Hopfield Neural Networks—A Survey

Humayun Karim Sulehria, Ye Zhang
School of Electronics and Information Engineering
Harbin Institute of Technology, Harbin
PR China

Abstract: - In this work we survey the Hopfield neural network, introduction of which rekindled interest in the neural networks through the work of Hopfield and others. Hopfield net has many interesting features, applications, and implementations and it comes in two flavors, digital and analog. A brief review of the model oriented towards pattern recognition is also considered. Some interesting variations of the network or neuron model are noted which are being considered by researchers may lead to better performance or overcome problems such as capacity of the network.

Key-Words: - Hopfield net, neurocomputing, pattern recognition, associative memory, fuzzy neuron, hysteretic neuron, capacity.

1 Introduction

Many types of neural network models exist, but in this survey we would like to consider an interesting model studied by Amari and formally introduced by Hopfield in 1982 that has a wide range of applications. This model is sometimes referred to as Amari-Hopfield model. Hopfield neural network is a single-layer, non-linear, autoassociative, discrete or continuous-time network that is easier to implement in hardware [9]. The motivation for study of this field is presented in the next paragraphs.

Artificial intelligence, neural computing, and pattern recognition share a common knowledge base comprising of multiple disciplines. Contemporary neurocomputing takes its models from the biological system [20]. Our brain has been the basic motivation in the endeavor to building intelligent machine in the field of artificial intelligence [21]. Earlier the AI experts were trying to solve problems they thought were the essence of human intelligence such as playing the game of chess while, ignoring problems such as vision understanding, thought to be easy. These problems were being solved on conventional computers using algorithms generated through the human knowledge, implying that a conventional computer is limited to problems for which we (humans) can find an algorithm [2]. Fault-tolerance [9] is also an added advantage. This is where the neural networks come in, trying to overcome the mismatch between conventional computer processing and the working of human brain. Neural network research has faced many ups and downs in its history. The idea of creating a network of neurons got a boost when McCulloch and Pitts presented their model of the artificial neuron laying the foundations. Hebb was responsible for presenting the concept of learning. Much work was done in the field to a point where simulations of the net could be performed on computers. This situation changed drastically when the Minsky and Papert book cast a shadow on the computation ability of neural networks. The interest was lost in the field but some researchers were working to overcome the problems presented by Minsky and Papert, such as Amari [3], Little [16], Nakano [19], and many others.

In [9], John J. Hopfield presented a neural network model that he proposed as a theory of associative memory, thus changing the status quo resulting in a sudden surge of research activities that is ongoing. John Hopfield brought his skills in physics to the world of neurobiology as part of a larger effort to better understand how the brain thinks. A Hopfield network has the following interesting features [20]: distributed representation, distributed, asynchronous control, content addressable memory, and fault tolerance.

We would be using this model in pattern recognition tasks, so this review would be helpful to determine and design a workable system. We describe the Hopfield net in more detail in the next sections. In section 2, we define the network and look at the neuron and learning models for it. In sections 3 and 4, we give a description of research and analysis related to the HNN and we discuss the diverse applications and implementations of the network. Some variations of the model also appear in literature, they are also noted in the next sections. Finally, we draw some conclusions from this survey and point out our future work in Section 5.

2 Hopfield Neural Network Model

Hopfield neural network, or Hopfield associative memory is related to an autocorrelator by Simpson [24] and defined as a "single–layer, symmetric, non-linear, autoassociative (recurrent [7]), nearest-neighbor pattern encoder that stores binary/bipolar spatial patterns $A_k = (a_1^k, \ldots, a_n^k)$, k = 1,2, …, m, using Hebbian learning," in the discrete case, while, for continuous-time case he defines it as, "single-layer, autoassociative, nearest-neighbor encoder that operates in continuous-time, stores arbitrary analog spatial patterns $A_k = (a_1^k, \ldots, a_n^k)$, k = 1,2, …, m." The network learns offline, asynchronously updating its processing elements (PE) and has the topology as shown in Fig. 1 below.

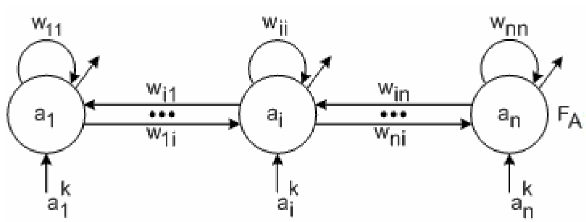

Fig. 1. Topology of the Discrete Autocorrelator (DA). This single-layered ANS paradigm has lateral and recurrent connections.

The input vector's components $A_k = (a_1^k, \ldots, a_n^k)$ feed directly into the F_A layer, and outputs can be read from the F_A layer at any time. To keep the presentation uncluttered, all connections are not shown—there is actually a lateral connection from each F_A PEs to every other F_A PEs, and a recurrent connection from every F_A PE to itself. Usually, the net is shown in a slightly different way, that is, without self-feedback loops. The asynchronous updating of units allows a function, known as an energy or Lyapunov function, to be found for the net. Lyapunov function is described later in this paper.

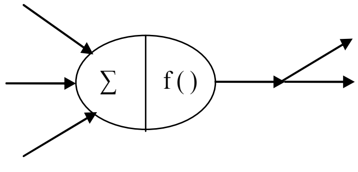

Fig. 2. A neuron as a simple threshold unit.

2.1 Neuron Model

As every neural network is built from simple units called neurons, nodes, or processing elements, etc, we first describe the neural model Fig. 2 used in the Hopfield neural network (HNN). Each input carries a signal that is added, Σ, together and after addition, the signal is processed through a threshold function f() producing an output signal(s). The neuron model used is the McCulloch-Pitts in the discrete case [25] with binary/bipolar values illustrated in Fig.3, and f() is step threshold function,

$$f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

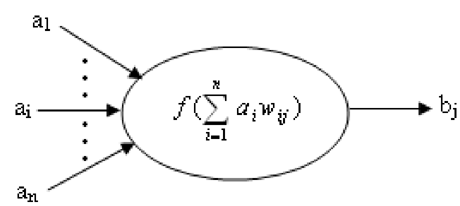

Fig. 3. A generic PE b_j.

The McCulloch-Pitts model was presented in 1943 and was a major boost to the neural network research at the time. This model uses binary/bipolar values, while in [10] Hopfield presented a continuous, deterministic neuron network of interconnected neurons with graded response that works as well as the two-state, i.e., bipolar one, the neuron essentially is analog here, that is the threshold function f() is a ramp or a sigmoid (S-shaped). Common sigmoid functions include logistics, tanh, and the augmented ratio of squares, the logistics function is for example

$$f(x) = (1 + e^{-x})^{-1} \tag{2}$$

Research on the McCulloch-Pitts model of neuron resulted in a new variation [7], reported by Lee and Lee, and is the Fuzzy neuron model. A brief introduction to fuzzy set terminology is in order. The values of fuzzy set members range from 0 to 1. The ith member's value of the fuzzy set $A_k = \{a_1, \ldots, a_n\}$ is $M_A(a_i^k)$. As an example, consider the three-member fuzzy set $A_k = \{0.5, 0.2, 0.8\}$, the value of A_k's 2nd element $M_A(a_2^k) = 0.2$. The basic fuzzy set operations are MIN and MAX which represent union and intersection, respectively. Using the fuzzy set of our example, MIN($M_A(a_1^k)$,$M_A(a_2^k)$) = min(0.5, 0.2) = 0.2. The neuron employing such a method of representation, it is a fuzzy neuron.

Recently a hysteretic neuron model as described in Fig. 4 has been proposed by Bhartikar and Mendel [5], based on the property of hysteresis in magnetic materials, in humans, etc. Hysteretic neuron models using signum functions have been proposed by Yanai and Sawada, and Keeler et al for associative memory. They demonstrated that their hysteretic model performed better than nonhysteretic neuron models, in terms of capacity, signal-to-noise ratio, recall ability, etc. Takefuji and Lee proposed a two-state (binary) hysteretic model. The Bhartikar and Mendel model is multivalued, has memory, and is adaptive.

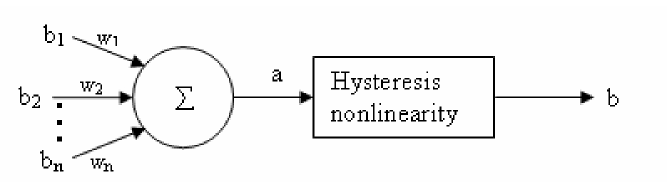

Fig. 4. Hysteretic neuron.

2.2 Learning model

Next we come to the learning or training models. These are basically used to train a neural network by adjusting synaptic weights [7]. Learning is defined to be any change in the memory W with respect to time, mathematically

$$\text{Learning} \equiv dW / dt \neq 0 \qquad (3)$$

Learning is classified into two categories, supervised and unsupervised, although aspects of each may co-exist in a given architecture. Supervised learning involves an external teacher and/or global information and is further classified into two subcategories: structured and temporal learning. For unsupervised learning, also referred to as self-organization, is a process in which there is no external teacher and it depends on local information and internal control. In the definition of Discrete HNN, Hebbian learning is described, which is credited to Donald Hebb for presenting the first learning concept, now commonly used in many neural network models. Hebbian learning, is the adjustment of a connection weight according to the correlation of the values of two or more PEs it connects.

The Hebb learning rule is a vector outer product rule and forms the weight matrix using the equation:

$$W = \sum_{k=1}^{m} [A_k^T A_k - I] \qquad (4)$$

where A_k is the input vector defined earlier, I is the n-by-n identity matrix.

Competitive learning or "winner-take-all" can also be used in the Analog HNN case. One example of competitive learning is the self-organized mapping or the Kohonen net. Winner-take-all HNN model has been discussed by [28]. In the simple form of competitive learning it is assumed that only one unit is active at a given time. This active unit is called the winner and is determined as the unit with the largest weighted sum net_i^k, where

$$net_i^k = w_i^T x^k \qquad (5)$$

and x^k is the current input and w_i is the weight matrix.

After selecting a neuron model, number of neurons needed for a particular application [15], some form of interconnectivity between neurons or PEs, determining the number of layers, and using a learning rule we get the NN, ready to use. To form an HNN we make full interconnectivity, the nodes are arranged in a single-layer. The HNN can use the binary/bipolar, analog or fuzzy inputs if we use the fuzzy logic for the threshold or activation function. As this net paradigm is simple to implement and easy to understand, it has been one of the most studied and applied of all existing nets.

3 Discussion and Analysis

Many researchers have worked on the HNN, the research work being initiated by Taylor [8]. Other significant contributions to the early development of associative memory include papers by Anderson, Kohonen, and Nakano, who independently proposed the idea of correlation matrix memory based on the outer product learning rule. Amari [4] presented the simple autocorrelator and several variants, and strictly analyzed the stability of these models using statistical neurodynamics, a method he pioneered. A description of functions very similar to Lyapunov functions, can be found in his treatment of autocorrelators using stochastic approximation. Later, Little [16], Little & Shaw [17] were involved in the neural network research. Hopfield's work is of particular importance as he established the isomorphism between Lyapunov energy function or Ising spin-glass and recurrent networks and was able to excite a large number of researchers to these powerful yet simple content-addressable associative memories. Hinton and Sejnowski gave the concept of Boltzmann learning to form Boltzmann machines by adding a noise model to the HNN.

Studies of HNN have been focused on network dynamics, memory capacity [9], [1], higher-ordered networks, error correction. Hopfield, Feinstein & Palmer [11] and Sasiela proposed a method for improving the storage capacity through the use of "unlearning" of information. Bayesian probabilities have been incorporated, HNN was compared to Sparse Distributed Memory, classification performance studied by Jacyna & Malaret [14], introduction of time-delays, introduction of hysteresis [5], comparison of HNN has been made to other neural network models for pattern classification by Lippmann [15] in his survey. In [13], the progress in research has been tracked. HNN strengths include total recall from partial or incomplete data, its stability under asynchronous conditions, and fault-tolerance. In [27], we find a variant of the HNN, that is, the multi-layer HNN model for pattern or object recognition, which converges to the single-layer model.

For the continuous-time HNN, see [24], [8], it was introduced by Cohen & Grossberg [6] and Hopfield [10], earlier work on analog HNN being done by Cowan, and Grossberg. It has been proven in literature that the HNN is a special case of the Cohen-Grossberg theorem. There have been several extensions and careful analyses to this type of HNN, such as Anderson & Abrahams, Platt and Hopfield, Hopfield and Tank [12]. Analog HNN strengths include its ability to handle data in continuous-time. Work of Hopfield and Grossberg was extended by Kosko to form his adaptive bidirectional associative memory (ABAM) as reported in a survey by Widrow and Lehr [26].

4 Applications and Implementations

The original formulation of the HNN showed the usefulness of the net as content-addressable memory using binary patterns. The applications of HNN include image [25] and speech processing, control, signal processing, database retrieval, fault-tolerant computing, pattern classification and recognition [20], automatic target recognition [24], olfactory processing, knowledge processing, while for the analog version we have applications such as image and signal processing, control, olfactory processing, pattern recognition [24], and in combinatorial optimization [12] problems. Many applications of the HNN in industry are described in (Table 1) [18]. In [23], the application of HNN in automatic target recognition is reviewed.

Table 1. Hopfield Neural Network Applications

Hopfield Neural Network Applications	
Application	**Type of HNN**
Image Processing	Discrete and Analog
Speech Processing	Discrete and Analog
Control	Discrete and Analog
Signal Processing	Discrete and Analog
Database Retrieval	Discrete
Fault-Tolerant Computing	Discrete
Pattern Classification	Discrete
Pattern Matching	Analog
Pattern Recognition	Discrete and Analog
Olfactory Processing	Discrete and Analog
Knowledge Processing	Discrete
Graph Coloring, Graph Flow & Graph Manipulation	Analog
Load Balancing & Programming Parallel Computers	Analog
ANS Programming	Analog
Data Deconvolution	Analog
Abductive Reasoning	Analog
Stress Measurement	Analog
Traveling Salesman, Scheduling & Resource Allocation	Analog

The simplicity of network model makes it easier to design and apply to wider range of problems and also easier to implement in VLSI. The authors want to implement the model on FPGAs. Binary/bipolar and analog versions of HNN have been implemented [24], where discrete-time net has more implementations performed then the continuous-time one, the analog having a more diverse implementation range. The implementation of HNN in hardware was described by

Hopfield in [9] for a binary model, and in [10] was described for the analog case. In [23], the application of HNN in automatic target recognition is reviewed.

Table 2. Hopfield Neural Network Implementations

Implementation	**Type of HNN**
Integrated Circuits	Discrete and analog
Coprocessor/Attached Processor	Discrete and analog
Bus-oriented Processor	Discrete and analog
Massively Parallel Computer	Discrete and analog
Optical/Electro-Optical	Discrete and analog

In [19] Nakano described an “Associatron” which was an earlier hardware implementation attempt, even before the HNN was formally identified. The “Associatron” composed of 180 neurons and was simulated by a digital computer. Prototype hardware was built that comprised of 25 neurons or 325 memory units and was used for trial purposes. Some implementations are highlighted in Table 2. Rietman [22] in his book describes some simple practical neural circuits that can be easily built and are based on the HNN model. As neural networks consist of simple nodes connected together, so these neural circuits can be extended to form sophisticated systems for pattern recognition. In [23], the author states that HNN can be precisely implemented in an all-optical design using computer-generated holograms.

5 Conclusions

The HNN can be used as an associative memory or for solving optimization problems through the use of energy function minimization. The model has several interesting features described in earlier sections, which give it a wide range of applications. Some new concepts such as the Hysteretic neuron model and Fuzzy neuron models have been studied. Similarly, new methods of learning such as Boltzmann learning can be used to train the HNN to further increase its range of applications and implementations.

We hope to develop a technique to improve the memory capacity parameter, important in pattern recognition applications and many researchers have worked on it. Our research would be oriented towards using this network model in the context of pattern recognition. This could lead us towards a simulation of and an implementation of the pattern recognition system.

The HNN has been extensively studied, extended, but still there is scope for research. In some applications it may be used in conjunction with other neural network models.

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