Modelling Municipal Creditworthiness by Hierarchical Structures of Fuzzy Inference Systems

PETR HÁJEK, VLADIMÍR OLEJ
Institute of System Engineering and Informatics
Faculty of Economics and Administration
University of Pardubice
Studentská 84, 532 10 Pardubice
CZECH REPUBLIC

Abstract: - The paper presents the design of municipal creditworthiness parameters. Municipal creditworthiness modelling is realized by fuzzy logic based systems. Therefore, current designs of hierarchical structures of Mamdani-type fuzzy inference systems and their analysis are introduced.

Key-Words: - Municipal creditworthiness, Mamdani-type fuzzy inference systems, hierarchical structures of fuzzy inference systems, classification.

1 Introduction

Municipal creditworthiness [1] is the ability of a municipality to meet its short-term and long-term financial obligations. It is assigned based on parameters (factors) relevant to the assessed object. High municipal creditworthiness shows a low credit risk, while low one shows a high credit risk.

Municipal creditworthiness evaluation is currently realized by methods combining mathematical-statistical methods and expert opinion [1,2]. Scoring systems [3], rating [4], rating-based models [1,5], default models [6] and unsupervised models [2,7] fall into these methods. Their output is represented either by a score evaluating the municipal creditworthiness (scoring systems) or by an assignment of the municipalities to the j-th class $\omega_j \in \Omega$, $\Omega = \{\omega_1,\omega_2, \dots ,\omega_j, \dots ,\omega_c\}$ according to their creditworthiness (rating, unsupervised models). Scoring systems are typical for easy calculation, low accuracy and inability to work with uncertainty and expert knowledge. Rating is an independent expert evaluation based on complex analysis of all known risk parameters of municipal creditworthiness. It is considered to be rather subjective. Municipalities are classified into classes $\omega_1,\omega_2, \dots ,\omega_c$ by rating-based models. The classes are assigned to the municipalities by rating agencies. The models showed low classification accuracy [1]. The objective of default models is to find the causes of municipalities default. Low number of municipalities makes the application of default models impossible. Municipalities are assigned to classes based on selected parameters similarity.

Therefore, the methods capable of processing and learning the expert knowledge, enabling their user to generalize and properly interpret, have been considered most suitable. The use of natural language is typical for municipal creditworthiness decision-making process. Natural language is characterized by semantic vagueness, therefore it cannot be transformed directly into mathematical formulas. Moreover, precise description of municipal creditworthiness parameters does not correspond to reality. The presented problem can be solved by fuzzy logic [8,9,10], enabling its user to model the meaning of natural language words. For example, fuzzy inference systems [11] are suitable for municipal creditworthiness evaluation. Evaluation of company and bank client creditworthiness illustrates their possible application [12,13,14].

Therefore, the paper presents the design of municipal creditworthiness parameters and their modelling by hierarchical structures of Mamdani-type fuzzy inference systems.

2 Municipal Creditworthiness Parameters Design

Economic, debt, financial and administrative categories [1,15] are listed among the common categories of parameters. The economic, debt and financial parameters are pivotal [15]. The differences in creditworthiness evaluation methods are presented in the used parameters and their weights. Methods in [1,15] assume a high fiscal autonomy of municipalities. This allows the municipalities to influence their revenues through local taxation and charges for municipal services. On the other hand, the municipalities in the Czech Republic have low fiscal autonomy. Therefore, the parameters of evaluation differ from the methods.

2.1 Economic Parameters Design

Economic parameters affect long-term municipal creditworthiness [16]. The municipalities with more diversified economy and more favourable social and economic conditions are better prepared for economic recession [1]. The economic growth, however, is able to enlarge public services, and consequently, to increase the indebtedness. Stable municipal economy can indicate economic stagnation. There is no synthetic parameter that would quantify the level of municipal economies. The economic parameters for municipal creditworthiness evaluation can be designed as follows:

$$\text{Parameter } p_1 = PO_r \,, \qquad (1)$$

where PO_r is population in the r-th year. Higher value of the parameter p_1 entails especially higher municipal tax revenues. Tax revenues depend on the number of inhabitants and on the coefficient, which indicates the size category of a given municipality. Larger municipalities have higher share in tax yield, because the more populated municipalities have higher spending for the infrastructure and other public goods. Higher population guarantees future municipal revenues for the creditors. At the same time, it decreases the credit risk [16].

$$\text{Parameter } p_2 = PO_r / PO_{r-s} \,, \qquad (2)$$

where R_{r-s} is population in the year r-s, and s is the selected period of time. The change in the number of inhabitants is a good criterion of the economic vitality of a municipality [15]. Economic growth of the municipality leads to the growing number of its inhabitants. Sudden growth of the parameter should be assessed prudently, because the real trend is not needed.

$$\text{Parameter } p_3 = U \,, \qquad (3)$$

where U is the unemployment rate in a municipality. The rate of unemployment evaluates general economic wealth of the municipality. Economic growth reduces the unemployment in a given municipality. Therefore, low rate of unemployment indicates good economic conditions. High unemployment rate entails higher expenses for social services. Jobs deficiency also reduces the price of real estates, while the budget revenues from the real estate tax decrease.

$$\text{Parameter } p_4 = \sum_{i=1}^{k} (PZO_i / PZ)^2 \,, \qquad (4)$$

where PZO_i is the employed population in a given municipality in the i-th economic sector, i=1,2, … ,k, PZ is the total number of employed inhabitant, and k is the number of the economic sectors. Parameter p_4 represents the concentration of employment in economic sectors and presents the measure of municipal economic concentration. Low value of parameter p_4 means a long - term flexibility of the municipal economy and protection against bankruptcy of one sector. According to [1], the parameter p_4 is the most considerable factor of municipal creditworthiness.

2.2 Debt Parameters Design

Debt parameters include the size and structure of the debt. Ratios are often used to measure both the debt of the municipality and its ability to pay off a debt service. However, using the ratios is only effective if the parameters for comparable municipalities are available. The comparison with the municipalities illustrates the current debt and financial situation of a given municipality. Based on the aforementioned facts, the debt parameters can be designed as follows:

$$\text{Parameter } p_5 = DS/OP \,, \qquad (5)$$

where $p_5 \in <0,1>$, DS is the debt service and OP stands for the periodical revenues. It is a crucial debt factor measuring the ability of the municipality to pay off the DS from regular budget revenues [15]. The debt service includes yearly interest and annuity payment. Periodical revenues are total revenues minus nonrecurring and capital revenues. The value of parameter p_5 above 0.15 can be considered a signal of the imminent debt trap.

$$\text{Parameter } p_6 = CD/PO \,, \text{ [Czech crowns]}, \qquad (6)$$

where CD is a total debt. The indicated parameter measures gross measure of the municipality indebtedness, i.e. the extent of the accrued debt per an inhabitant. Its absolute value is not predicative itself. It is necessary to compare the value with those of other municipalities in the region, or with the whole country [1].

$$\text{Parameter } p_7 = KD/CD \,, \qquad (7)$$

where $p_7 \in <0,1>$ and KD is a short-term debt. It analyses the debt structure. A short-term debt is designed to meet the short-term engagements resulting from the insufficient cash flow. The short-term debt should be paid off during a fiscal year. If KD is intended to cover a budget deficit or to finance the capital projects, it should be considered alarming, since it negatively influences the credit risk [17]. Interest rates of short-term debt are usually floating rates. This may result in the inability to pay the debt service.

2.3 Financial Parameters Design

Financial parameters inform about the scope of budget implementation. Their values are extracted from the municipality budget. Financial parameters for municipal creditworthiness evaluation can be designed as follows:

$$\text{Parameter } p_8 = OP/BV \,, \qquad (8)$$

where $p_8 \in R^+$ and BV are current expenditures. The parameter p_8 reports on the quality of the budget implementation. If it is constantly greater than 1, i.e. current budget is in excess, and at the same time a growing trend is indicated, the municipality is in good financial state. Good financial standing enables the municipality to use common surplus to finance its engagements. On this account, the parameter is regarded a key factor in [17].

$$\text{Parameter } p_9 = VP/CP, \quad (9)$$

where $p_9 \in <0,1>$, VP are own revenues and CP are total revenues. Higher proportion of own revenues in total revenues entails higher fiscal autonomy of the municipality. Consequently, higher fiscal autonomy leads to lower indebtedness. According to [17], the size of the fiscal autonomy affects the municipality decision-making management. Municipal management chooses a combination of the VP and the debt on public goods financing. The higher is the fiscal autonomy of the municipality, the smaller the need for the debt as a financing tool.

$$\text{Parameter } p_{10} = KV/CV, \quad (10)$$

where $p_{10} \in <0,1>$ and KV are capital expenditures, CV are total expenditures. Higher value of the parameter indicates capital activity of the municipality and a good common management enabling its further development [17]. This hypothesis complies with the inter-generation theory of justice, where both the contemporary as well as future users of public goods should take part in capital expenses.

$$\text{Parameter } p_{11} = IP/CP, \quad (11)$$

where $p_{11} \in <0,1>$ and IP are capital revenues. The debt is primarily intended to finance the capital (investment) expenditures (projects). The higher is the parameter p_{11}, the smaller the need of the next indebtedness to finance capital projects.

$$\text{Parameter } p_{12} = LM/PO, \text{ [Czech crowns]}, \quad (12)$$

where LM is the size of municipal liquid assets. The municipalities control their own assets. These are often used as bank's credit collateral. The banks grant a credit only on condition, that the collateral assets are liquid enough, i.e. cashable in a short time. The liquid assets of the municipality include suitably situated extensive land properties, commercial buildings, agricultural land properties and assets for commercial use being in possession of the municipality.

3 Design of Hierarchical Structures of Fuzzy Inference Systems for Municipal Creditworthiness Evaluation

General structure of fuzzy inference system (FIS) is presented in Fig. 1 [9,11,18]. It contains the fuzzification process by means of input membership functions, construction of base rules (BRs) or automatic extraction of rules from the input data, application of operators (AND, OR, NOT) in rules, implication and aggregation within rules and the defuzzification process of obtained outputs to the crisp values. Based on the general structure of FIS, three fundamental types of FIS can be designed, i.e. Mamdani-type [11], Takagi-Sugeno-type [19] and Tsukamoto-type [20].

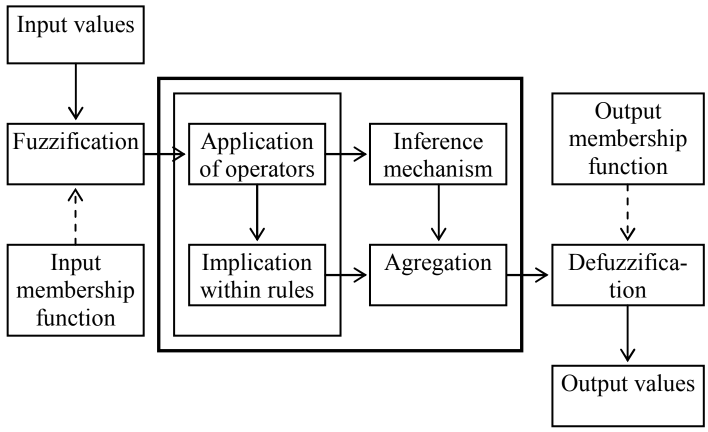

Fig. 1 General structure of fuzzy inference system

Let $x_1, x_2, \dots, x_i, \dots, x_n$ be the input variables defined in the reference sets $X_1, X_2, \dots, X_i, \dots, X_n$ and let y be the output variable defined in the reference set Y. Then FIS has n input variables and one output variable. Each set X_i, $i = 1,2, \dots, n$, can be divided into p_j, $j = 1,2, \dots, m$, the fuzzy sets $\mu_1^{(i)}(x), \mu_2^{(i)}(x), \dots, \mu_{pj}^{(i)}(x), \dots, \mu_m^{(i)}(x)$. Individual fuzzy sets $\mu_1^{(i)}(x), \mu_2^{(i)}(x), \dots, \mu_{pj}^{(i)}(x), \dots, \mu_m^{(i)}(x)$, $i = 1,2, \dots, n$; $j = 1,2, \dots, m$ represent the assignment of linguistic variables relating to sets X_i. The set Y is also divided into p_k, $k = 1,2, \dots, o$ the fuzzy sets $\mu_1(y), \mu_2(y), \dots, \mu_{pk}(y), \dots, \mu_o(y)$. The fuzzy sets $\mu_1(y), \mu_2(y), \dots, \mu_{pk}(y), \dots, \mu_o(y)$ represent the assignment of linguistic variables for the set Y. Then the Mamdani-type FIS rule can be put as follows [11,20]

IF x_1 is $A_1^{(i)}$ AND x_2 is $A_2^{(i)}$ AND … AND x_n is $A_{pj}^{(i)}$ THEN y is B, (13)

where:
- $i = 1,2, \dots, n$, $j = 1,2, \dots, m$,
- $A_1^{(i)}, A_2^{(i)}, \dots, A_{pj}^{(i)}$ represent linguistic variables corresponding to fuzzy sets $\mu_1^{(i)}(x), \mu_2^{(i)}(x), \dots, \mu_{pj}^{(i)}(x), \dots, \mu_m^{(i)}(x)$,
- B represents linguistic variable corresponding to fuzzy sets $\mu_1(y), \mu_2(y), \dots, \mu_{pk}(y), \dots, \mu_o(y)$, $k=1,2, \dots, o$.

Let's have a given Mamdani-type FIS. Then the number of rules in this FIS is defined according to the relation

$$N_{PP} = k^m, \quad (14)$$

where:
- N_{PP} is number of rules,
- k is number of membership functions in FIS,
- m is number of input variables.

Due to a great number of m, FIS can be ineffective with regard to the increase of N_{PP}. The design of the FIS hierarchical structures is one of the ways leading to the decrease of rules number N_{PP} [21,22,23]. The BRs reduction lowers the computational cost and makes FIS interpretation possible. Fuzzy inference system is easily to interpret if the following conditions are met [24]:

- FIS has low number of rules N_{PP} with low number of variables in antecedent.
- FIS has low number of membership functions for individual variables k.
- There are no weights assigned to rules.
- The same language expressions are represented by the same membership functions.

The number of rules in hierarchical structure of fuzzy inference system (HSFIS) is defined

$$N_{PP} = \left(\frac{m-t}{t-1}+1\right)k^{t}, \tag{15}$$

where t is number of variables contained in each layer. The fundamental types of HSFIS [21], i.e. cascade, tree and combination of tree and cascade structure, are used in the design of the HSFIS for municipal creditworthiness evaluation. The HSFIS model for municipal creditworthiness evaluation is presented in Fig. 2.

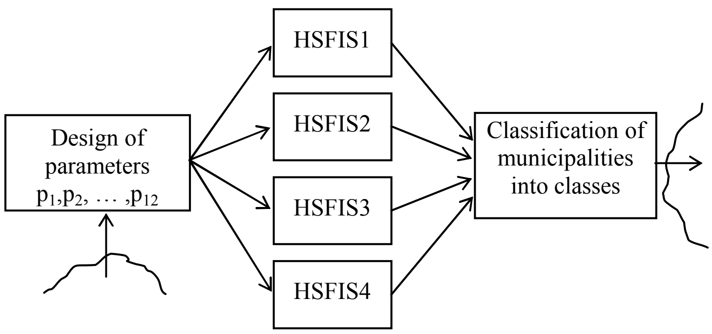

Fig. 2 HSFIS model

The decrease of rules number N_{PP} is obtained by the design of this model. In addition to rules reduction, the model design should reproduce the expert's decision-making by municipal creditworthiness evaluation with the intent to consider the similarity and mutual relations of parameters $p_1,p_2, \dots ,p_{12}$. The designs of HSFIS for municipal creditworthiness evaluation are presented in Fig. 3 (HSFIS1-cascade structure), Fig. 4 (HSFIS2-tree structure 1), Fig. 5 (HSFIS3-tree structure 2) and Fig. 6 (HSFIS4-combination of tree and cascade structure).

Design of HSFIS1

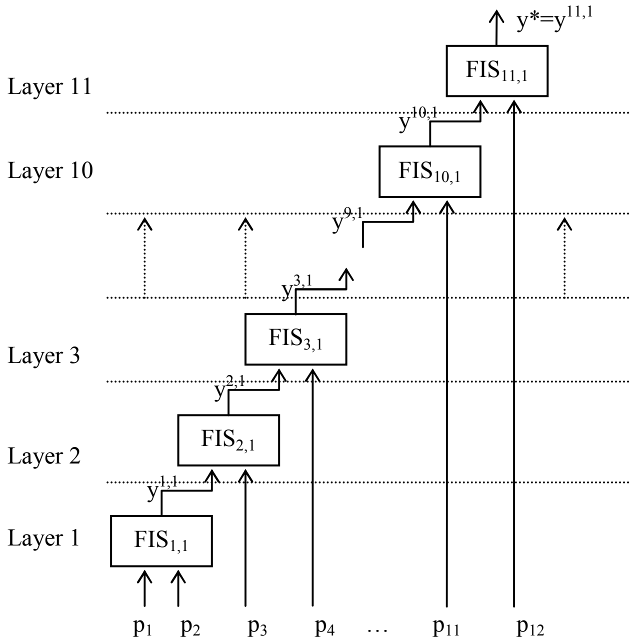

Fig. 3 Design of HSFIS1 for municipal creditworthiness evaluation

Legend: $p_1,p_2, \dots ,p_{12}$ are input variables, $y^{1,1},y^{2,1}, \dots ,y^{11,1}$ are outputs of subsystems $FIS_{1,1},FIS_{2,1}, \dots ,FIS_{11,1}$, L=11 is the number of HSFIS1 layers.

The HSFIS1 model can be formalized by BRs $R^{h_{1,1}}, R^{h_{2,1}}, \dots, R^{h_{11,1}}$ and outputs $y^{1,1},y^{2,1}, \dots ,y^{10,1},y^{*}$ of single subsystems HSFIS1 this way:

Layer 1: $FIS_{1,1}$: $R^{h_{1,1}}$: IF p_1 is $A_1^{h_{1,1}}$ AND p_2 is $A_2^{h_{1,1}}$ THEN $y^{1,1}$ is $B^{h_{1,1}}$,

Layer 2: $FIS_{2,1}$: $R^{h_{2,1}}$: IF $y^{1,1}$ is $B^{h_{1,1}}$ AND p_3 is $A_3^{h_{2,1}}$ THEN $y^{2,1}$ is $B^{h_{2,1}}$,

… (16)

Layer 11: $FIS_{11,1}$: $R^{h_{11,1}}$: IF $y^{10,1}$ is $B^{h_{10,1}}$ AND p_{12} is $A_{12}^{h_{11,1}}$ THEN y^{*} is $B^{h_{11,1}}$,

$$y^{1,1}(B^{h_{1,1}}) = \frac{\sum_{j=1}^{p} y_j^{1,1} \times \mu_{B^{h_{1,1}}}(y_j^{1,1})}{\sum_{j=1}^{p} \mu_{B^{h_{1,1}}}(y_j^{1,1})}, \tag{17}$$

$$y^{2,1}(B^{h_{2,1}}) = \frac{\sum_{j=1}^{p} y_j^{2,1} \times \mu_{B^{h_{2,1}}}(y_j^{2,1})}{\sum_{j=1}^{p} \mu_{B^{h_{2,1}}}(y_j^{2,1})}, \tag{18}$$

…

$$y^*(B^{h_{11,1}}) = \frac{\sum_{j=1}^{p} y_j^{11,1} \times \mu_{B^{h_{11,1}}}(y_j^{11,1})}{\sum_{j=1}^{p} \mu_{B^{h_{11,1}}}(y_j^{11,1})}, \quad (19)$$

where: - $p_1,p_2, \dots ,p_{12}$ are input parameters,

- $A_1^{h_{1,1}}, A_2^{h_{1,1}}, \dots, A_{12}^{h_{11,1}}$ represent linguistic variables corresponding to the fuzzy sets $\mu_1^{h_{1,1}}(p_j), \mu_2^{h_{1,1}}(p_j), \dots, \mu_{12}^{h_{11,1}}(p_j)$,
- $B^{h_{1,1}}, B^{h_{2,1}}, \dots, B^{h_{11,1}}$ represent linguistic variables corresponding to the fuzzy sets $\mu^{h_{1,1}}(y^{1,1}), \mu^{h_{2,1}}(y^{2,1}), \dots, \mu^{h_{11,1}}(y^*)$,
- $\mu_{B^{h_{1,1}}}(y_j^{1,1}), \mu_{B^{h_{2,1}}}(y_j^{2,1}), \dots, \mu_{B^{h_{11,1}}}(y_j^{11,1})$ are membership functions values of the aggregated fuzzy set for values $y_j^{1,1}, y_j^{2,1}, \dots, y_j^{11,1}$ from the reference sets.

Design of HSFIS2

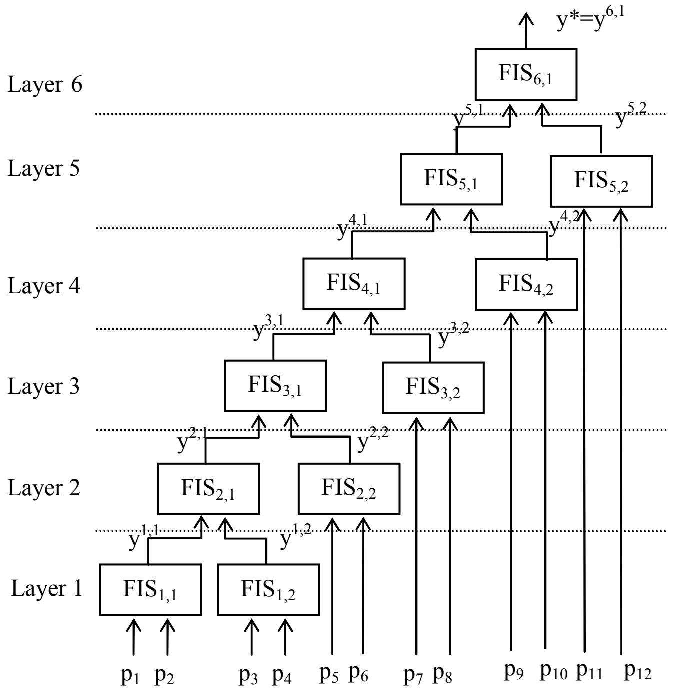

Fig. 4 Design of HSFIS2 for municipal creditworthiness evaluation

Legend: $p_1,p_2, \dots p_{12}$ are input variables, $y^{1,1},y^{1,2}, \dots ,y^{6,1}$ are outputs of subsystems $FIS_{1,1},FIS_{1,2}, \dots ,FIS_{6,1}$, L=6 is the number of HSFIS2 layers.

The HSFIS2 model can be formalized by BRs $R^{h_{1,1}}, R^{h_{1,2}}, \dots, R^{h_{6,1}}$ and outputs $y^{1,1},y^{1,2}, \dots ,y^{5,1},y^*$ of single subsystems HSFIS2 this way:

Layer 1: $FIS_{1,1}$: $R^{h_{1,1}}$: IF p_1 is $A_1^{h_{1,1}}$ AND p_2 is $A_2^{h_{1,1}}$ THEN $y^{1,1}$ is $B^{h_{1,1}}$,

$FIS_{1,2}$: $R^{h_{1,2}}$: IF p_3 is $A_3^{h_{1,2}}$ AND p_4 is $A_4^{h_{1,2}}$ THEN $y^{1,2}$ is $B^{h_{1,2}}$,

Layer 2: $FIS_{2,1}$: $R^{h_{2,1}}$: IF $y^{1,1}$ is $B^{h_{1,1}}$ AND $y^{1,2}$ is $B^{h_{1,2}}$ THEN $y^{2,1}$ is $B^{h_{2,1}}$,

$FIS_{2,2}$: $R^{h_{2,2}}$: IF p_5 is $A_5^{h_{2,2}}$ AND p_6 is $A_6^{h_{2,2}}$ THEN $y^{2,2}$ is $B^{h_{2,2}}$,

… (20)

Layer 6: $FIS_{6,1}$: $R^{h_{6,1}}$: IF $y^{5,1}$ is $B^{h_{5,1}}$ AND $y^{5,2}$ is $B^{h_{5,2}}$ THEN y^* is $B^{h_{6,1}}$,

$$y^{1,1}(B^{h_{1,1}}) = \frac{\sum_{j=1}^{p} y_j^{1,1} \times \mu_{B^{h_{1,1}}}(y_j^{1,1})}{\sum_{j=1}^{p} \mu_{B^{h_{1,1}}}(y_j^{1,1})}, \quad (21)$$

$$y^{1,2}(B^{h_{1,2}}) = \frac{\sum_{j=1}^{p} y_j^{1,2} \times \mu_{B^{h_{1,2}}}(y_j^{1,2})}{\sum_{j=1}^{p} \mu_{B^{h_{1,2}}}(y_j^{1,2})}, \quad (22)$$

…

$$y^*(B^{h_{6,1}}) = \frac{\sum_{j=1}^{p} y_j^{6,1} \times \mu_{B^{h_{6,1}}}(y_j^{6,1})}{\sum_{j=1}^{p} \mu_{B^{h_{6,1}}}(y_j^{6,1})}, \quad (23)$$

where: - $p_1,p_2, \dots ,p_{12}$ are input parameters,

- $A_1^{h_{1,1}}, A_2^{h_{1,1}}, \dots, A_{12}^{h_{6,1}}$ represent linguistic variables corresponding to fuzzy sets $\mu_1^{h_{1,1}}(p_j), \mu_2^{h_{1,1}}(p_j), \dots, \mu_{12}^{h_{6,1}}(p_j)$,
- $B^{h_{1,1}}, B^{h_{1,2}}, \dots, B^{h_{6,1}}$ represent linguistic variables corresponding to fuzzy sets $\mu^{h_{1,1}}(y^{1,1}), \mu^{h_{1,2}}(y^{1,2}), \dots, \mu^{h_{6,1}}(y^*)$,
- $\mu_{B^{h_{1,1}}}(y_j^{1,1}), \mu_{B^{h_{1,2}}}(y_j^{1,2}), \dots, \mu_{B^{h_{L,1}}}(y_j^{6,1})$ are membership functions values of aggregated fuzzy set for values $y_j^{1,1}, y_j^{1,2}, \dots, y_j^{6,1}$ from the reference sets.

Design of HSFIS3

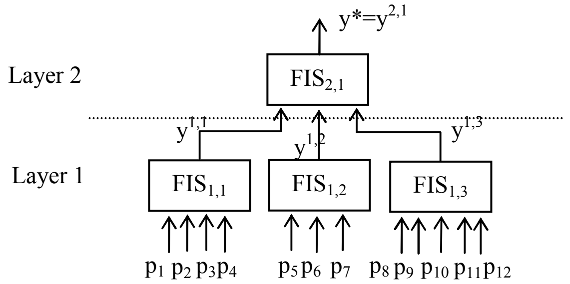

Fig. 5 Design of HSFIS3 for municipal creditworthiness evaluation

Legend: $p_1, p_2, \ldots, p_{12}$ are input variables, $y^{1,1}, y^{1,2}, \ldots, y^{2,1}$ are outputs of subsystems $FIS_{1,1}, FIS_{1,2}, \ldots, FIS_{2,1}$, L=2 is the number of HSFIS3 layers.

The HSFIS3 model can be formalized by BRs $R^{h_{1,1}}, R^{h_{1,2}}, \ldots, R^{h_{2,1}}$ and outputs $y^{1,1}, y^{1,2}, \ldots, y^*$ of single subsystems HSFIS3 this way:

Layer 1: $FIS_{1,1}$: $R^{h_{1,1}}$: IF p_1 is $A_1^{h_{1,1}}$ AND p_2 is $A_2^{h_{1,1}}$ AND p_3 is $A_3^{h_{1,1}}$ AND p_4 is $A_4^{h_{1,1}}$ THEN $y^{1,1}$ is $B^{h_{1,1}}$,

$FIS_{1,2}$: $R^{h_{1,2}}$: IF p_5 is $A_5^{h_{1,2}}$ AND p_6 is $A_6^{h_{1,2}}$ AND p_7 is $A_7^{h_{1,2}}$ THEN $y^{1,2}$ is $B^{h_{1,2}}$,

… (24)

Layer 2: $FIS_{2,1}$: $R^{h_{2,1}}$: IF $y^{1,1}$ is $B^{h_{1,1}}$ AND $y^{1,2}$ is $B^{h_{1,2}}$ AND $y^{1,3}$ is $B^{h_{1,3}}$ THEN y^* is $B^{h_{2,1}}$,

$$y^{1,1}(B^{h_{1,1}}) = \frac{\sum_{j=1}^{p} y_j^{1,1} \times \mu_{B^{h_{1,1}}}(y_j^{1,1})}{\sum_{j=1}^{p} \mu_{B^{h_{1,1}}}(y_j^{1,1})}, \tag{25}$$

$$y^{1,2}(B^{h_{1,2}}) = \frac{\sum_{j=1}^{p} y_j^{1,2} \times \mu_{B^{h_{1,2}}}(y_j^{1,2})}{\sum_{j=1}^{p} \mu_{B^{h_{1,2}}}(y_j^{1,2})}, \tag{26}$$

…

$$y^{*}(B^{h_{2,1}}) = \frac{\sum_{j=1}^{p} y_j^{2,1} \times \mu_{B^{h_{2,1}}}(y_j^{2,1})}{\sum_{j=1}^{p} \mu_{B^{h_{2,1}}}(y_j^{2,1})}, \tag{27}$$

where: - $p_1, p_2, \ldots, p_{12}$ are input parameters,

- $A_1^{h_{1,1}}, A_2^{h_{1,1}}, \ldots, A_{12}^{h_{2,1}}$ represent linguistic variables corresponding to fuzzy sets $\mu_1^{h_{1,1}}(p_j), \mu_2^{h_{1,1}}(p_j), \ldots, \mu_{12}^{h_{2,1}}(p_j)$,
- $B^{h_{1,1}}, B^{h_{1,2}}, \ldots, B^{h_{2,1}}$ represent linguistic variables corresponding to fuzzy sets $\mu^{h_{1,1}}(y^{1,1}), \mu^{h_{1,2}}(y^{1,2}), \ldots, \mu^{h_{2,1}}(y^*)$,
- $\mu_{B^{h_{1,1}}}(y_j^{1,1}), \mu_{B^{h_{1,2}}}(y_j^{1,2}), \ldots, \mu_{B^{h_{2,1}}}(y_j^{2,1})$ are membership functions values of the aggregated fuzzy set for values $y_j^{1,1}, y_j^{1,2}, \ldots, y_j^{2,1}$ from the reference sets.

Design of HSFIS4

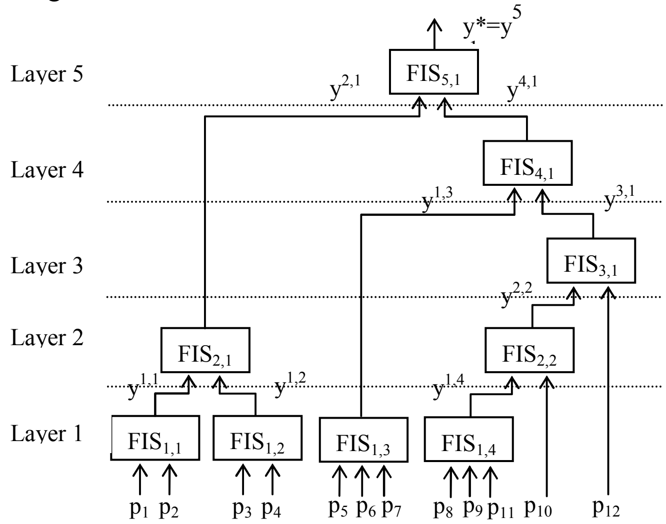

Fig. 6 Design of HSFIS4 for municipal creditworthiness evaluation

Legend: $p_1, p_2, \ldots, p_{12}$ are input variables, $y^{1,1}, y^{1,2}, \ldots, y^{5,1}$ are outputs of subsystems $FIS_{1,1}, FIS_{1,2}, \ldots, FIS_{5,1}$, L=5 is the number of HSFIS4 layers.

The HSFIS4 model can be formalized by BRs $R^{h_{1,1}}, R^{h_{1,2}}, \ldots, R^{h_{5,1}}$ and outputs $y^{1,1}, y^{1,2}, \ldots, y^*$ of single subsystems HSFIS4 this way:

Layer 1: $FIS_{1,1}$: $R^{h_{1,1}}$: IF p_1 is $A_1^{h_{1,1}}$ AND p_2 is $A_2^{h_{1,1}}$ THEN $y^{1,1}$ is $B^{h_{1,1}}$,

$FIS_{1,2}$: $R^{h_{1,2}}$: IF p_3 is $A_3^{h_{1,2}}$ AND p_4 is $A_4^{h_{1,2}}$ THEN $y^{1,2}$ is $B^{h_{1,2}}$,

… (28)

Layer 5: $FIS_{5,1}$: $R^{h_{5,1}}$: IF $y^{2,1}$ is $B^{h_{2,1}}$ AND $y^{4,1}$ is $B^{h_{4,1}}$ THEN y^* is $B^{h_{5,1}}$,

$$y^{1,1}(B^{h_{1,1}}) = \frac{\sum_{j=1}^{p} y_j^{1,1} \times \mu_{B^{h_{1,1}}}(y_j^{1,1})}{\sum_{j=1}^{p} \mu_{B^{h_{1,1}}}(y_j^{1,1})}, \quad (29)$$

$$y^{1,2}(B^{h_{1,2}}) = \frac{\sum_{j=1}^{p} y_j^{1,2} \times \mu_{B^{h_{1,2}}}(y_j^{1,2})}{\sum_{j=1}^{p} \mu_{B^{h_{1,2}}}(y_j^{1,2})}, \quad (30)$$

…

$$y^{*}(B^{h_{5,1}}) = \frac{\sum_{j=1}^{p} y_j^{5,1} \times \mu_{B^{h_{5,1}}}(y_j^{5,1})}{\sum_{j=1}^{p} \mu_{B^{h_{5,1}}}(y_j^{5,1})}, \quad (31)$$

where: - $p_1,p_2, \dots ,p_{12}$ are input parameters,

- $A_1^{h_{1,1}}, A_2^{h_{1,1}}, \dots, A_{12}^{h_{5,1}}$ represent linguistic variables corresponding to fuzzy sets $\mu_1^{h_{1,1}}(p_j), \mu_2^{h_{1,1}}(p_j), \dots, \mu_{12}^{h_{5,1}}(p_j)$,
- $B^{h_{1,1}}, B^{h_{1,2}}, \dots, B^{h_{5,1}}$ represent linguistic variables corresponding to fuzzy sets $\mu^{h_{1,1}}(y^{1,1}), \mu^{h_{1,2}}(y^{1,2}), \dots, \mu^{h_{5,1}}(y^*)$,
- $\mu_{B^{h_{1,1}}}(y_j^{1,1}), \mu_{B^{h_{1,2}}}(y_j^{1,2}), \dots, \mu_{B^{h_{5,1}}}(y_j^{5,1})$ are membership functions values of the aggregated fuzzy set for values $y_j^{1,1}, y_j^{1,2}, \dots, y_j^{5,1}$ from the reference sets.

The numbers and shapes of input and output membership functions and BRs are defined for the designed models HSFIS1, HSFIS2, HSFIS3 and HSFIS4.

4 Analysis of the Results

The comparison of given models according to the number of rules is presented in Table 1. The designed models are compared to Mamdani-type FIS. As the results show, the HSFIS1 and HSFIS2 models contain the lowest number of rules. The input parameters $p_1,p_2, \dots ,p_{12}$ form the following common categories, the economic (p_1,p_2,p_3,p_4), debt (p_5,p_6,p_7) and financial ($p_8,p_9,p_{10},p_{11},p_{12}$) parameters. In the design of HSFIS model, it is possible to take into account the membership of the parameters to the categories mentioned herein. Consequently, the interpretability of the model is improved. The HSFIS1 and HSFIS2 models do not reflect the affiliation of these parameters. Therefore, it is impossible to reproduce expert decision-making in the field of municipal creditworthiness evaluation. The parameters affiliation to categories (economic, debt and financial) was reflected in the design of HSFIS3 and HSFIS4 models. As the number of membership functions k increases, the design of HSFIS3 model starts to be too complicated due to increasing number of rules N_{PP}. The HSFIS4 model contains low number of rules N_{PP} even for great number of membership functions k. As a consequence, this model is suitable for municipal creditworthiness modelling.

Table 1 Comparison of models HSFIS

	FIS	HSFIS1	HSFIS2	HSFIS3	HSFIS4
Number of FIS	1	11	11	4	9
Number of layers L	1	11	6	2	5
N_{PP} for k=3	0.53x10^6	99	99	378	117
N_{PP} for k=4	16.8x10^6	176	176	1 408	240
N_{PP} for k=5	244 x10^6	275	275	4 000	425
N_{PP} for k=6	2.18x10^9	396	396	9 504	684

Classification of municipalities into classes $\omega_1,\omega_2, \dots ,\omega_6$ by HSFIS models and their frequency is presented in Fig. 7 (HSFIS1), Fig. 8 (HSFIS2), Fig. 9 (HSFIS3) and Fig. 10 (HSFIS4). In term of creditworthiness, the best municipalities are assigned to class ω_1, the worst ones to class ω_6. The models HSFIS1, HSFIS2, HSFIS3 and HSFIS4 classify the municipalities so that the classes ω_3 and ω_4 have the highest frequencies. The average municipalities dominate in the data sample. The designed fuzzy sets and BRs of the models HSFIS1, HSFIS2, HSFIS3 a HSFIS4 make the interpretation of the generated classes possible.

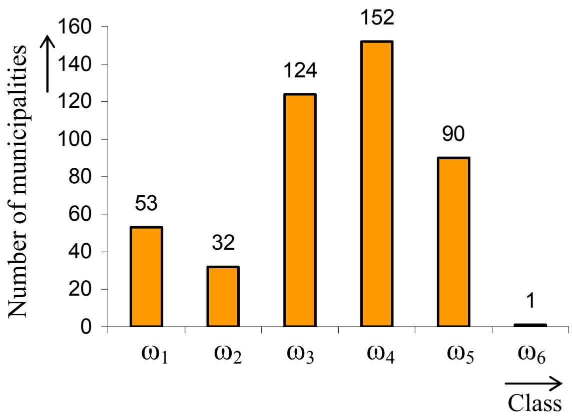

Fig. 7 Classification of municipalities into classes by HSFIS1

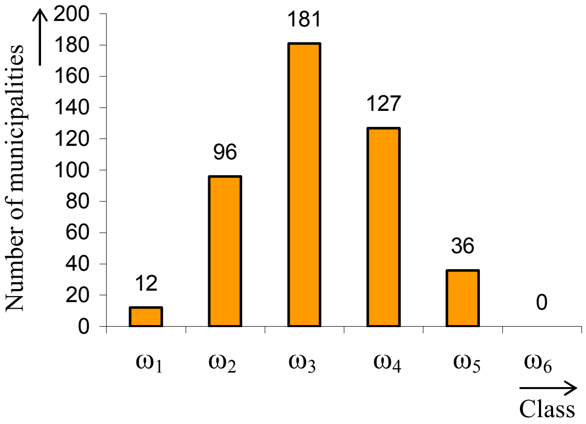

Fig. 8 Classification of municipalities into classes by HSFIS2

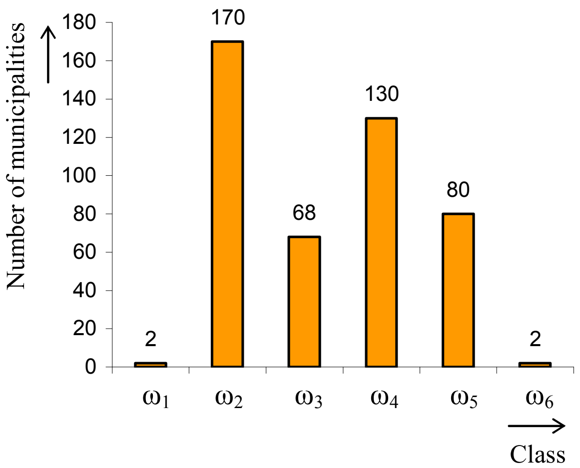

Fig. 9 Classification of municipalities into classes by HSFIS3

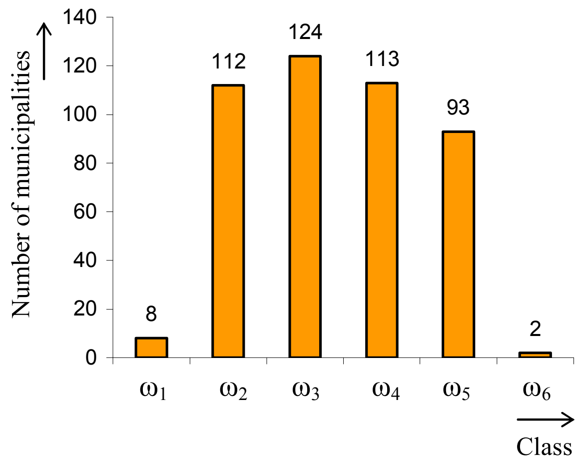

Fig. 10 Classification of municipalities into classes by HSFIS4

5 Conclusion

The paper presents the design of municipal creditworthiness parameters. Further, the HSFIS1, HSFIS2, HSFIS3 and HSFIS4 models are designed to realize the municipal creditworthiness evaluation. The designed HSFIS models are suitable instruments for municipal creditworthiness evaluation due to their effectiveness and easy interpretation. The lowest number of rules has been achieved by HSFIS1 and HSFIS2 models. The parameters p_1,p_2, … ,p_{12} affiliation to categories (economic, debt and financial) was reflected in the design of HSFIS3 model. As a result of this procedure, the effectiveness of this model decreases. The HSFIS4 is considered to be the most suitable of the presented models, because it contains a low number of rules N_{PP} and, at the same time, it models an expert's decision-making process in a given field.

The HSFIS models were carried out in Matlab Simulink in MS Windows XP operation system.

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