_i^*\right| \tag{30}$$

therefore this can concluded that

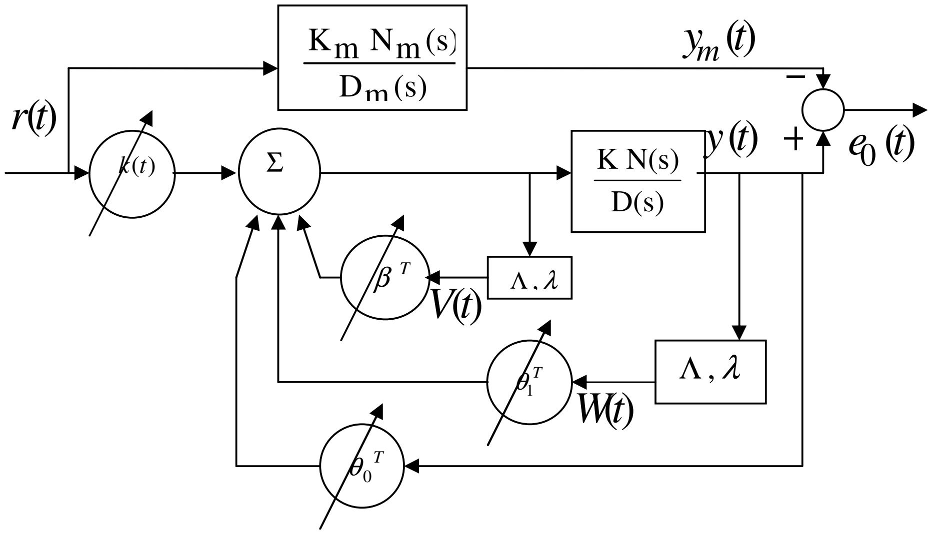

Figure 1. Block diagram of the system.

$$\dot{\Gamma} = -\frac{1}{2}e^T Qe - \sum_{i=1}^{n-1}[\bar{\sigma}_i\left|\frac{1}{k}v_i\ e_0\right| + \sigma_i^* \frac{1}{k}v_i\ e_0] \tag{31}$$

the terms in the summation are positive, therefore $\dot{\Gamma}\langle 0$ and regarding (26) asymptotic stability is proved.

2.4 Transient behavior

In the following section, we analyse the transient response. This will be shown that the hyper-surface

$$\mathbf{S} = \bar{h}^{\mathbf{T}}\mathbf{e} = 0 \tag{32}$$

is a sliding surface, which fulfilled the following conditions

$$\frac{1}{2}\frac{\mathbf{d}}{\mathbf{dt}}\mathbf{S}^2 = \mathbf{S}\,\dot{\mathbf{S}}\langle 0, \quad \text{for } |\mathbf{S}|\rangle\, 0 \tag{33}$$

regarding relation (23) we can write

$$\frac{1}{2}\frac{\mathbf{d}}{\mathbf{dt}}\mathbf{S}^2 = \mathbf{S}\,\dot{\mathbf{S}} = \mathbf{S}\,\bar{h}^{\mathbf{T}}\bar{A}\,\mathbf{e} + \mathbf{S}\,\bar{h}^{\mathbf{T}}\bar{B}(\beta-\sigma^*)\mathbf{V} + \mathbf{S}\,\bar{h}^{\mathbf{T}}\bar{B}\,\phi^{\mathbf{T}}\boldsymbol{\omega} \tag{34}$$

using relations (30) and regarding (32), ($\mathbf{S} = \mathbf{e}_0$), also (25), ($\bar{h}^{\mathbf{T}} = k\bar{B}^{\mathbf{T}}\mathbf{P}$), yields

$$\frac{1}{2}\frac{\mathbf{d}}{\mathbf{dt}}\mathbf{S}^2 = \mathbf{S}\,\dot{\mathbf{S}} =$$

$$= |e_0|\{\alpha_1\ \|e\|\ - B^T P\,\mathrm{B}\sum_{i=1}^{n-1}[(\bar{\sigma}_i - |\sigma_i^*|)|\ |v_i|\} + e_0\,\bar{B}\,\phi^T k\bar{B}^T p\omega \tag{35}$$

Now if define a signal η as: $\eta = \phi^T\omega$

Then regarding to $h^T = kB^T P$ we can write

$$\frac{1}{2}\frac{\mathbf{d}}{\mathbf{dt}}\mathbf{S}^2 = \mathbf{S}\,\dot{\mathbf{S}} =$$

$$= |e_0|\{\alpha_1\|e\|\ - B^T \boldsymbol{P}\,B\sum_{i=1}^{n-1}[(\bar{\sigma}_i - |\sigma^*|)|kV_i| + B^T \boldsymbol{P}\,B\ \ |k\eta|\,]\}$$

$$\langle\, |e_0|\{\alpha_1\|e\| - |\mathrm{k}|B^T P\,\mathrm{B}\,\Delta\sigma_{\mathrm{m}}\|\mathrm{V}\| + B^T P\,\mathrm{B}\,|k\bar{\eta}|\}$$

in which

$$\Delta\sigma_m = \min_i(\bar{\sigma}_i - |\sigma_i^{\ *}|)\ ,\ \bar{\eta} = \sup_{t\rangle 0}|\eta(t)|$$

therefore if for all times $\mathbf{t}\rangle\mathbf{t}_0$ we have

$$\|V\|\rangle\,\mu\,\rangle\,\frac{\bar{\eta}}{\Delta\sigma_{\mathrm{m}}}$$

then there exists a time $\mathbf{T}\rangle\mathbf{t}_0$ such that for all times $\mathbf{t}\rangle\mathbf{T}$ and with condition $|\mathbf{e}_0|\rangle 0$ we have

$$\frac{1}{2}\frac{\mathbf{d}}{\mathbf{dt}}\mathbf{S}^2 = \mathbf{S}\,\dot{\mathbf{S}}\langle\, 0 \tag{37}$$

this means that for all times $\mathbf{t}\rangle\mathbf{T}$, the surface $\mathbf{S} = \bar{h}^{\mathbf{T}}\mathbf{e} = 0$ will be a sliding surface. Also regarding (23) and (32) we can write

$$\dot{\mathbf{S}} = \bar{h}^{\mathbf{T}}\dot{\mathbf{e}} = \bar{h}^{\mathbf{T}}\bar{A}\,\mathbf{e} + \bar{h}^{\mathbf{T}}\bar{B}(\beta-\beta^*)^{\mathbf{T}}\mathbf{V} + \bar{h}^{\mathbf{T}}\bar{B}\,\phi^{\mathbf{T}}\boldsymbol{\omega} \tag{38}$$

and consider to (35) this can concluded that for times $\mathbf{t}\rangle\mathbf{t}_1$ we have

$$|\dot{\mathbf{S}}|\rangle\,\alpha \tag{39}$$

Equations (37) and (39) show that S tends to zero in a finite time, and sliding mode take place on the surface $\mathbf{S} = \bar{h}^{\mathbf{T}}\mathbf{e} = 0$.

3 Simulation and results

Simulation results have been presented for an unstable second order system. Responses for both, mixed adaptation adaptive stabilizer and conventional one are presented.

Consider an unstable second order system as

$$\dot{\mathbf{X}} = \begin{bmatrix} 0 & 1 \\ \mathbf{a}_1 & \mathbf{a}_2 \end{bmatrix} \mathbf{X} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mathbf{u} , \quad \mathbf{y} = \begin{bmatrix} 5 & 1 \end{bmatrix} \mathbf{X} \qquad (40)$$

or in transfer function form

$$\mathbf{G}(\mathbf{s}) = \frac{\mathbf{s}+5}{\mathbf{s}^2 + \mathbf{a}_2\,\mathbf{s} + \mathbf{a}_1} \qquad (41)$$

we assume that the parameters $\mathbf{a}_1$ and $\mathbf{a}_2$ of the system actually varied as $\mathbf{a}_1 = 1 + \sin(\pi\, t)$ and $\mathbf{a}_2 = -2 + \sin(\pi\, t)$, but designer designer only know the range of variations as $0 \le a_1 \le 2$ and $-3 \le a_2 \le -1$. The reference model is chosen as

$$\dot{\mathbf{X}}_{\mathbf{m}} = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} \mathbf{X}_{\mathbf{m}} , \quad \mathbf{y}_{\mathbf{m}} = \begin{bmatrix} 1 & 1 \end{bmatrix} \mathbf{X}_{\mathbf{m}} \qquad (42)$$

or in transfer function form

$$\mathbf{G}_{\mathbf{m}}(\mathbf{s}) = \frac{\mathbf{s}+1}{\mathbf{s}^2 + 2\mathbf{s} + 2} \qquad (43)$$

consider to equations (6), (7), (9), (24), (28), and (30), we can write

$$\dot{V} = -V + u \qquad (44)$$

$$\dot{W} = -W + y \qquad (45)$$

$$u = \beta V + \sigma_0\, \mathrm{y} + \sigma_1\, \mathrm{w} + kr \qquad (46)$$

$$\beta = -\bar{\sigma}\, \mathrm{sgn}(V\, e_0) \qquad (47)$$

$$\dot{\sigma}_0 = -e_0\, \mathrm{y} \qquad (48)$$

$$\dot{\sigma}_1 = -e_0\, \mathrm{w} \qquad (49)$$

$$\dot{k} = -e_0\, r \qquad (50)$$

Simulation results have been presented in Figure 2.1 for step input r = 8 , and Figure 2.2 for sinusoidal input $r = 0.5 + 1.5 \sin 3.2\pi t$.

4 Conclusion

A mixed variable structure - conventional model reference adaptive controller for single input single output systems have been proposed, designed and analyzed. Adaptation law is mixed of variable structure type for numerator parameters and conventional type for denominator parameters of transfer function. Therefore only the bounds on numerator parameters of transfer function are required. Global exponential stability is proved based on Lyapunov criterion. Transient behavior is analyzed using sliding mode theory.

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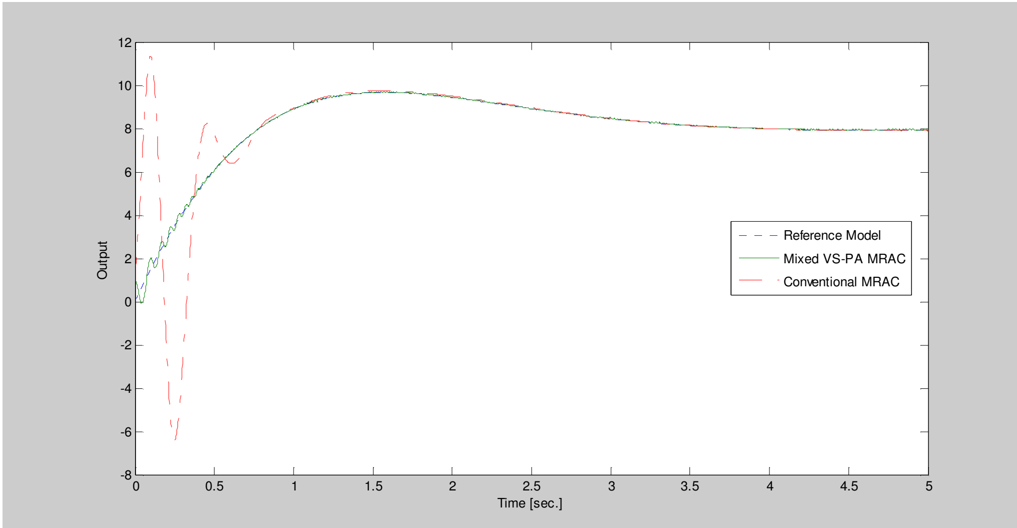

Figure 2.1. Simulation results, step input r=8, initial conditions: $\mathbf{X}(0) = [1 \quad 0]^{\mathbf{T}}$, $\mathbf{X}_{\mathbf{m}}(0) = [0 \quad 0]^{\mathbf{T}}$.

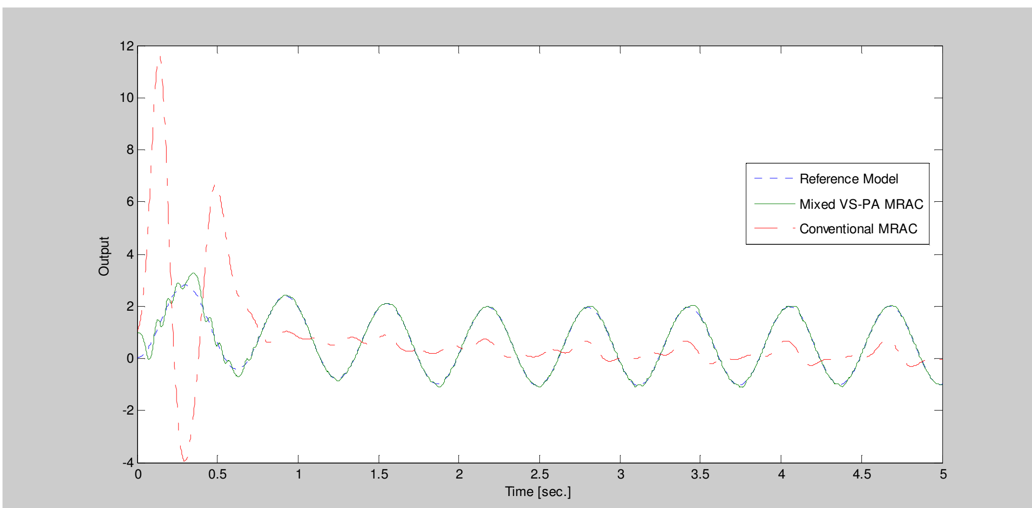

Figure 2.2. Simulation results, sinusoidal input $r = 0.5 + 1.5\sin 3.2\pi t$, initial conditions: $\mathbf{X}(0) = [1 \quad 0]^{\mathbf{T}}$, $\mathbf{X}_{\mathbf{m}}(0) = [0 \quad 0]^{\mathbf{T}}$.