# Multi-market Bidding Strategy of Power Suppliers

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*Abstract*: In electricity markets based on a sealed auction, power suppliers can develop optimal bidding strategies by estimating the bidding behaviors of their competitors. By application of combined order statistics, non-basic auction theory, and the Monte Carlo simulation method, the successful bidding probability of power suppliers is obtained. Based on the successful bidding probability and maximum expected profit, an optimum bidding strategy model is put forward. After that, considering the electricity volume and price in the contract market and the spot market, a Cournot Game model is used to get the bidding function of the power suppliers in the spinning reserve market. An example supports the validity of the bidding model.

*Key-Words:* Multi-market, Spinning reserve market, Bidding strategy, Game theory, Cournot model, Non-basic auction theory

## 1 Introduction

In recent years many scholars have studied the application of Game Theory to the bidding strategy of power suppliers. Based on a game theoretic matrix model, Ferrero denotes the candidate's bidding strategy as a discrete quantum. When the bidding strategy features a few discrete points, a matrix of power suppliers' revenue can be structured when they adopt different mixed strategies, and then a balanced mixed bidding strategy can be found which corresponds to the optimum bidding strategy [1]. Kwang-Ho establishes a revenue matrix reflecting many power suppliers' participation in bidding and having a mixed strategy equilibrium solution [2]. Ferrero describes competition in a power pool as a problem of non-cooperative game theory. Each participant has only incomplete information and knows only its own operating cost and nothing of its opponents'. The problem is solved through a Nash equilibrium and translated into an imperfect-information game, and the optimum price is at the Nash equilibrium point [3]. Considering transmission constraints in a power pool market, Correia establishes a Bayesian equilibrium model under incomplete and non-symmetric information and finds the optimum bidding strategy solution through a Monte Carlo model [4]. Hou Jingzhou puts forward a stochastic model of bidding strategy and establishes the revenue matrix by considering the incompleteness of market information and estimating the opponents' lowest bid. Adopting a min/max decision rule, the model directly chooses the optimum bidding strategy for a single period from the revenue matrix, reducing the status number [5]. Torre proposes a three-step method to get a Nash equilibrium solution of power suppliers' bidding [6]. Wu Zhiyong puts forward a math model of optimum bidding by power suppliers, and gives a solution based on game theory and probability theory, which solves many tough multi-person game and incomplete information problems by introducing the virtual opponent and the method of parameter estimation [7]. Chen Qi-an establishes a game theoretic bidding strategy model of power suppliers under relatively wide assumption. The research shows that when electricity demand volume in the market is less than or equals to the amount of electricity offered into the grid, the optimum bidding by power suppliers will slightly fluctuate between the unit generation cost and the price cap in the power market [8]. Hao has supposed that each power supplier offers the same amount of electricity, and uses auction theory to obtain a bidding game model of power suppliers. However, because in a real electricity market the bidding volume is far larger than one unit and the power generator has incomplete information on the costs of its opponents in the bidding game, the optimum bidding strategy is even more complicated [9]. Adopting first-price sealed-bid auction theory, Ren Yu-long studies an incomplete information bidding strategy model of power suppliers when their bidding volume varies and the contract market is considered. The model takes into account unit cost, the probability of successful bidding, electricity contract volume, contract price and expected market-clearing price [10]. Liu Ya-an analyzes the correlation between unit profit, electricity clearing volume and bidding strategy coefficient, and gets the optimum bidding strategy of a single unit and the optimum readjustment method when bidding goes beyond that point, and also proposes a model of a joint bidding strategy between different units of a single power supplier[11].

Power suppliers have a big impact on the clearing price the more they have the characteristics of a monopoly. In that case the winning price is bid upward by them to obtain monopoly profit. In the mechanism of a non-basic auction, the market supervisor defines the benchmark bidding range prior to bidding in order to reduce the market power of electricity suppliers. So the theory of non-base auction is applied to analyze bidding strategies in this paper.

Combined order statistics, non-basic auction theory, and Monte Carlo simulation method are applied to obtain the probability of successful bidding by power suppliers. Bases on the probability of successful bidding and the maximum expected profit, an optimum bidding strategy model is put forward. After that, considering the electricity volume and price in both the contract market and the spot market, a Cournot game model is used to get the bidding function of power suppliers in the spinning reserve market.

## 2 The Spot Market Bidding Strategy Model

### 2.1 Bidding Probability in a Non-Basic Auction System

In a non-basic auction system, it is supposed that the expected value of bidding is based on an approximately normal distribution, and the bidding must be the function of the expected value. It is supposed that the bidding function is a linear transformation. Suppose $b_{-i}$ denotes the bids by other bidders, and $b_{-i} \sim N(u_{-i}, \sigma_{-i}^2)$.

Suppose bid $b$ is at the price level $k$, that is, $b > b_i, i = 1,2,...,k-1$ ,and $b < b_j$ , $j = k+1, k+2,..., N$ . When $(1-k)\bar{b} \le b \le (1+k)\bar{b}$ , the bid is valid. According to the optimum solution, all lower bids are invalid, namely, $b_i < (1-k)\bar{b}, i = 1,2,...,k-1$ . In this case, the probability of successful bidding is obtained by:

$$\begin{aligned} h_k(b) = \Pr((1-k)\bar{b} \le b \le (1+k)\bar{b}, \\ b_i < (1-k)\bar{b} \big| b > b_i, b < b_j) \\ i = 1,2,...,k-1,\ j = k+1, k+2,..., N \end{aligned} \quad (1)$$

The probability that bidding $b$ is at price level $k$ is calculated by:

$$g_k(b) = C_{N-1}^{k-1} \cdot [F(b)]^{k-1}[1 - F(b)]^{N-k} \quad (2)$$

When bid $b$ is at the price cap, the probability $h_N(b) = 0$ .

As the above cases are independent, the probability of successful bidding can be obtained by:

$$P(b) = \sum_{k=1}^{N} h_k(b) g_k(b) \tag{3}$$

## 2.2 Basic Assumption of the Model

In a single-clearing-price power exchange with $n$ power suppliers, the bidding strategy of supplier $i$ is analyzed. If $c_i$ is the unit cost of power supplier $i$, then $c_i$ is the lowest unit price for electricity that power supplier $i$ is ready to offer. $b_i$ is its bid, and the market clearing price is $w$. We assumed that all participating power suppliers are rational, and other power suppliers don't know the value of $b_i$, but know only that $b_i$ is a random variable drawn from a distribution over [$b_1$ □ $b_2$] whose density function is defined by $f(b)$.

## 2.3 Probability of Successful Bidding with Incomplete Information

### 2.3.1 Probability of Unsuccessful Bidding

If power supplier $i$ fails bids unsuccessfully, while $m$ suppliers are successful, then the bids by at least $m$ power suppliers are lower than $b_i$. As the transaction volume of each power supplier and the electricity demand in the corresponding period are known, $m$ can be calculated.

Let $L_i(b_i)$ be the probability that a bidder fails in the auction. This is a cumulative binomial distribution function:

$$\begin{cases} L_i(b_i) = \sum_{j=1}^{m} h_j(b_i) g_j(b_i) \\ \gamma q_m + \sum_{k=1}^{m-1} q_k = Q \end{cases} \tag{4}$$

where $\gamma (0 < \gamma \le 1)$ denotes the proportion of the electricity bid by $m$ that is dispatched to the grid by $m$.

### 2.3.2 Probability of Successful Bidding

(1) If $b_i = w$, where $w$ is the highest winning bid.

In this case, the volume of electricity dispatched to the grid at $w$ may not equal the total volume bid, and the volume dispatched to the grid has several possible values. Suppose $\eta_j (j = 1, ..., m)$ is the proportion of the electricity bid at $j$ that was actually dispatched to the grid; that is, $q_{ig} = \eta_j q_i$; so, if $x_j$ power suppliers bid successfully, then there are $x_j - 1$ power suppliers with bids lower than $b_i$; that is, power supplier $i$ is at the price level of $x_j$.

If $b_i = w$, suppose $H_i^j(b_i)$ is the probability of $\eta_j$, which is calculated as follows:

$$\begin{cases} H_i^j(b_i) = h_{x_j}(b_i) g_{x_j}(b_i) \\ \eta_j q_i + \sum_{k=1}^{x_j - 1} q_k = Q \end{cases} \tag{5}$$

Then, if supplier $i$ is successful in bidding, the successful bidding probability of $H_i(b_i)$ is obtained by:

$$H_i(b_i) = \sum_j H_i^j(b_i) \tag{6}$$

(2) If $b_i < w$

Suppose $R_i(b_i)$ is the probability of successful bidding, and $b_i < w$. According to the relations between the probabilities in the three cases, we get:

$$R_i(b_i) = 1 - L_i(b_i) - H_i(b_i) \quad (7)$$

## 2.4 Bidding Strategy Model

The payoff for the bidder is $q_i[\eta b_i - c_i + d_i q_i (1-\eta^2)/2]$, if $b_i$ wins the auction but not on the margin, and is $q_i[w - c_i]$ if bid $b_i$ is on the margin. Let $u_i$ be the revenue function of power supplier *i*, and

$$u_i(b_i, w, c_i) = \begin{cases} q_i[w-c_i] & if b_i < w \\ q_i[\eta b_i - c_i + d_i q_i (1-\eta^2)/2] & if b_i = w \\ 0 & if b_i > w \end{cases} \quad (8)$$

The common objective for bidders is to maximize their expected payoffs. Assuming the winning price is $w$, the payoff for Bidder $i$ can be seen from(8).Thus the expected payoff function is the sum of above two terms weighted by their probabilities of occurrence:

$$\begin{aligned} \max \pi_i(b_i) &= q_i(w - c_i) \cdot R_i(b_i) + \\ & q_i[\eta b_i - c_i + d_i q_i (1-\eta^2)/2] \cdot H_i(b_i) \end{aligned} \quad (9)$$

At the optimal solution for the bid $b_i$, the following necessary differential condition should be satisfied:

$$\begin{aligned} \frac{d\pi_i(b_i)}{db_i} &= \eta H_i(b_i) + (w - c_i)\frac{dR_i(b_i)}{db_i} + \\ & [\eta b_i - c_i + d_i q_i (1-\eta^2)/2]\frac{dH_i(b_i)}{db_i} = 0 \end{aligned} \quad (10)$$

Denoting ( $dH_i(b_i)/db_i$ ) and ( $d\mathrm{R}_i(b_i)/db_i$ ) as $H'$, $R'$, and suppressing the subscripts, we can simplify (10) as follows:

$$\eta H + \eta b H' = c(H' + R') - \frac{dq(1-\eta^2)}{2} H' - wR' \quad (11)$$

Integrating (11) over the price range from $b$ to $b_2$ yields the following:

$$\begin{aligned} \eta b H = \int_b^{b_2} (H' + R')c dc - \frac{dq(1-\eta^2)/2}{2} \times \\ [H(b_2) - H(b)] - w[R(b_2) - R(b)] \end{aligned} \quad (12)$$

The probability of winning at or below the margin is zero for the highest possible bid $b_2$. Thus the following boundary conditions at $b = b_2$ exist:

$$H_i(b_i = b_2) = R_i(b_i = b_2)$$

Applying the integration-by-parts formula and the boundary condition, we obtain the following result from (12)

$$b = \frac{dq(1-\eta^2)}{4\eta} + \frac{wR(b)}{\eta H(b)} - \frac{c[H(b) + R(b)]}{\eta H(b)} \quad (13)$$

## 3 Model of Bidding in the Reserve Market

When the electricity volume of the spot market and contract market are calculated, power supplier *i* can calculate the available spinning reserve. Assuming that the dispatched-reserves proportion is $\Psi$, and that the volume and price in the contract market of power supplier $i$ are $q_f$ and $p_f$ respectively, $q_g$ and $q_g$ and $p_g$ can be determined beforehand. $q_{i\max}$ and $q_{i\min}$ are the maximum and minimum output of power supplier *i*. In this case, assuming the inverse demand function of the reserve market is a linear function, we can calculate it by:

$$p_R = x - m(\sum_{i=1}^{N} Q_i) \quad (14)$$

where $p_R$ is the price in the reserve market, $x$ and $m$ are the intercept and slope of the inverse demand function in the spinning reserve market, $Q_i$ is the reserve capacity of power supplier $i$ in the spinning reserve market, and $N$ is the number of power suppliers.

Considering the impact of the spot market and the contract market on the spinning reserve market, the maximum profit of power supplier $i$ is calculated by:

$$\max \pi_i = q_g p_g + q_f p_f + Q_i p_R - c_i(q_g + q_f + \Psi Q_i) \quad (15)$$
$$s.t \quad q_{i\min} \le q_g + q_f \le q_{i\max} - Q_i$$

The profit-maximizing condition is:

$$\frac{\partial \pi_i}{\partial Q_i} = \frac{\partial p_R}{\partial Q_i} Q_i + p_R - \frac{\partial c_i(q_g + q_f + \Psi Q_i)}{\partial Q_i} = 0 \quad (16)$$
$$s.t \quad q_{i\min} \le q_g + q_f \le q_{i\max} - Q_i$$

Suppose

$$c_i(q_i) = a_i + d_i q_i^2 / 2 \quad (17)$$

where $a_i$ and $d_i$ represent the cost coefficients of generating sets. By formulae (14), (16), and (17), we get:

$$-mQ_i + p_R - \Psi(a_i + d_i(q_g + q_f + \Psi Q_i)) = 0 \quad (18)$$
$$s.t \quad q_{i\min} \le q_g + q_f \le q_{i\max} - Q_i$$

Then the price in the spinning reserve market is obtained by rearranging formula (18):

$$p_R = mQ_i + \Psi(a_i + d_i(q_g + q_f + \Psi Q_i)) \quad (19)$$

# 4 Example

To facilitate the calculation, all power suppliers are assumed to predict their rivals' bidding price with uniform distribution in the over the interval [9$/(MWh),21$/(MWh)], and the contract market is not considered. Assuming there are three power suppliers in the market, the parameters are shown in Table 1. The load is 250 MW; therefore, with $n = 3$, $x_j = 2$.

Table 1 Parameters of generating sets

| supplier | $a$ | $d$ | $q_{\min}$ (MW) | $q_{\max}$ (MW) |
|---|---|---|---|---|
| 1 | 3.5 | 0.05 | 50 | 200 |
| 2 | 4 | 0.04 | 50 | 150 |
| 3 | 5 | 0.03 | 50 | 150 |

Suppose $w = 15$ $/(MWh); the optimal output and the corresponding bidding prices can be calculated by formula (13) in the respective bidding markets. Data and the results are shown in Table 2.

Table 2 Parameters in the spot markets

| supplier | $R(b_i)$ | $H(b_i)$ | $q_g$ (MW) | $p_g$ $/MWh |
|---|---|---|---|---|
| 1 | 0.2675 | 0.5001 | 111.87 | 13.91 |
| 2 | 0.2998 | 0.4897 | 128.96 | 13.23 |
| 3 | 0.3547 | 0.4765 | 141.87 | 12.49 |

The bids into the spinning reserve market are calculated by formula (19) .The results are shown in Table 3.

Table 3 Bidding parameters in the reserve market

| suppliers | $\Psi$ | $x$ | $m$ | $p_R$ $/MWh |
|---|---|---|---|---|
| 1 | 0.32 | 25 | 0.02 | 18.59 |
| 2 | 0.32 | 25 | 0.02 | 10.22 |
| 3 | 0.32 | 25 | 0.02 | 9.91 |

# 5 Conclusion

Bidding strategy models of the spot market and the spinning reserve market based on incomplete information are put forward, the models are applied, and the bids of power suppliers in the current spot market and the spinning reserve market are calculated. Power suppliers can calculate the volume and price in the spot market and reserve market by the models so as to gain more profit. At the same time, the bidding method in the spinning reserve market is of practical significance to guarantee the security and reliability of the electric system, because it ensures that power suppliers get a rational profit rate from the spinning reserve market.

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