A Genetically Tuned Optimal PID controller

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Abstract- In this paper an optimal PID controller is proposed. To adjust the parameters of the controller a fitness function in terms of transient, steady state parts of response and control energy characteristics of system is introduced. A genetic algorithm is employed to minimize the fitness function to achieve a satisfactory response for system as well as minimizing energy. The results are verified by some simulations.

Key words: PID controller, Optimal control, Genetic algorithm, Energy

1. Introduction

Genetic algorithms (GA) are one of the efficient tools that are employed in solving optimization problems[1]. The basic idea of genetic algorithm is as follow[2][3]: the genetic pool of a given population potentially contains the solution, or a better solution, to a given optimization problem. This solution is not active because the genetic combination on which it relies is split between several subjects. Only the association of different genomes can lead to the solution. Optimization in genetic algorithm is based on optimization of a fitness function which is a function of environment individuals or genes. Each new generation is generated by applying Crossover and Mutation operand on old generation. Then in new generation good genes that lead to better fitness function have more chance to survive. So, after some generations the optimal solution will be attained.

Because of the simplicity and robustness, PID controllers are frequently used controllers in industries[4][5]. Parameter adjustment of PID controllers is an old challenge in the field of control system design. Some of methods have been proposed to select the PID coefficients, but they are not completely systematic methods and result in a poorly tuned controller that needs some trial and error[6]. So far, finding new methods to automatically select PID parameters was interest of researches[7]. On the other hand, existence algorithms mostly consider the output response parameters and have no consideration about optimization of control energy[4][8]...

Genetic algorithm can be a good candidate to be employed for optimization of control energy in PID controllers [9][10][11][12]. In this paper, we will introduce a new method to select coefficients of an optimal PID controller.

In next section the problem discussed in this paper is defined then in section III previous related works is viewed. In forth section the idea of genetically tuned optimal PID controller is proposed and then the idea is verified by simulation results in section V. The last section contains conclusions and future works.

2. Problem definition

The control model that will be studied in this paper is a common model as illustrated in Fig. 1. The controller is a PID controller and we have:

\[ u(t) = K_e e(t) + K_p \dot{e}(t) + K_i \int e(t) dt \]  (1)
where $K_p$, $K_I$, $K_D$ are parameters of controller and $e(t)$ and $u(t)$ are error and control signals respectively (see Fig.1).

The aim of optimal control problem is to select control parameters such that a good output response is attained as well as minimum energy is spent. Maximum overshoot and steady state error corresponding to step response parameterize the concept of "good response" and time integral of control signal parameterizes the concept of "energy".

The problem that we will deal in this paper is to propose a genetic algorithm approach for automatically selecting of controller parameters. Here definition of fitness function is the most important challenge. Then, to be comparable to previous works, an ordinary genetic algorithm will be employed to find the solution.

3. Related works

Some earlier works that have used evolutionary algorithms to parameter adjustment of controllers, have been utilized one of the following fitness functions [13][14]:

$$f(K_p, K_I, K_D) = A(1-e_{ss}) + B(1-O_{max})$$ (2)

$$f(K_p, K_I, K_D) = A(e_{ss}) + B(O_{max})$$ (3)

where $O_{max}$ denotes the maximum overshoot and $e_{ss} = \lim_{t \to \infty} e(t)$ denotes the steady state error of response. The fitness function (2) should be maximized and the fitness function (3) should be maximized during optimization algorithm. $A$ and $B$ are the parameters $(A+B=1)$ that should be manually tuned according to the understudy plant.

As it is clear, these fitness functions are proposed to achieve small amount for maximum overshoot and small amount steady state error, as much as possible. Utilizing such functions may cause large amounts of control energy that by itself may raise the cost of control and even hurt the control mechanism in practical cases.

4. Optimal PID controller

To obtain an optimal controller, the energy effort of system should be taken in account. For this purpose the fitness function of optimization mechanism must be modified. We enter a term concerning to energy of control signal to fitness function (3) and obtain new fitness function as:

$$f(K_p, K_I, K_D) = A(e_{ss}) + B(O_{max}) + C(E_o)$$

$$A + B + C = 1$$ (4)

where $E_o$ denotes the energy of control signal. Parameters $A,B,C$ should be manually tuned according to the understudy plant and importance priority of response characteristics.

Now, if optimization procedure is executed to minimize (4), an optimal PID controller will be expected.

The optimization algorithm that we employ is a common version of genetic algorithm. We can introduce the following computing procedure based on genetic algorithm for optimal selection of the parameters of controller:

**Algorithm:**

1. Randomly choose the genetic pool of parameters $K_p, K_I, K_D$

2. Compute the finesses of all genetic strings, taking (4) as fitness function.

3. Choose the best subset of the population of the parameters: $K_p, K_I, K_D$

4. Generate new strings using the subset chosen in step 3 as parents and the "single point crossover" and "mutation" as operators.

5. Verify the fitness of the new population members.

6. Repeat the steps 3 to 5 until the fixed amount of fitness is attained.

This algorithm is illustrated in Fig. 2. For details the reader can refer to the texts about genetic algorithm such as [2][3],[15][16] and also HELP documents of MATLAB.

5. Simulation Results

A numerical simulation was implemented using MATLAB. We selected transfer function of plant as:
and performed the genetic algorithm to optimize fitness function (4). To comparison purposes we applied the same genetic algorithm on fitness function (3) which is proposed in a previous work [14]. The characteristics of applied GA were as follow:

- **Population** = 20
- **Scaling function** = Rank
- **Selection** = Stochastic Uniform
- **Crossover** = 0.85

Table 1. PID controllers corresponding to fitness functions (3) and (4) and step response characteristics for each controller

<table>
<thead>
<tr>
<th>Fitness function (3)</th>
<th>$e_{ss}$</th>
<th>$O_{max}$ (%)</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[K_P, K_D, K_I] =$</td>
<td>6.10e-6</td>
<td>7.1</td>
<td>112</td>
</tr>
<tr>
<td>[6.4381, 0.3608, 5.9471]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fitness function (4)</th>
<th>$e_{ss}$</th>
<th>$O_{max}$ (%)</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[K_P, K_D, K_I] =$</td>
<td>4.46e-4</td>
<td>0.12</td>
<td>45</td>
</tr>
<tr>
<td>[2.06381, .00033 ,2.44784]</td>
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</tr>
</tbody>
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Fig. 2. Genetic algorithm employed for parameter adjustment of PID controller

$$G(s) = \frac{1}{(s + 1)(s + 4)}$$

(5)

and performed the genetic algorithm to optimize fitness function (4). To comparison purposes we applied the same genetic algorithm on fitness function (3) which is proposed in a previous work[14]. The characteristics of applied GA were as follow:

- **Elite Count** = 2
- **Generation** = 100
- **Mutation function** = Gaussian
- **Scale** = 1
- **Shrink** = 1.5
- **Migration Direction** = Forward
- **Fraction** = 0.2
- **Interval** = 20
- **Iteration** = 32

Fig. 3. Output response of simulated system with two controllers:
- **Dashed-Blue**: A PID controller selected according to fitness function (3)- Previous method
- **Solid-Black**: An optimal PID controller selected according to fitness function (4)- Proposed method

Fig. 4. Error signal of simulated system with two controllers:
- **Dashed-Blue**: A PID controller selected according to fitness function (3)- Previous method
- **Solid-Black**: An optimal PID controller selected according to fitness function (4)- Proposed method
Obtained PID parameters and corresponding response characteristics are shown in Table 1. In the table energy of control signal is calculated from:

\[ \text{energy} = \int_{t_0}^{t_f} \left| u(t) \right|^2 dt \]  

where \( t_f \) denotes transient time, i.e. integration is done over transient period of control signal. In addition, plots of control, error, and output signals for both fitness functions are depicted in Figs 3, 4 and 5 respectively.

It is clear that inserting energy term into fitness function causes to a little increase in steady state error, but a significant decrease occurred in control energy (less than half) and also maximum overshoot of response is improved. In fact weakness in steady error of output signal is a cost that is paid for decreasing energy consuming in system.

6. Conclusions

The aim of this paper was parameter tuning of an optimal PID controller using genetic algorithms. To reach that goal a fitness function in terms of transient, steady state and control energy characteristics of system is introduced. Minimization of such a fitness function by genetic algorithm causes a satisfactory steady state error and maximum over shoot as well as less control energy in comparison with the similar fitness functions.

In the proposed method some parameters of fitness function (i.e. \( A, B, C \) in (4)) should be tuned manually. Proposition of some algorithms to automatically tune these parameters will be accomplished in future works.