A New Modeling Approach of MIMO linear Systems using the generalized Orthonormal basis functions

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Abstract: - In this paper we propose a new modeling technique for LTI multivariable systems using the generalized Orthonormal basis functions with ordinary poles. Once the model structure is built we proceed to update the membership set of the resulting model parameters through the execution of unknown but bounded error identification algorithms. This updating aims to synthesize a robust control strategy.

Key-Words: -Modeling, Multivariable systems, Parameter estimation, Generalized Orthonormal Basis Functions, UBBE.

1 Introduction

The generalized Orthonormal basis functions [1], [2], [3], [4] regroup the common FIR, Laguerre and Kautz bases [5], [6] which are special cases of this complete construction. Consequently all type of linear, causal and stable systems can be represented by the generalized Orthonormal basis functions.

Modeling of MIMO (Multi-Input Multi-Output) linear systems using the generalized Orthonormal bases filters supposes in [7], [8], [9] that the poles of these bases are fixed which limited their performances. In this paper we surmounted these difficulties by the use of generalized Orthonormal basis functions with *ordinary poles*. A new simplified formulation of the state space representation of these bases has been elaborated which permits the modeling of linear MIMO systems and the problem of parameter estimation is solved by this new modeling approach. We also notice that the optimal poles of these bases are estimated by a nonlinear optimization method.

In contrast to the classic identification approach that leads to the determination of a vector of parameters and when no statistical information on the noise is available at the exception is bounded and of known boundary, the UBBE (unknown but bounded error) approach can be used. With this hypothesis the coefficients set of the linear combination is called the parameter uncertainty domain. It is often considered under a convex polytope which can be delimited by the induction of its vertices, its arrests or its faces. This set is compatible with the measures, the model structure and the error bounds.

Because of the complexity of the convex polytope, some authors searched to approach this form in a recursive way, either by simpler geometric form as an ellipsoid [10], an orthotope [11], a parallelotope [12] or a limited complexity polytope [13]. It is necessary to notice that the region can be empty in the case where the bounded error or the initial region, were badly selected.

This paper is organized as follows: In the second paragraph the problem formulation of the new state space representation of the generalized Orthonormal basis functions and the block diagram of MIMO linear systems represented on these bases are presented. The third paragraph presents the new modeling approach of MIMO systems. A preview concept of the UBBE approach is given in the fourth paragraph. The fifth paragraph summarizes some simulation results and finally a conclusion is given in the last paragraph.

2 Problem formulation

We consider a MIMO linear system with r input sequences $\{u_1(k), u_2(k), \cdots, u_r(k)\}$ and m output sequences $\{y_1(k), y_2(k), \cdots, y_m(k)\}$ described by its transfer matrix $G(q^{-1})$ of dimension $(m \times r)$.

Each elementary transfer function $G_{ij}(q^{-1})$ (i=1,2,…,m ; j=1,2,…,r) can be decomposed on the generalized Orthonormal bases filters as follows :

$$G_{ij}(z) = \sum_{n=0}^{N_{i,j}} g_n^{i,j} \mathcal{B}_n^{i,j}(z, \underline{\xi_{ij}}) \tag{1}$$

with:

$\{g_n^{i,j}\}$, $\underline{\xi_{i,j}}$ and $N_{i,j}$ are respectively the set of the Fourier coefficients, the poles vector and the truncating order of the network (*i,j*).

$\{\mathcal{B}_n^{i,j}(z,\underline{\xi_{i,j}})\}$ represent the generalized Orthonormal basis functions given in [1] and defined by:

$$\mathcal{B}_n^{i,j}(z)=\frac{\sqrt{1-\left|\xi_n^{i,j}\right|^2}}{z-\xi_n^{i,j}}\prod_{k=0}^{n-1}\left(\frac{1-\overline{\xi}_k^{i,j}z}{z-\xi_k^{i,j}}\right) \tag{2}$$

where:

$\xi_k^{i,j}$ and its conjugated $\overline{\xi}_k^{i,j}$ are the poles of the filter *k*.

The block diagram of the figure 1 shows the network (*i,j*) of the generalized Orthonormal base filters.

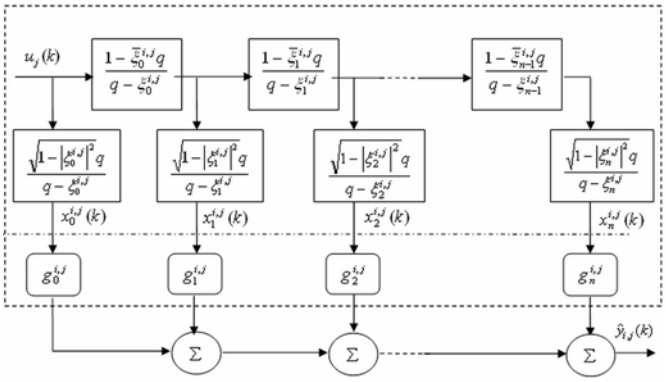

Fig.1: Network (*i,j*) of the generalized orthonormal base filters

The network (*i,j*) of the generalized Orthonormal bases filters can be described by a state space representation which is **reformulated** in a simplified version and **rewritten** in matrix form as follows:

$$\begin{cases}X_{i,j}(k+1)=A_{i,j}X_{i,j}(k)+B_{i,j}u_j(k)\\ \hat{y}_{i,j}(k)=\theta_{i,j}^T X_{i,j}(k)\end{cases} \tag{3}$$

with:

$X_{i,j}(k)$ is a state vector of dimension $(N_{i,j}+1)$:

$$X_{i,j}(k)=\left[x_0^{i,j}(k)\; x_1^{i,j}(k)\cdots x_{N_{i,j}}^{i,j}(k)\right]^T \tag{4}$$

where:

$$x_n^{i,j}(k)=Z^{-1}\left\{\mathcal{B}_n^{i,j}(z,\underline{\xi_{ij}})\right\}u_j(k) \tag{5}$$

$A_{i,j}$ is a matrix of dimension $(1+N_{i,j})\times(1+N_{i,j})$:

$$A_{i,j}(p,q)=\begin{cases}\xi_{p-1}^{i,j} & if \quad p=q\\ a_{i,j}(p,q) & if \quad p\succ q\\ 0 & if \quad p\prec q\end{cases} \tag{6}$$

Where:

$$a_{i,j}(p,q)=(-1)^{p+q+1}\alpha_{p-1}^{i,j}(1-\xi_{q-1}^{i,j}\overline{\xi}_{q-1}^{i,j})\prod_{l=q+1}^{p-1}\alpha_{l-1}^{i,j}\overline{\xi}_{l-1}^{i,j} \tag{7}$$

$B_{i,j}$ and $\theta_{i,j}$ are vectors of dimensions $(1+N_{i,j})$:

$$B_{i,j}(p)=(-1)^{p+1}\alpha_{p-1}^{i,j}\prod_{l=1}^{p-1}\alpha_{l-1}^{i,j}\overline{\xi}_{l-1}^{i,j} \quad (p=1,\cdots,n+1) \tag{8}$$

$$\theta_{i,j}=\left[g_0^{i,j}\; g_1^{i,j}\cdots g_{N_{i,j}}^{i,j}\right]^T \tag{9}$$

where:

$$\alpha_l^{i,j}=\frac{\sqrt{1-\left|\xi_l^{i,j}\right|^2}}{\sqrt{1-\left|\xi_{l-1}^{i,j}\right|^2}}\ (l\succ 0)\ \text{and}\ \alpha_0^{i,j}=\sqrt{1-\left|\xi_0^{i,j}\right|^2} \tag{10}$$

According to (1) and (3), the MIMO linear system can be decomposed on the generalized Orthonormal bases filters as shown in figure 2:

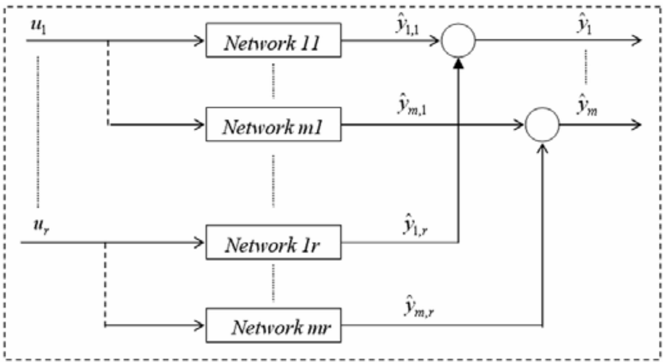

Fig.2: Network of the generalized Orthonormal bases filters for a MIMO system

3 Modeling of MIMO systems

To model the MIMO linear system represented on the generalized Orthonormal basis functions, we use the block diagram of the figure 2. We determine the state space representation of a MISO system and we deduce the MIMO representation.

According to the figure 2, we can write:

$$\hat{y}_i(k) = \sum_{j=1}^{r} \hat{y}_{i,j}(k) = \sum_{j=1}^{r} \theta_{i,j}^T X_{i,j}(k)$$
$$= \theta_{i,1}^T X_{i,1}(k) + \theta_{i,2}^T X_{i,2}(k) + \cdots + \theta_{i,r}^T X_{i,r}(k)$$

Where again:

$$\hat{y}_i(k) = \left[\theta_{i,1}^T\ \theta_{i,2}^T \cdots \theta_{i,r}^T\right] \begin{bmatrix} X_{i,1} \\ X_{i,2} \\ \vdots \\ X_{i,r} \end{bmatrix} = \Theta_i^T X_i(k) \tag{11}$$

Let's define the following state vector:

$$X_i(k) = \begin{bmatrix} X_{i,1}(k) \\ X_{i,2}(k) \\ \vdots \\ X_{i,r}(k) \end{bmatrix} = \left[X_{i,1}^T\ X_{i,2}^T \cdots X_{i,r}^T\right]^T \tag{12}$$

$$\dim X_i(k) = \sum_{j=1}^{r} (N_{i,j} + 1) \tag{13}$$

By using (3), (11) and (12), the state space representation of a MISO system can be written as:

$$\begin{cases} X_i(k+1) = A_i X_i(k) + B_i u(k) \\ \hat{y}_i(k) = \Theta_i^T X_i(k) \end{cases} \tag{14}$$

with:

$$A_i = diag(A_{i,1}, A_{i,2}, \cdots, A_{i,r}) \tag{15}$$

$$\dim A_i = \sum_{j=1}^{r} (1 + N_{ij}) \times \sum_{j=1}^{r} (1 + N_{ij}) \tag{16}$$

$$B_i = diag(B_{i,1}, B_{i,2}, \cdots, B_{i,r}) \tag{17}$$

$$\dim B_i = \left(\sum_{j=1}^{r} (1 + N_{i,j}) \right) \times r \tag{18}$$

$$\Theta_i = \left[\theta_{i,1}^T\ \theta_{i,2}^T \cdots \theta_{i,r}^T\right]^T \tag{19}$$

$$\dim \Theta_i = \sum_{j=1}^{r} (1 + N_{i,j}) \tag{20}$$

where:

$A_{i,j}$, $B_{i,j}$, and $\theta_{i,j}$ are defined in (6), (8), and (9).

By using (12) and (14), we define the following state vector:

$$X(k) = \begin{bmatrix} X_1(k) \\ X_2(k) \\ \vdots \\ X_m(k) \end{bmatrix} \tag{21}$$

$$\dim X(k) = \sum_{i=1}^{m} \sum_{j=1}^{r} (N_{i,j} + 1) \tag{22}$$

The state space representation of a MIMO system can then be written as:

$$\begin{cases} X(k+1) = AX(k) + Bu(k) \\ \hat{y}(k) = \Theta X(k) \end{cases} \tag{23}$$

The matrixes A, B, and Θ are given by:

$$A = diag(A_1, A_2, \cdots, A_m) \tag{24}$$

$$\dim(A) = \sum_{i=1}^{m} \sum_{j=1}^{r} (1 + N_{i,j}) \times \sum_{i=1}^{m} \sum_{j=1}^{r} (1 + N_{i,j}) \tag{25}$$

$$B = \left[B_1^T\ B_2^T \cdots B_m^T\right]^T \tag{26}$$

$$\dim(B) = \left(\sum_{i=1}^{m} \sum_{j=1}^{r} (1 + N_{i,j}) \right) \times r \tag{27}$$

$$\Theta = diag(\Theta_1^T, \Theta_2^T, \cdots, \Theta_m^T) \tag{28}$$

$$\dim(\Theta) = m \times \left(\sum_{i=1}^{m} \sum_{j=1}^{r} (1 + N_{i,j}) \right) \tag{29}$$

where:

A_i, B_i, and Θ_i are defined in (15), (17), and (19).

We deduce from (28), (29) and figure 2 that the identification problem of the MIMO linear system can divide in several identification problems of MISO (Multi-Input Single-Output) subsystems. Therefore, we interest in the follow paragraph to the identification of a MISO subsystem represented on the generalized Orthonormal basis functions.

4 Concept of UBBE approach

We consider the bounded error in (11). The model output of a MISO subsystem can be written:

$$y_i(k) = X_i(k)\Theta_i + e_i(k) \tag{30}$$

X_i and $\Theta_i(k)$ are defined respectively in (12) and (19). where $e_i(k)$ is the modeling error summarizing the approximation infinite serial of the general Orthonormal base by a finite serial and the additional measure noise. This error is assumed to be unknown but bounded and of known boundary. For a reason of simplicity the born δ_i is chosen symmetric.

We proceed to the identification of the vector $\hat{\Theta}_i$ by determining the parameter uncertainty region of the general Orthonormal basis functions.

From (30) we can write:

$$y_i(k) - \delta_i \le X_i(k)\Theta_i \le y_i(k) + \delta_i \tag{31}$$

The inequalities (31) generate at each time instant k tow hyperplanes H_{k1} and H_{k2} in the parameters space of the vector Θ_i. These two hyperplanes are normal to the state vector $X_i(k)$ and defined by:

$$\begin{aligned} H_{k1} &= \{\Theta_i / X_i(k)\Theta_i = y_i(k) + \delta_i\} \\ H_{k2} &= \{\Theta_i / X_i(k)\Theta_i = y_i(k) - \delta_i\} \end{aligned} \tag{32}$$

Each hyperplane H_{kj} $(j = 1,2)$ generates a negative half space and a positive half space. The vector Θ_i of parameters satisfying the double inequality (31) belongs to the domain defined by the intersection of the positive half closed spaces H_{k1}^{+} and H_{k2}^{+}.

The membership set of the vector Θ_i, obtained following the acquirement of L measures, must therefore satisfy all the constraints associated to these measures, either:

$$S_L = \bigcap_{k=1}^{L} H_{k1}^{+} \cap H_{k2}^{+} \tag{33}$$

In this way UBBE approach consists to determine at each instant k the smallest set of parameters $S_L \subset R^n$ consistent with the measurements, the model structure and the error bounds. This set is a *convex polytope*.

5 Simulation Results

To illustrate the utility of the new modeling approach, we consider a MIMO linear system with two input sequences and two output sequences with transfer matrix representation given by:

$$H(z^{-1}) = \begin{bmatrix} \dfrac{z^{-1}(1-0.525z^{-1})}{1-0.825z^{-1}} & \dfrac{z^{-1}(-0.515+0.895z^{-1})}{(1-0.315z^{-1})(1+0.575z^{-1})} \\ \dfrac{-z^{-1}(0.712+0.810z^{-1})}{1+0.715z^{-1}} & \dfrac{z^{-1}(0.510+0.475z^{-1})}{(1+0.225z^{-1})(1-0.695z^{-1})} \end{bmatrix} \tag{34}$$

The input signals are uniformly distributed sequences and the model errors assumed to be bounded with bounds $\delta_1 = 3.75$ and $\delta_2 = 4.76$. The truncating orders and the optimal poles for MISO subsystems yielded the following results:

$$N_{i,jopt} = 1 \quad (i = 1,2; \;\; j = 1,2)$$

$$\underline{\xi_{1opt}} = [0.825 \;\; 0 \;\; 0.315 \;\; \text{-}0.575]^T$$

$$\underline{\xi_{2opt}} = [\text{-}0.715 \;\; 0 \;\; 0.695 \;\; \text{-}0.225]^T$$

We proceed to estimate the uncertainty region of parameters from observable sequences of data (figure 3).

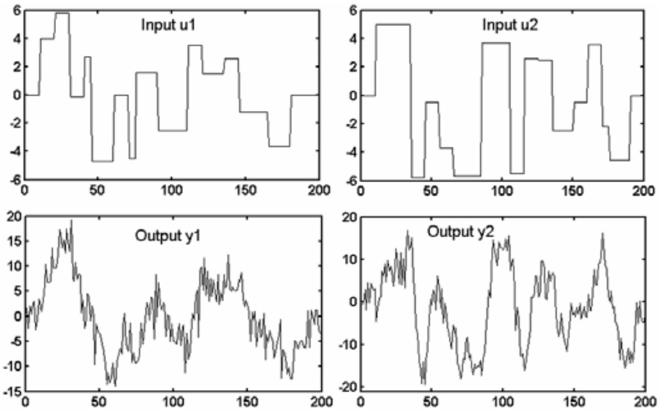

Fig.3: Input-Output signals of the system

The performances to evaluate concern either the uncertainty region or the updating algorithm performances. The former is determined by the final volume (FV) or the uncertainty intervals and its center. The latter are defined by the convergence time (CT), the updating rate (UR) and the mean computing time (MCT). The following tables summarize the performances of uncertainty regions of parameters and the updating algorithm for a signal to noise ratio SNR=10. We limit only to present the simulation results for the ellipsoidal approach.

Table1: Performances of the method

	Final volume	UR (%)	CT	MCT
Subsystem 1	0.2901	40.7035	201	0.00055
Subsystem 2	1.6993	32.1608	201	0.00094

Table 2: Region centers and Uncertainty intervals

$\hat{\Theta}_1$	$\Delta\hat{\Theta}_1$	$\hat{\Theta}_2$	$\Delta\hat{\Theta}_2$
0.9223	0.7658	-0.4216	5.3519
-0.4666	1.8685	-0.8059	2.2835
-0.1446	1.0189	1.0295	0.6960
1.1262	2.1958	0.2398	1.5178

We notice that the ellipsoid center is considered as the best valuator of the parameter vector. To validate this new modeling approach, we trace in figures 4 and 5 the model and estimated outputs valued by the ellipsoid centers.

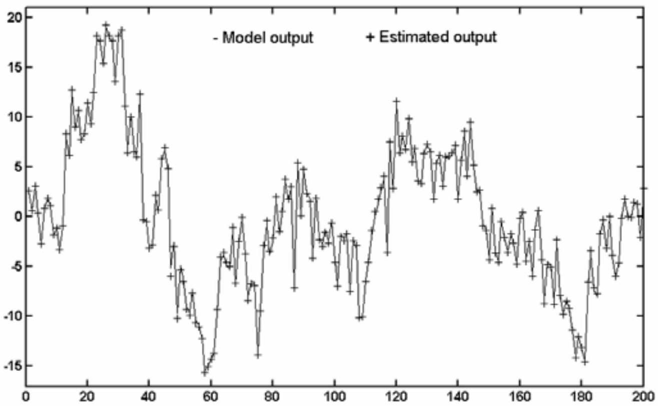

Fig.4: model and estimated outputs of the fist subsystem

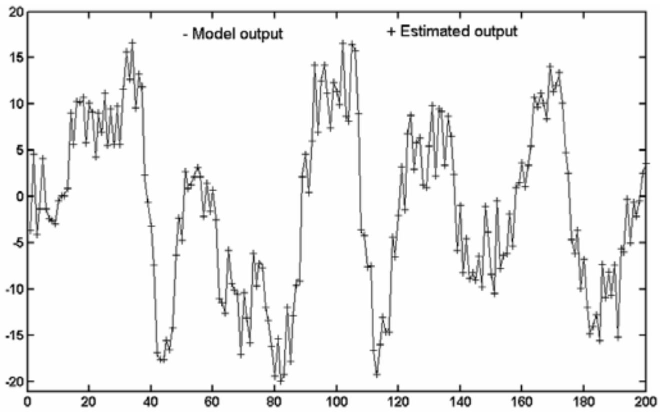

Fig.5: model and estimated outputs of the second subsystem

6 Conclusion

This paper has presented a new modeling approach of MIMO linear systems represented on the generalized Orthonormal basis functions with ordinary poles. The parameter uncertainty domain has been provided using the UBBE approach. So that it enables to synthesize a robust predictive control.

References

[1] B. Ninness and F. Gustafsson, A unifying construction of orthonormal bases for system identification, *IEEE Transactions on Automatic Control*, TAC-42 (4), 1997, pp. 515-521.

[2] Chul-Min Bae and Jong-Il Bae, System identification using generalized orthonormal basis, *Proc. of SPIE, Systems Control,* vol. 6052, 60520S, 2005.

[3] B. Ninness, H. Hjalmarsson, and F. Gustafsson, The fundamental role of general orthogonal bases in system identification, *IEEE Trans. on Automatic Control,* TAC-44(7), 1999, pp. 1384-1407.

[4] Van den Hof, P.M.J., Heuberger, P.S.C., and Bokor, J., System identification with generalized orthonormal basis functions, *Automatica,* 31, 1995, pp. 1821-1834.

[5] B. Wahlberg, System identification using Laguerre models, *IEEE Transactions on Automatic Control*, TAC-36, 1991, pp. 551-562.

[6] B. Wahlberg, System identification using Kautz models, *IEEE Transactions on Automatic Control*, TAC-39, 1994, pp. 1276-1282.

[7] B. Ninness, J. Gómez and S. Weller, MIMO system identification using orthonormal basis functions, *In Proceedings of the 34th IEEE Conf. on Decision and Control,* New Orleans, 1995, pp. 703-708.

[8] B. Ninness, J. C. Gómez, Asymptotic Analysis of MIMO systems estimates by the use of orthonormal bases, *Preprints 13th triennial IFAC world congress.* San Francisco, CA, 1996, pp. 363-368.

[9] T.J. de Hoog, Szabo Z., P. Heuberger, P.M.J. Van den Hof, J. Bokor, Minimal partial realization from generalized orthonormal basis function expansions, *Automatica,* Vol. 38, Issue 4, 2002, pp. 655-669.

[10] G. Favier and L. Arruda, Review and Comparison of Ellipsoidal Bounding Algorithms, *In M. et al., editor, Bounding Approaches to system identification,* Plenum Press, New York, chap. 4, 1996, pp. 43-68.

[11] H. Messaoud and G. Favier, Recursive determination of parameter uncertainty intervals for linear models with unknown but bounded errors, *In Proc. 10th IFAC Symposium on System Identification,* Copenhagen, Denmark, 1994, pp. 365-370.

[12] A. Vicino and G. Zappa, Sequential approximation of feasible parameter sets for identification with set membership uncertainty, *IEEE Transactions on Automatic Control,* vol. 41 N°6, 1996, pp. 774-784.

[13] H. Messaoud and S. Maraoui, Limited complexity bounding error algorithms, Design and comparative study, *Submitted to IFAC, System Structure and Control,* Prague, Czech Republic, 2001, pp. 29-31.