Optimal design of AQM controllers

D.P. IRACLEOUS E. MANOLAKOS
Computer Science Department
IST Studies in association with University of Hertsfordshire
72 Pireos Str., 183 46, Moschato, Athens
GREECE

Abstract: - Active queue management (AQM) algorithms have been proposed to leverage TCP protocol inherent congestion algorithms which usually fail to maintain the queue length to an average level under large or bursting workloads. The main task of AQM algorithms is to detect network congestion at its early stages and start drop packets before the congestion affects the network throughput in a high degree.
In this paper, an new router controller is proposed based on optimal control theory and is compared to the most important AQM algorithms according to their functionality, characteristics and their parameters. Furthermore, illustrative simulation experiments are conducted to evaluate overall system performance queue length response over different number of flows at different time intervals. The proposed algorithm results significant improvement of performance of the network and faster respond to external loads.

Key-Words: - AQM, router models, network congestion, optimal control.

1 Introduction

Transport control protocol (TCP protocol) [1-2] tries to provide a reliable data transfer service for the internet. It is connection oriented, which means that before any data exchange between two parties take place, a connection first must be established.

The TCP protocol deals with congestion by implementing the Tail Drop algorithm (TD algorithm). According to the TD algorithm, a router discards packets only when a buffer overflow occurs, until there is again a free buffer space. The congestion window, which is included in every TCP connection, specifies a limit on a TCP sender transmission rate at a given time. The Additive Increase/ Multiplicative Decrease (AIMD) mechanism increments the congestion window by one packet per RTT (Round Trip Time) in case congestion is not presented (e.g. sender received the appropriate ACK packets). In case congestion is present, the congestion window is modified.

AQM algorithms are classified as reactive where decisions will take place according to current congestion and proactive where decisions take place on the expected congestion. Reactive algorithms decide whether to drop a packet or not according to average queue length, packet loss rate and link utilization, packet class, and control theory techniques.

Several control methods have been applied to the problem. Modeling of the system usually results to a time delay system model [3] due to the presence of the network.
In this paper a supervisory controller is proposed to maintain the queues of the controlled routers at a desired length size.

2 Problem formulation

Let the overall TCP system is modeled by a nxn, time invariant, multi-input, linear system described by the following

$$x(k+1) = Ax(k) + Bu(k) \tag{1}$$

where $x \in R^n$ is the state vector , $u \in R^m$ is the input vector and A, B are constant system and input matrices with dimensions nxn and nxm respectively. Applying on system (1) the state feedback control law

$$u(k) = Kx(k) \tag{2}$$

the following closed-loop form is obtained

$$x(k+1) = (A+BK)\, x(k). \tag{3}$$

The existing model describes the linear relations among the state variables W(t) and q(t) and input variable p(t)

$$\delta\dot{W} = \frac{2N}{C^2 R_0^2}\delta W - \frac{C^2 R_0^2}{2N}\delta p(t - R_0) \tag{4}$$

$$\delta\dot{q} = \frac{N}{R_0}\delta W \tag{5}$$

where W(t) (packets) is congestion window size, queue length q(t) around the equilibrium point , and packet dropping probability p(t).
Also N is number of TCP connections, C (packets/s) is the link capacity , and finally R_0 is the round-trip delay in seconds. The state variables (δW, δq) is perturbation from the equilibrium point (W_0, q_0).
RED algorithm [9] calculates the input p(t) to system (4) and (5). In this paper the RED controller is replaced by the proposed optimal regulator described in next section.

3 Controller design

A simple solution to the optimal eigenvalue assignment problem was developed. It is proven that for an n-order system with m independent inputs the problem is split into the two sequential stages where the first stage n-m eigenvalues are almost arbitrarily are assigned and the next stage the remaining m eigenvalues are assigned in such a way that linear quadratic optimal criteria are simultaneously satisfied.
The method is the discrete case problem [6] of the corresponding continuous case [5].
In the following, without loss of generality, we assume that the input matrix of system (1) has the following form

$$B = \begin{bmatrix} I_m \\ 0 \end{bmatrix}. \tag{6}$$

Also, the state matrix A and the feedback gain-matrix K be correspondingly decomposed as follows

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{7}$$

and K=[K1 K2]. (8)

In this case using theorem 2 [6] the feedback gain matrix is given as

$$K_1 = \left(S A_{22}^{-1} A_{21} - \frac{a}{a+1} \left(A_{11} - A_{12} A_{22}^{-1} A_{21} \right) \right) \left(I + X A_{22}^{-1} A_{21} \right)^{-1} \tag{9}$$

and

$$K_2 = S - K_1 X \tag{10}$$

where

- the arbitrary mx(n-m) matrix X assigns the (n-m) eigenvalues of the matrix $A_{22} + A_{21}X$
- the matrix S is given as

$$S = -A_{11}X + XA_{21}X + XA_{22} - A_{12} \tag{11}$$

- and a is a free tuning parameter, used to guarantee stability.

4 Illustrative example

Let a simple network of two routers. The overall system is described by a state vector defined as

$$x = \begin{bmatrix} \delta W_1 & \delta q_1 & \delta W_2 & \delta q_2 \end{bmatrix} \tag{12}$$

and

$$u = \begin{bmatrix} p_1 & p_2 \end{bmatrix} \tag{13}$$

where the subscript denotes the router number.
In the following a numeric example is given

$$A = \begin{bmatrix} 1.1 & 1 & -0.2 & 0.3 \\ 0.2 & 0.3 & -0.2 & -0.1 \\ -0.1 & 0 & 0.2 & 0 \\ 0 & 0.2 & 0 & 0.2 \end{bmatrix} \tag{14}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{15}$$

The input matrix has already the desired form of eq. (6). The system is open loop unstable with the following eigenvalues.

1.3352, 0.1500 ± 0.1936i, 0.1648 (16)

Choosing

$$X = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix} \tag{17}$$

$$S = \begin{bmatrix} 0.2 & -0.8 \\ 0.2 & 0.1 \end{bmatrix} \tag{18}$$

a=5 (19)

The closed loop system eigenvalues are:
0.2000, -0.0762, 0.3000 and 0.0929
which are obviously stable and a suitable quadratic index is satisfied [6].

5 Conclusion

The paper presents a computational simple algorithm for optimal eigenvalue assignment to linear models of network congestion systems.
In the next step of the research, the method will be validated using a detailed network model in a software package, like NS2[8].

References:

[1] RFC 793 - Transmission Control Protocol, IETF RFC 793.

[2] W. Stevens, TCP slow-start, congestion avoidance, fast retransmit, and fast recovery algorithms, IETF RFC 2001.

[3] C.V. Hollot ,V. Misra, D. Towsley and W.B. Gong, On designing improved controllers for AQM routers supporting TCP flows, *Proceedings of IEEE INFOCOM 2001, April 2001*, pp. 1726-1734.

[4] J. Aweya, M. Ouelette and D.Y. Montuno, A control theoretic approach to active queue management, *Computer Networks*, vol.36, 2001, pp. 203-235.

[5] Nooshin Bigdeli and Mohammad Haeri, AQM controller design for networks supporting TCP vegas: A control theoretical approach, *ISA Transactions*, Available online 30 July 2007, In Press.

[6] D.P. Iracleous, A.T. Alexandridis, "New results to the inverse optimal control problem for discrete-time linear systems", *Journal of Applied Mathematics and Computer Science*, Vol.8, No. 3, 1998, pp. 517-528..

[7] D.P. Iracleous, A.T. Alexandridis, "A simple solution to the optimal eigenvalue assignment problem", *IEEE Trans. on Automatic Control*, Vol. 44, No. 9, Sept. 1999, pp. 1746-1749.

[8] The Network Simulator – ns, http://nsnam.isi.edu/nsnam/index.php/Main_Page [Date Accessed: 23/7/2007].

[9] T.J. Ott, T.V. Lakshman and L. Wong, SRED: Stabilized RED, Proceedings of IEEE, *INFOCOM 1999*, March 1999, pp. 1346-1355.