Design of Chebyshev FIR Filter Based On Antenna Theory Approach

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Abstract: In this paper we will present a novel method for designing the Chebyshev FIR filter which will take help of the techniques used in antenna theory.

Key–Words: FIR filter, Chebyshev polynomials.

1 Introduction

In this paper we will present an approach to realize FIR filters using Chebyshev Polynomials. Chebyshev polynomials play a vital role in antenna as well as in signal processing theory. The FIR filter design has also been disscussed previously [2-10], these papers discuss approximation methods, while the approach we will discuss in this paper gives exact design of FIR filter in Chebyshev sense. The Dolph-Chebyshev distribution of currents feeding the elements of a linear array comprising an antenna gives a sharp main lobe and small side lobes all of which have the same power level. In this paper we will present a method by which we can design a low-pass filter with linear phase and,
(i) A given pass band to stop band ratio,
(ii) A given pass band to stop band transition, and to some extent
(iii) The frequency band of the pass band.

Taking the case of a linear equispaced antenna array with n elements, labelled from left to right.

$$|E| = |A_0e^{j0}+A_1e^{j\psi}+A_2e^{j2\psi}+\ldots+A_{n-2}e^{j(n-2)\psi}+A_{n-1}e^{j(n-1)\psi}| \quad (1)$$

$$\psi = \beta d cos\phi + \gamma \quad (2)$$

where,
$|E|$ =is the magnitude of the far field,
$\beta = 2\pi/\lambda$,
λ is the free space wavelength,
d is the spacing between elements,
ϕ is the angle from the normal to the linear array,
γ is the progressive phase shift from left to right, and $A_0, A_1, A_2, \ldots$ are complex amplitudes which are proportional to the current amplitudes.

If we substitute $z = e^{j\psi}$ and write Equation (1) as

$$H(z) = A_0 + A_1z + A_2z^2 + \ldots + A_{n-2}z^{n-2} + z^{n-1} \quad (3)$$

This equation represents an FIR filter. Where, H(z) is impulse response of the filter with $z = e^{j\omega}$, $A_0, A_1, A_2, \ldots$ represents amplitudes at the corresponding frequencies.

Now we will design our FIR filter based on the antenna design [1].
The Chebyshev polynomial is given by

$$T_m(x) = cos(mcos^{-1}x) \qquad 0 < |x| < 1$$
$$T_m(x) = cosh(mcosh^{-1}x) \qquad 1 < |x| \quad (4)$$

Following the steps outlined in [1] we can state that

$$b = 10^{(attenuation\ in\ dB)/20} \quad (5)$$

$$\omega_s = 2cos^{-1}\left[1/\left\{cosh(\overline{1/m}cosh^{-1}b)\right\}\right] \quad (6)$$

$$\omega_p = 2cos^{-1}\left[\frac{cosh\left\{(1/m)cosh^{-1}(b/\sqrt{2})\right\}}{cosh(\overline{1/m}cosh^{-1}b)}\right] \quad (7)$$

where,
m is the order of the filter,
b is the absolute value of attenuation in the stop band,
ω_s is the stopband frequency,
ω_p is the passband frequency.

The manner in which we use Equations (5) to (7) is as follows:
(a) Use Equation (5) to obtain 'b' for a desired level of stop band,
(b) Calculate ω_s and ω_p from Equations (6) and (7).

The location of zeros, ω_m, on unit circle can be calculated by the following equation ([1])

$$\omega_m = 2cos^{-1}\left\{cos(\omega_k)/cosh(\overline{1/m}cosh^{-1}b)\right\} \quad (8)$$

Where, $\omega_k = (2k-1)\pi/2m$,
and $k = 0 \ldots m$.
Using the relation $z_m = e^{jw_m}$, we can write Equation (3) as follows

$$H(z) = (z - z_1)(z - z_2)\ldots(z - z_m) \quad (9)$$

where,
$z_1, z_2, \ldots$ are location of zeros,
H(z) is the frequency response in z-transform domain.
Replacing z by $e^{j\omega}$ and z_m's by $e^{j\omega_m}$'s in Equation (9)

$$H(z) = (e^{j\omega} - e^{j\omega_1})(e^{j\omega} - e^{j\omega_2})\ldots(e^{j\omega} - e^{j\omega_m}) \quad (10)$$

2 Filter Design

Now let us consider a design problem.
Let us design a Chebyshev FIR filter of order 6 with side bands 40dB down from the pass band.

Following the design steps outlined from Equation (5) to Equation (7) we can say that
m=6,
$b = 10^{40/20} = 100$
from Equation (6) and Equation (7) we can calculate
$\omega_p = 0.5622$
$\omega_s = 1.5732$
the values of the $\omega_m{}'^s$ can be calculated by using Equation (8)
$\omega_1 = 1.64; \omega_2 = 2.0958; \omega_3 = 2.7739; \omega_4 = 3.5093; \omega_5 = 4.1874; \omega_6 = 4.6431$
We can write H(z) as

$$H(z) = (z - e^{j1.64})(z - e^{j2.0958})(z - e^{j2.7739})(z - e^{j3.5093})(z - e^{j4.1874})(z - e^{j4.6431})$$

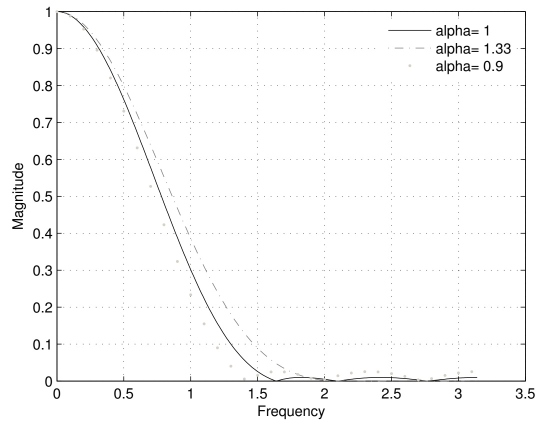

Figure 1: Magnitude Response of 6th Order FIR Filter

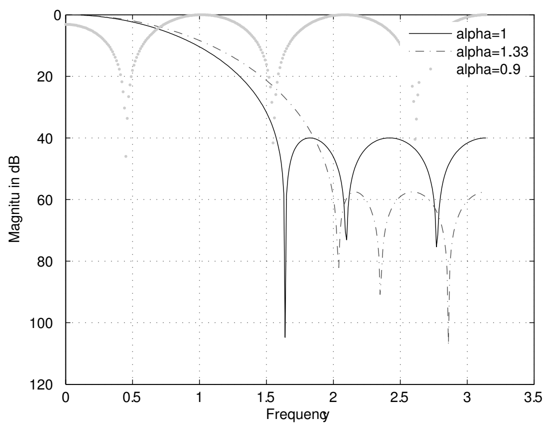

Figure 2: Magnitude response of 6th order FIR filter in dB

The dark continuous lines ($\alpha = 1$) in Figures (1), (2),and (3) show the magnitude response, magnitude response in dB, and phase response of above mentioned FIR filter respectively. While Figures (4) and (5) represents the magnitude response of 4th and 20th order FIR filters respectively, with side band 40dB down and they are designed following the above mentioned proce-

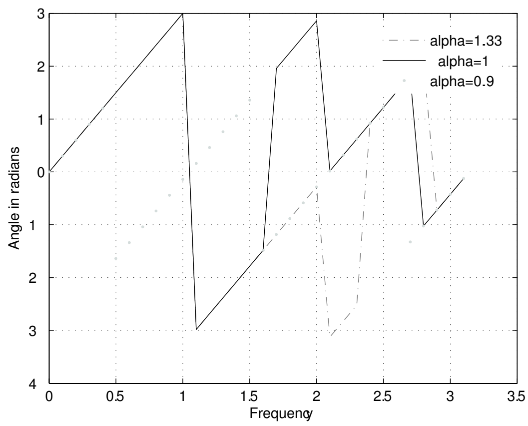

Figure 3: Phase Response of 6th order FIR filter

dure. Figures (6) and (7) show the magnitude responses in dB.

As it is clear from the Figures (1), (4), and (5) that the width of the pass band decreases as the order of filter increases, and the transition band becomes more steep. The dark continuous lines of magnitude response in dB curves shown in Figures (2), (6), and (7) make it more clear.

Figure (3) shows the phase response of the 6th order FIR filter designed above, which clearly shows its linear nature. The phase response of 4th and 20th order filter will also be linear and it can be varified easily by following the same steps.

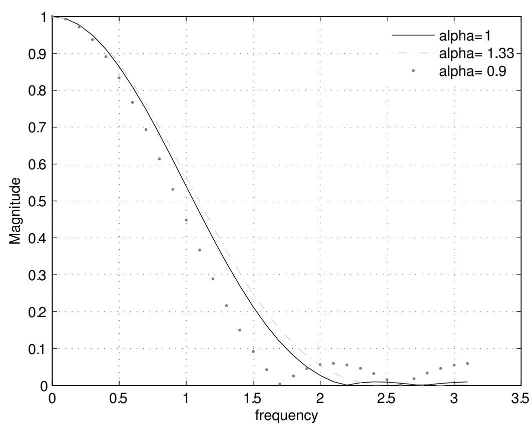

Figure 4: Magnitude response of 4th order filter

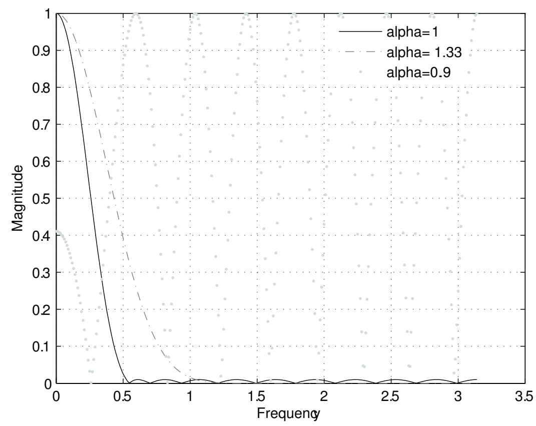

Figure 5: Magnitude response of 20th order FIR filter

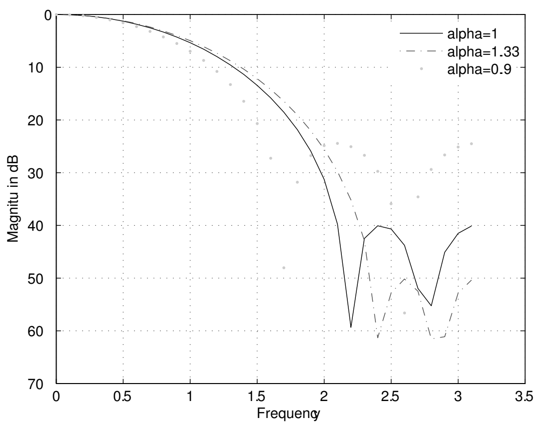

Figure 6: Magnitude response of 4th order filter in dB

3 The Modified Chebyshev Filter

Now, we can introduce a new parameter to change the filter characteristics. In the original Chebyshev polynomial (Equation (4)) we will multiply a new parameter $'\alpha'$ with parameter 'x'. Thus, now Equation (4) becomes

$$T_m(\alpha x) = cos(mcos^{-1}\alpha x) \qquad 0 < |x| < 1$$
$$T_m(\alpha x) = cosh(mcosh^{-1}\alpha x) \qquad 1 < |x| \quad (11)$$

Then ω_s and ω_p are given by the equations:

$$\omega_s = 2cos^{-1}\left[1/\left\{cosh(\overline{1/m}cosh^{-1}b)\right\}\right] \qquad (12)$$

$$\omega_p = 2cos^{-1}\left[cosh\left\{(1/m)cosh^{-1}(b/\sqrt{2})\right\}/\left\{cosh(\overline{1/m}cosh^{-1}b)\right\}\right] \qquad (13)$$

When we will multiply 'x' with $'\alpha'$, it will make change in ω_s only and ω_p will remain same, as

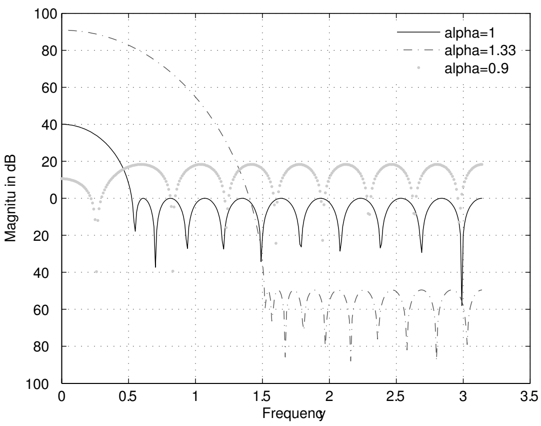

Figure 7: Magnitude response of 20th order filter in dB

$'\alpha'$ will be present in both numerator as well as denominator.
So, now ω_s will be

$$\omega_s = 2cos^{-1}\left[1/\alpha\left\{cosh(\overline{1/m}cosh^{-1}b)\right\}\right] \quad (14)$$

and ω_p will be same as in Equation (13).
We can calculate the location of zeros by

$$\omega_m = 2cos^{-1}\left[cos(\omega_k)/\left\{\alpha(cosh(\overline{1/m}cosh^{-1}b))\right\}\right] \quad (15)$$

Where,
$\omega_k = (2k-1)\pi/2m$,
and k=0...m.
Now, we can write H(z) as

$$H(z) = (e^{j\omega} - e^{j\omega_1})(e^{j\omega} - e^{j\omega_2})\dots(e^{j\omega} - e^{j\omega_m}) \quad (16)$$

with new values of ω_m's calculated using Equation (15)

Plotting the magnitude response of the design problem for 6th order again with the new parameter taken into account, we get the plots shown in Figure (1), "dash followed by dot" and "dots" for $\alpha > 1$ and $\alpha < 1$ respectively. It is evident from the figure that the bandwidth of our filter is increased in case of $\alpha > 1$. Figure (2) shows that the side bands are further below than they were with $\alpha = 1$. When $\alpha < 1$ we can easily conclude from Figure (1) and Figure (2) that bandwidth is reduced with an increase in the stopband level. Figure (3) shows that our modified Chebyshev FIR filter has linear phase characteristics, and on the other hand it confirms that the filter linear phase even after introducing the new parameter α.

4 Conclusion

In conclusion we can say that
(a) We have described two new low pass filters which are of the Chebyshev type and whose stopband levels can be designed by us.
(b) We can further change the passband frequency to some extent by using the parameter $'\alpha'$.
(c) The phase response of the original and modified filters are linear in nature.
(d) To design a band-pass filter, band reject, ar high pass filter wecan use the standard frequency transformations.

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