New Figure of Merit for Color Reproduction Ability of Color Imaging Devices using the Metameric Boundary Descriptor

SIMON SCHULZ
Hamburg University of Technology
Vision Systems
Harburger Schlossstrasse 20
21079 Hamburg, GERMANY

ROLF-RAINER GRIGAT
Hamburg University of Technology
Vision Systems
Harburger Schlossstrasse 20
21079 Hamburg, GERMANY

Abstract: A measure for the color reproduction ability of a color imaging device is important for comparison purposes as well as for the sensor design process. Several figures of merit which describe this ability in a single measure have been proposed in the past. All these measures have shortcomings in that their error measure is defined in a non-representative color space for human color perception, or they do not consider measurement noise, or they rely on a specific color correction function. We introduce a new figure of merit for color reproduction ability of digital imaging devices. This new approach uses the metameric boundary descriptor (MBD) to quantify the limit of color reproduction for this sensor. It is therefore independent of the color correction method used. The error measure is defined in a perceptually uniform color space (*CIELab*) and takes measurement noise into account. Only the spectral sensitivities of an imaging device are needed. The advantages over existing quality measures as well as the plausibility of the assumptions required are discussed. Quality measures for 5 different cameras are presented.

Key–Words: Image quality, Optical filters, Spectral Sensitivity, Colorimetry, Quality Measure
December 19, 2006

1 Introduction

Whenever the recorded image of a color imaging device (e.g. a digital color camera) is to be presented to a human, the ability to reconstruct the percieved colors of the scene from the device's output data is of great interest. A single figure of merit (FOM) would be of great help for choosing the correct device, optimizing the spectral sensitivies in the sensor design process and to be able to compare the performance of different devices for such an application.

The human visual color perception can be described in a tristimulus system. There are three different types of cones in the eye with different spectral sensitivities. Based on these spectral sensitivities the Commission Internationale de l'Eclairage (CIE) has defined the color matching functions for the standard observer and the color spaces *CIEXYZ* and *CIELab*. Whereas the *CIEXYZ* color space describes the human vision from the physical perspective, the *CIELab* color space was defined to be uniform in terms of percieved color differences. The Euclidean distance of two colors in *CIELab* closely matches the perceived color difference between these colors. Therefore it is usefull to develop a figure of merit in *CIELab* color space.

2 Related work

Previously published figures of merit may be divided into two groups: the first group tries to measure the goodness as a geometrical difference between the human visual subspace and the subspace of the sensor. The second group measures the goodness as an average color error after an optimized color correction is applied to the sensor's output.

2.1 Geometric Difference FOM

The figures of merit based on geometrical differences use calculations in the spectral vector space. Each spectrum $x(\lambda)$ is therefore given as a vector of n sample values $\vec{x} \in \mathbb{R}^n_+$.

The response $\vec{a} = (X, Y, Z)^T$ of the human eye observing a surface with reflectance spectrum $\vec{r}$ under the illumination $\vec{b}$ can be expressed as

$$\vec{a} = A^T \mathcal{D}(\vec{b})\,\vec{r} = A_{\vec{b}}{}^T \vec{r} \qquad (1)$$

where the color matching functions for the standard observer are stacked into the matrix $A = (\bar{\mathbf{x}}, \bar{\mathbf{y}}, \bar{\mathbf{z}}) \in \mathbb{R}_+^{\mathbf{n}\times\mathbf{3}}$ and vector $\vec{b}$ is tranformed into a diagonal matrix containing the elements of $\vec{b}$ by the operator $\mathcal{D}(\bullet)$. Replacing the matrix A in eqn. 1 with the matrix $\Omega \in \mathbb{R}_+^{n\times m}$ of the stacked spectral sensitivities $\vec{\omega}_i,\ i = 1, \ldots, m$ of an m-channel sensor, we get the sensor response $\vec{c}$ of the same stimulus:

$$\vec{c} = \Omega^T \mathcal{D}(\vec{b})\,\vec{r} = \Omega_{\vec{b}}{}^T \vec{r} \qquad (2)$$

Using vector space $V \in \mathbb{R}^n$, we can define a subspace $\mathrm{HVSS} := \mathrm{Span}(\bar{\mathbf{x}}, \bar{\mathbf{y}}, \bar{\mathbf{z}}) = \mathrm{Span}(\mathbf{A}), \mathrm{HVSS} \subset \mathbf{V}$ to be the human visual subspace, which is determided by the columns of A as basis vectors. Analogously we define the system visual subspace to be $\mathrm{SVSS} := \mathrm{Span}(\Omega)$

Neugebauer's q-factor [1] uses a projection $\mathcal{P}_{A_{\vec{b}}}(\vec{\omega}_i)$ of a single spectral sensitivity $\vec{\omega}_i$ of a sensor onto the $\mathrm{HVSS}_{\vec{b}}$ under the illumination $\vec{b}$. If we consider the length of the projection $\mathcal{P}(\bullet)$ in proportion to the length of the spectral sensitivity $\vec{\omega}_i$, we get a representation of the part of the color energy that is recoverable from such a sensor channels response:

$$q(\vec{\omega}_i) = \frac{\left\| \mathcal{P}_{A_{\vec{b}}}(\vec{\omega}_i) \right\|^2}{\|\vec{\omega}_i\|^2} \qquad (3)$$

The projection $\mathcal{P}_{A_{\vec{b}}}(\bullet)$ may be expressed by the pseudo inverse $(A_{\vec{b}}{}^T)^{\ominus}$:

$$\begin{aligned} \mathcal{P}_{A_{\vec{b}}}(\vec{\omega}_i) &= \underbrace{A_{\vec{b}}(A_{\vec{b}}{}^T A_{\vec{b}})^{-1}}_{=(A_{\vec{b}}{}^T)^{\ominus}} A_{\vec{b}}{}^T \vec{\omega}_i \\ &= (A_{\vec{b}}{}^T)^{\ominus} A_{\vec{b}}{}^T\, \vec{\omega}_i \qquad (4) \end{aligned}$$

Note that on the other hand using eqn. 4 we could try to reconstruct the reflectance spectrum $\vec{r}$ from the sensor responses $\vec{c}$

$$\begin{aligned} \vec{r} &= A_{\vec{b}}(\Omega_{\vec{b}}{}^T \Omega_{\vec{b}})^{-1} \Omega_{\vec{b}}{}^T \vec{c} + \mathrm{Kern}(\Omega_{\vec{b}}{}^T) \\ &= (\Omega_{\vec{b}}{}^T)^{\ominus} \vec{c} + \mathrm{Kern}(\Omega_{\vec{b}}{}^T) \qquad (5) \end{aligned}$$

As long as $n > m$, $\mathrm{Kern}(\Omega_{\vec{b}}{}^T) \neq 0$ holds true. This means that the reflectance spectrum $\vec{r}$ cannot be reconstructed from the sensor responses $\vec{c}$ without error.

The major drawback of Neugebauer's FOM is it's restiction to a single sensor channel only. Even the seperate calculation of the q-factors for all sensor channels doesn't help, as the joined performance of the sensor channels is of interest. Therefore Vora [2] extended Neugebauer's q-factor to be used for multiple channel sensors. Vora calculates a measure for the ovelap of the $\mathrm{SVSS}_{\vec{b}}$ and the $\mathrm{HVSS}_{\vec{b}}$ using an orthonormal basis O and N respectively for both. Under the assumption of reflectance spectra $\vec{r}_i$ beeing composed of statistically independent, identically distributed random variables, he defines $\nu(A_{\vec{b}}, \Omega_{\vec{b}})$ to be his FOM:

$$\nu(A_{\vec{b}}, \Omega_{\vec{b}}) = \frac{1}{\alpha} \sum_{i=1}^{\alpha} \gamma_i{}^2 (O^T N) \qquad (6)$$

where $\gamma_i(O^T N)$ denotes the ith singular value of $O^T N$.

The Color Quality Factor (CQF) proposed by Sharma [6] uses Neugebauer's FOM but in the oposite direction, calculating projections (e.g. $\mathcal{P}_{\Omega_{\vec{b}}}(\bar{x})$) of the human visual "channels" onto the SVSS.

2.2 Average Color Error FOM

The second class of FOMs use the mean color error on a set of reflectance spectra in a CIE color space. The sensor responces $\vec{c}$ are mapped to *CIEXYZ* using a linear transformation. The resulting color differences between reference colors and corrected colors are taken as measurement of quality. Shimano's Q_{st} and Q_{sf} metrics [3, 4] consider the minimized color error in *CIEXYZ* space. Tajima's indexes [5] asume object color spectral characteristics to be composed of few principle components only. Hung's CRI and Sharma-Trussel's FOM [6] rate the color reproduction ability using an error measure in a perceptually uniform color space using local or global linearization techniques. They even take signal independent recording noise into account, which none of the previous mentioned methods does. Quan [7] futher extends this FOM to the so called Unified measure of goodness (UMG) taking signal dependent recording noise into account.

The results of these FOMs depend on the set of reflectance spectra used. Therefore these sets should be choosen with care. The reflectance spectra of the GretagMacbeth ColorChecker are commonly used due to their representativeness for much larger sets [8] as well as the widespread use of this target in the color imaging area.

3 Metameric Boundary Descriptor FOM

An resonable FOM should meet the following requirements: The figure of merit should rate the quality of color reproduction ability in a perceptual relevant error measure as color reproduction for a human observer is the focus. Recording noise is an issue in all practical use of imaging sensors and should therefore be considered in the FOM calculation. The FOM should not rely on a specific color correction technique. Incorporating a specific color correction technique into the FOM would result in an quality measure that rates the joint performance of the device with this specific correction technique. A statement for color reproduction ability of the specific device alone could therefore not be given.

The motivation to introduce a new FOM is the limitation of the previously presented FOMs:

The geometric difference FOMs all suffer from their error measure not beeing defined in a perceptual uniform color space. The error is quantified in the spectral vector space which represents the physical layer of human vision. For our figure of merit to be relevant we rather need to define our error in a perceptual uniform color space.

Some average color error FOMs use error measures defined in linearized perceptual uniform color spaces. Sharma-Trussell's FOM [6] and Quan's UMG [7] additionally consider recording noise. But all previous color error FOMs rely on an optimized linear color correction function in order to transform the sensor output values $\vec{c}$ into the estimate $\hat{\vec{a}}$ of percieved colors.

Using the Metameric Boundary Descriptor (MBD) suggested by Urban [9, 10] we may define a error measure in the percepually mostly unifrom *CIELab* color space that is completely independent of the color correction function used. It only describes the limitation of color reproduction itself using a theoretical, best possible color correction function. For a given sensor output $\vec{c}$ we can interpret the MBD as a measure of uncertainty with respect to the percieved color that led to this sensor output. Recording noise can easily be taken into account.

The MBD describes the metameric subspace $M^{\vec{c}}_{\text{XYZ}}$ of a sensor output $\vec{c}$. The metameric subspace $M^{\vec{c}}_{\text{XYZ}}$ contains all percieved colors $\vec{a} = A^T_{\vec{b}_a}\vec{r}$ under viewing illumination $\vec{b}_a$ that may leed to the sensor output $\vec{c}_{\vec{b}_e} = \Omega^T_{\vec{b}_e}\vec{r}$ under recording illumination $\vec{b}_a$:

$$M^{\vec{c}}_{\mathit{CIEXYZ}} := \{A^T_{\vec{b}_a}\vec{r} \mid \forall\vec{r} : \vec{c}_{\vec{b}_e} = \Omega^T_{\vec{b}_e}\vec{r}\} \qquad (7)$$

If we use the well known tranformation $\mathcal{L}$: *CIEXYZ* $\mapsto$ *CIELab* [11], we can express this metameric subspace in a perceptually uniform color space:

$$M^{\vec{c}}_{\mathit{CIELab}} := \{\mathcal{L}(A^T_{\vec{b}_a}\vec{r}) \mid \forall\vec{r} : \vec{c}_{\vec{b}_e} = \Omega^T_{\vec{b}_e}\vec{r}\} \qquad (8)$$

Not all reflectances $\vec{r}$ that comply eq. (8) are necessarily physically reasonable. Assuming the refectances $\vec{r}$ to be smooth, non negative ($r_i \geq 0, i = 1, \ldots, n$) and bounded ($r_i \leq 1, i = 1, \ldots, n$), we can reduce the volume of the metameric subspace to realistic description of the color reproduction ability for the camera for a given sensor output $\vec{c}$.

In order to extract a illustrative error measure from the MBD method, we consider the average volume $\bar{\mathcal{V}} = \frac{1}{k}\sum_{i=0}^{k}\mathcal{V}(M^{\vec{c}_i}_{\mathit{CIELab}})$ of the metamer subspaces over the sensor responces $\vec{c}_i$ to a set of k representative reflectance spectra. The error measure τ_{CIELab} is defined as the radius of a globe with equal volume $\bar{\mathcal{V}}$:

$$\tau_{\mathit{CIELab}} = \sqrt[3]{\frac{3}{4\pi}\bar{\mathcal{V}}} = \sqrt[3]{\frac{3}{4\pi}\frac{1}{k}\sum_{i=0}^{k}\mathcal{V}(M^{\vec{c}_i}_{\mathit{CIELab}})} \qquad (9)$$

τ_{CIELab} may be interpreted as the mean color error ΔE_{ab} that is to be expected using the sensor with an optimal color correction. τ_{CIELab} is an illustrative FOM due to ΔE_{ab} beeing a well known error measure in the color imaging community.

Recording noise can easyly be integrated in eq. (2) by adding a vector of random variables $\vec{\epsilon}$: $\vec{c} = \Omega_{\vec{b}}{}^T\vec{r} + \vec{\epsilon}$. This leeds to a modified description of the metameric subspace:

$$M^{\vec{c}}_{\mathit{CIELab}} := \{\mathcal{L}(A^T_{\vec{b}_a}\vec{r}) \mid \forall\vec{r} : \vec{c}_{\vec{b}_e} = \Omega^T_{\vec{b}_e}\vec{r} + \vec{\epsilon}\} \qquad (10)$$

Whereas the noise ϵ_i is only bounded $-\Delta\epsilon < \epsilon_i < \Delta\epsilon$ but a priori unknown.

The metamer subspace is described in the MBD by storing points in the *CIELab* color space that are equally distributed over the surface of the metamer subspace. These points are calculated by solving linear optimization problems. We refer to the publications of Urban [9, 10] for futher details.

We can calculate the MBD FOM for several combinations of recording and viewing illuminations $(\vec{b}_{e_1}, \ldots, \vec{b}_{e_p})$ and $(\vec{b}_{a_1}, \ldots, \vec{b}_{a_q})$ respectively. The results may be presented in a quality matrix $\mathbf{T}$ as proposed in [7]:

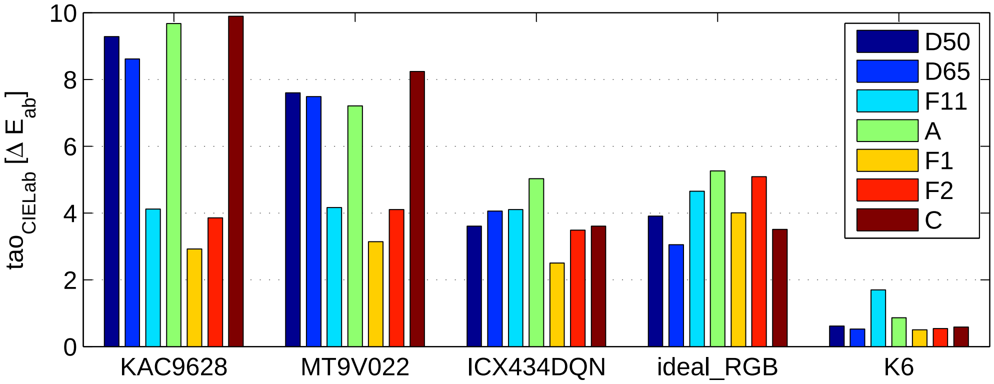


Figure 1: MDB FOM for viewing illumination D50, various recording illuminations and five different sensors

$$\mathbf{T} = \begin{bmatrix} \tau_{11} & \tau_{12} & \cdots & \tau_{1q} \\ \tau_{21} & \tau_{22} & \cdots & \tau_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ \tau_{p1} & \tau_{p2} & \cdots & \tau_{pq} \end{bmatrix} \quad (11)$$

where τ_{ij} is the quality measure defined in eq. (9) for recording illumination $\vec{b}_{e_i}$ and viewing illumination $\vec{b}_{a_j}$. We can even formulate a single quality factor using a weighted sum of the matrix elements:

$$\tau = \sum_{i=1}^{p}\sum_{j=1}^{q} w_{ij}\tau_{ij} \text{ while } \sum_{i=1}^{p}\sum_{j=1}^{q} w_{ij} = 1 \quad (12)$$

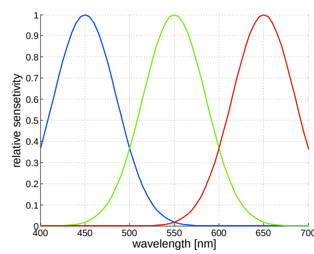
The weights w_{ij} may be chosen according to the importance of this specific combination of recording and viewing illumination for the application.

4 Results

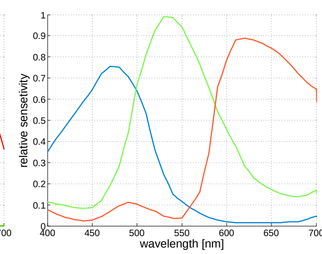
In order to evaluate the newly proposed FOM, we calculated the MBDs for five different sensors using the reflectances of the GretagMacbeth ColorChecker. The reflectances of the ColorChecker were used due to their well known representativeness of much larger spectral sets [8].

The spectral sensitivities of the sensors used are shown in figure 2. For comparison purposes we have included an "ideal" 3-channel sensor with Gaussian shaped spectral sensitivities as well as an experimental 6 channel sensor [12] denoted as K6. An example of a metameric subspace for the $8th$ field of the GretagMacbeth ColorChecker recorded and viewed under D50 illumination is shown in figure 3. The K6 sensor was used to generate this MBD.

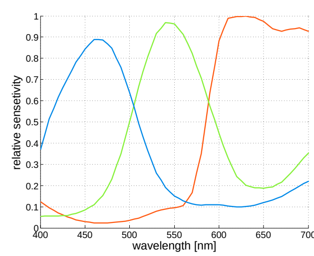
The results of the MBD FOM calculation for the five sensors are shown in figure 1. The different



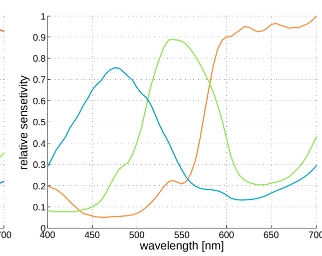
(a) "Ideal" RGB Sensor



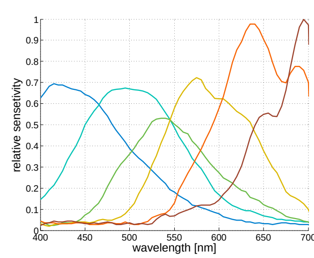
(b) Sony ICX434DQN Sensor



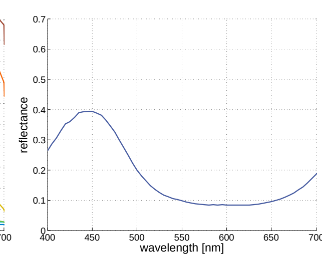
(c) Micron MT9V022 Sensor



(d) Kodak KAC9628 Sensor



(e) Experimental 6 channel sensor



(f) Reflectance spectrum of $8th$ ColorChecker field

Figure 2: Spectral sensitivies of the five sensors used for MBD FOM and example reflectance

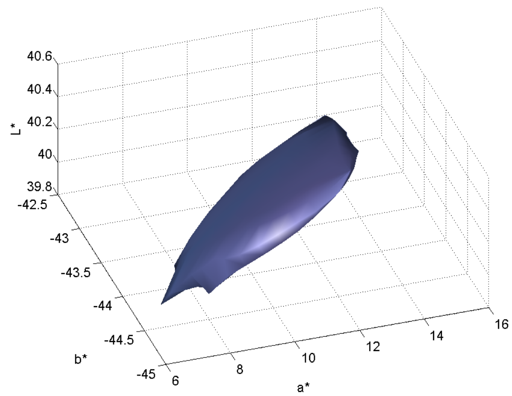


Figure 3: Example: MBD for the $8th$ field of the ColorChecker, using experimental 6 channel sensor (fig. 2) under illumination D50

τ_{CIELab} presented are all calculated using viewing illumination D50 and various recording illuminations.

We see that the resulting MBD FOM for the sensors differ by great amount. The color reproduction ability depends much on the combination of the recording and viewing illumination.

We can now compare the performance of sensors directly: Looking at recording illumination A for the sensors e.g. ICX434DQN and KAC9628, we can say that first sensor performs twice as good as the second one in terms of color reproduction ability. This is true due to τ_{CIELab} beeing defined in a perceptually uniform color space.

We can also clearly see the advantage in color reproduction ability of multiple channel sensors with a higher number of carefully chosen sensor channels in the example of the experimental 6 channel sensor K6 in figure 1.

An exemplary sample of a quality matrix $\mathbf{T}$ is given in table 1 for the Kodak sensor KAC9628.

		recording illumination						
		D50	D65	F11	A	F1	F2	C
viewing illumination	D50	9.28	8.61	4.12	9.67	2.92	3.85	9.89
	D65	9.53	8.79	4.26	8.86	3.17	3.88	10.22
	F11	11.00	10.35	2.45	10.36	6.51	6.32	12.09
	A	8.61	8.27	3.64	9.06	2.70	3.03	9.11
	F1	9.78	9.02	4.47	8.56	2.76	3.87	10.39
	F2	9.69	8.98	4.31	9.35	3.81	2.74	10.93
	C	9.65	8.73	4.34	9.05	2.95	3.75	9.80

Table 1: Quality matrix for sensor KAC9628 in ΔE_{ab}-units

The resulting unified quality factor $\tau = \sum_{i=1}^{p}\sum_{j=1}^{q} w_{ij}\tau_{ij}$ depends on the weights w_{ij} choosen for a specific application.

5 Conclusion

We presented a new figure of merit for color reproduction ability of digital imaging devices. This FOM is defined in a perceptual uniform color space, takes recording noise into consideration and quantifies the ability to reproduce colors independently of the color correction method used. The resulting τ_{CIELab} values are illustratively given in ΔE_{ab} units of the expected mean color error using a specific sensor with the best possible color correction function.

The results presented in this paper correspond to the experiences in color reproduction ability with the sensors used.

References:

[1] H. E. J. Neugebauer, Quality factor for filters whose spectral transmittances are different from the color mixture curves, and its application to color photography *J. Opt. Soc. Am. A.*, Vol. 46, No. 10, 1956, pp. 821–824.

[2] P.L. Vora and H.J. Trussell, Measure of goodness of a set of color-scanning filters *J. Opt. Soc. Am. A.*, Vol. 10, No. 7, July 1993, pp. 1499–1508.

[3] N. Shimano, Colorimetric evaluation of color image acquisition systems I *J. Electr. Photo. Japan*, Vol. 29, No. 5, 2000, pp. 506–516.

[4] N. Shimano, Colorimetric evaluation of color image acquisition systems II *J. Electr. Photo. Japan*, Vol. 29, No. 5, 2000, pp. 517–525.

[5] J. Tajima, New quality measures for a set of color sensors *IS&T/SID Conf. Color Science, Systems and Appl.*, Springfield, VA, 1996, pp. 25–28.

[6] G. Sharma and H.J. Trussell, Figures of Merit for Color Scanners *IEEE Trans. Img. Proc.*, Vol. 6, No. 7, July 1997, pp. 990–1001.

[7] S. Quan, N. Ohta, R.S. Berns and X. Jiang, Unified Measure of Goodness and Optimal Design of Spectral Sensitivity Functions *J. of Imaging Science and Technology*, Vol. 46, No. 6, 2002, pp. 485–497.

[8] M.J. Vhrel and H.J. Trussell, Color correction using principal components *Color Science and Application*, Vol. 17, No. 5, 1992, pp. 328–338.

[9] P. Urban and R.-R. Grigat, The Metameric Boundary Descriptor Method for Color Correction *Journal of Imaging Science and Technology*, Vol. 49, No. 5, 2005, pp. 418–430.

[10] P. Urban and R.-R. Grigat, Visualization and Quantity Estimation of Mismatch Gamuts of Metamers viewed under different Illuminants 10^{th} *Conf. Int. Colour Association*, 2005, pp. 827–830.

[11] G. Wyszecki and W.S. Stiles, Color Science: Concepts and Methods, Quatitative Data and Formulae, 2nd. ed. *New York: Wiley*, 1982.

[12] P.G. Herzog, D. Knipp, H. Stiebig and F. Koenig, Colorimetric characterization of novel multiple channel sensors for imaging and metrology, *J. of Electronic Imaging*, Vol. 8, No. 4, 1999, pp. 342–353.