A Design of the Matched Filter for the Passive Radar Sensor

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Abstract: - This paper discusses a method of designing a matched filter for a passive microwave sensor. This sensor receives pulse signals transmitted by a target active radar. Received signals can be categorized into two types due to either the pulse signals or noise. The method first estimates an autocorrelation function of the received signals. The autocorrelation of the pulse signals is then extracted from the autocorrelation of the received signals by exploiting the difference between the statistics of the pulse signals and the noise. A power spectrum of the pulse signals is obtained by taking discrete Fourier-transform of its autocorrelation. The matched filter is designed from the knowledge of this power spectrum. The spectrum of the matched filter is identical to the spectrum of the pulse signals transmitted by the radar, and then signal-to-noise ratio (SNR) is improved by matching the filter with the pulse signals. Performance of the proposed method is theoretically analyzed in term of SNR. Moreover, simulations verified that the proposed method correctly designs the matched filter and achieves expected performance.

Key-Words: - signal estimation, stochastic process, pulse signal, matched filter, signal-to-noise ratio, passive sensor

1 Introduction

A passive microwave sensor is used for finding a direction with a target active radar which transmits pulses. This sensor system receives radio waves, and then distinguishes the arriving pulse signals, that are transmitted by the target active radar, from received radio waves by comparing received pulse signals and the information such as a carrier frequency and a pulse reputation interval (PRI) collected in advance [1].

In order to discriminate arriving signals from received signals, it is necessary to decrease noise components. Most techniques for improving signal-to-noise ratio (SNR) in radar signal processing cannot apply to the passive sensor because a statistic of the pulse signals transmitted by the target active radar is unknown. An integration processing is used in the passive sensor even though a statistic of arriving signals is unidentified.

A conventional matched filter in a radar receiver is designed based on the information of radar's transmitting pulse waveform. Its spectrum is identical to a spectrum of the radar's transmitting pulse waveform [2]. The conventional matched filter technique dose not apply for the passive sensor because the waveform of arriving signals is unknown.

This paper proposes a new method of designing a matched filter for the passive radar sensor. This filter is designed from only the received signals transmitted by the target active radar. The proposed method first estimates an autocorrelation function of received signals. The autocorrelation of pulse signals is then extracted from the autocorrelation of the received signals by exploiting the difference between the statistics of the pulse signals and noise. A power spectrum of the pulse signals is obtained by taking discrete Fourier-transform (DFT) of its autocorrelation, and then a matched filter is designed from the knowledge of the power spectrum of the pulse signals. A spectrum of the matched filter is identical to the spectrum of the pulse signals transmitted by the target active radar, and the filter improves SNR by matching the pulse signals [3].

This paper is organized as follows. In Chapter 2, the passive radar sensor system is briefly explained and the received signals are formulated as a stochastic process. Chapter 3 describes the proposed methods of estimating the power spectrum of the pulse signals, and of a procedure for exploiting the estimated power spectrum for increasing SNR. In Chapter 4, the performance of the proposed method is theoretically analyzed in term of SNR using a Gaussian pulse waveform for pulse signals, and compared with the integration processing as a conventional method. Additionally, the proposed method is evaluated via computer simulation.

2 Received signals on the sensor system

2.1 Representation on a stochastic process

The conventional radar system comprises of a transmitter and a receiver, while the passive sensor system is only composed of a receiver. This sensor receives unidentified arriving signals and discriminates target pulse signals by comparing the target information collected in advance [1]. Figure 1 shows a block diagram of the passive sensor system. The arriving carrier frequency signals transmitted by the target radar are first received via a wide-band receiving antenna. The received carrier frequency signals are then converted into a base-band frequency at a wide-band demodulator based on the information on the radar's carrier frequency, and then the base-band frequency signals are sampled. The sampled discrete signals are multiplied by a series of windows. A matched filter is designed from the window output by using the proposed method, and then the received signal norm at the matched filter output is calculated.

Figure 2(a) shows sampled pulse waveforms of arriving signals. A pulse waveform of sampled signals repeats with the radar's PRI denoted as T. Figure 2(b) shows a series of windows to be multiplied to the sampled signals. The duration of each window is set at the time T corresponding to the radar's PRI, and the window multiplication divides the received samples into blocks of M samples.

The sampler samples the base-band signals with a time interval T/M, and produces M samples during the

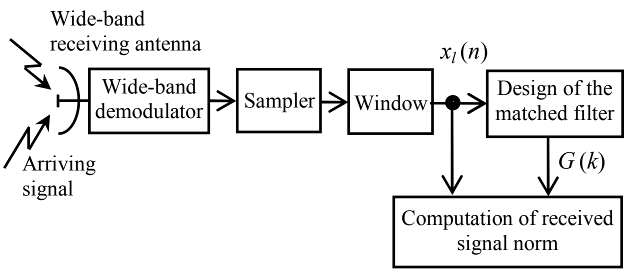


Fig. 1. A block diagram of the passive sensor system.

time T. If the samples are denoted as $x(n)$, the output of lth window is written as

$$x_l(n)=x(lM+n),\ 0\leq n\leq M\text{-}1. \quad (1)$$

The window output is arranged in the M-dimensional vector as

$$\mathbf{x}_l=[\ x_l(0)\ \ x_l(1)\ \ \cdot\ \cdot\ \cdot\ \ x_l(M\text{-}1)]. \quad (2)$$

We take L of these vectors and a numerical average of the vectors as

$$\mathbf{y}=\left[y(0)\quad y(1)\quad \cdots \quad y(M-1)\right]=\frac{1}{L}\sum_{l=0}^{L-1}\mathbf{x}_l\ . \quad (3)$$

Received signals acquired by the passive sensor are either the arriving signals or the noise, and then the arriving signals are expressed by the sum of the pulse signals and additive noise [4,5,6]. Received signals can be categorized into two types due to either the pulse signals or the noise, and $\mathbf{y}_T$ and $\mathbf{y}_N$ are assumed as stochastic processes corresponding to received signals due to the target pulse signals and the noise respectively [5,6]. Therefore $\mathbf{y}$ can be regarded as a stochastic process. When L is large enough, $\mathbf{y}$ approaches to

$$\mathbf{y}=P_T\mathbf{y}_T+P_N\mathbf{y}_N, \quad (4)$$

where P_T and P_N are probabilities of occurrence of the pulse signals and the noise.

2.2 Power spectrum of pulse signals

The autocorrelation function of $\mathbf{y}$ is defined as

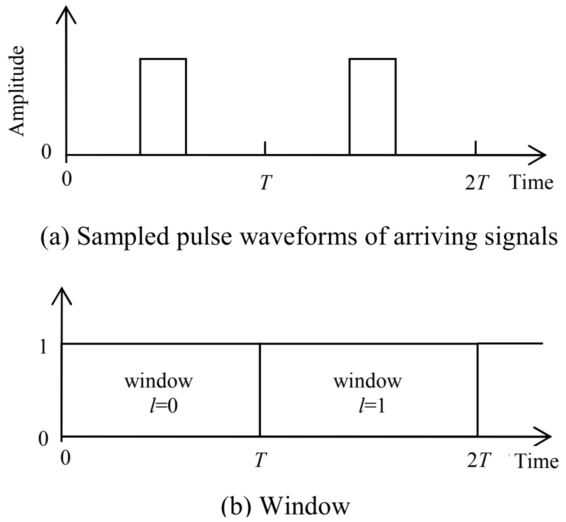


Fig. 2. Arriving signals and window function.

$$R_y(\tau)=E\{y(n)y(n+\tau)\},\ -M/2 \le \tau \le M/2-1, \qquad (5)$$

where $E\{\cdot\}$ denotes the expectation operation [5]. Because the length of the vectors is finite, the autocorrelation function depends on the variable n. However, to avoid complexity in the following analysis, we concede that y is stationary, and that the autocorrelation function given by (5) is well-defined. The power spectrum of the process is defined as the M-point DFT of the autocorrelation function,

$$S_y(k)=\sum_{\tau=-M/2}^{M/2-1} R_y(\tau)\exp(-j2\pi\tau k/M),\ 0 \le k \le M-1. \qquad (6)$$

Based on the probability theory [5,6], the proposed method assumes the followings;

hypothesis 1: the processes $\mathbf{y}_T$ and $\mathbf{y}_N$ are statically independent,

hypothesis 2: the process $\mathbf{y}_N$ is zero mean white Gaussian.

By (4) and the above hypotheses, an autocorrelation function of received data is obtained as

$$R_y(\tau)=P_T^2 R_T(\tau)+P_N^2\sigma_N^2\delta(\tau), \qquad (7)$$

where $R_T(\tau)$ is the autocorrelation function of the pulse signal $\mathbf{y}_T$, σ_N^2 is the variance of the process $\mathbf{y}_N$, and $\delta(\tau)$ is the unit impulse function. From the interpretation of Eq.(7), the autocorrelation function of the received signals $R_y(\tau)$ is given as

$$R_y(\tau)=P_T^2 R_T(\tau),\ \tau \neq 0. \qquad (8)$$

This means that the shape of $R_T(\tau)$, $\tau \neq 0$ is identical to the shape of $R_y(\tau)$, $\tau \neq 0$ apart from the constant multiplication by P_T^2. The value of $R_T(\tau)$ at $\tau=0$ can be estimated from its known neighbors in $R_y(\tau)$ by using the Lagrange interpolation polynomial explained at the next section [7]. The power spectrum of the pulse signals $S_T(k)$ is estimated by taking DFT of the estimate of $R_T(\tau)$.

3 Processing in the sensor system

3.1 Estimation of the power spectrum of the pulse signal

In order to proceed with estimating the power spectrum of the pulse signals described at the end of the previous chapter, we first compute the $M \times M$ matrix $\mathbf{B}$ from the received signals by

$$\mathbf{B}=\frac{1}{L}\begin{bmatrix} \mathrm{x}_0^T \\ \mathrm{x}_1^T \\ \vdots \\ \mathrm{x}_{L-1}^T \end{bmatrix}^T \begin{bmatrix} \mathrm{x}_0 \\ \mathrm{x}_1 \\ \vdots \\ \mathrm{x}_{L-1} \end{bmatrix}, \qquad (9)$$

where the superscript ^{T} denotes the transpose operation. Evidently, the matrix $\mathbf{B}$ approximates an autocorrelation matrix of received signals when L is sufficiently large. The (m,n)th entry of this matrix is given by

$$b_{m,n}=\frac{1}{L}\sum_{l=0}^{L-1} x_l(m)x_l(n)\cdot \qquad (10)$$

The autocorrelation of the received signals $R_y(\tau)$ is approximated by exploiting these entries. From (7), the autocorrelation of the pulse signals $R_T(\tau)$ when $\tau \neq 0$ is estimated from $R_y(\tau)$, but $R_T(0)$ is not directly estimated. Accordingly, we use the second order Lagrange interpolation polynomial to estimate $R_T(0)$ from its known neighbors in $R_y(\tau)$ [7]. Using the values of $R_y(\tau)$, $\tau=1,2,3$, on the interpolation polynomial [7], the autocorrelation $R_T(0)$ is obtained as

$$R_T(0)=3R_y(1)-3R_y(2)+R_y(3). \qquad (11)$$

Once $R_T(\tau)$ is given, the power spectrum of the pulse signals $S_T(k)$ is estimated by taking DFT of the estimate of $R_T(\tau)$.

When there is no pulse signal or when the number of pulse signals is not enough within L windows, the method fails to estimate the autocorrelation of the pulse signals, and then the power spectrum of the pulse signals can not be accurately estimated. This dependence of the estimate on the number of pulse signals is analyzed in Chapter 4 via the computer simulation.

3.2 Computation of the received signal norm

After the power spectrum of the pulse signals is estimated, it is now ready to process the received signals to improve SNR using the estimated power spectrum $\hat{S}_T(k)$. We prepare the matched filter $G(k)$ given as

$$G(k)=\sqrt{\frac{M}{\sum_{m=0}^{M-1}\left|\hat{S}_T(m)\right|}}\sqrt{\left|\hat{S}_T(k)\right|}\,. \tag{12}$$

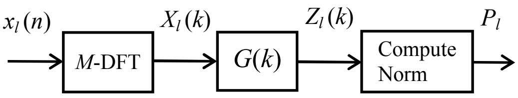


Fig. 3. Steps for computing the received signal norm.

The matched filter $G(k)$ is normalized so that the sum of $|G(k)|^2$, $0 \le k \le M$-1, becomes M. Using the matched filter $G(k)$, we perform the steps described in Fig. 3. The M-point DFT $X_l(k)$, $0 \le k \le M-1$, of $x_l(n)$, $0 \le n \le M-1$, is computed, and then each DFT value is multiplied by the matched filter $G(k)$ to obtain its output $Z_l(k)$ as

$$Z_l(k)=G(k)X_l(k),\ 0 \le k \le M-1. \tag{13}$$

Finally, the norm of this matched filter output is calculated by

$$P_l=\frac{1}{M}\sum_{k=0}^{M-1}\left|Z_l(k)\right|^2\,. \tag{14}$$

When the norm P_l takes a larger value, the received signals at the time of window l are regarded as the arriving signals, and then this signals are distinguished based on the target radar information.

4 Evaluation of the method

4.1 Improvement ratio of SNR

The received signal $x_l(n)$ can be regarded as either the pulse signal or the white Gaussian noise. According to this observation, in the place of P_l, we assign two random variables,

$$Q_i=\frac{1}{M}\sum_{k=0}^{M-1}\left|G(k)Y_i(k)\right|^2,\ i=T,N, \tag{15}$$

where $Y_T(k)$ and $Y_N(k)$ are the M-point DFTs of $y_T(n)$ and $y_N(n)$ respectively. Therefore Q_T and Q_N are the norm of the matched filter output. Evidently, phase components of the matched filter do not affect the computation of the norm. From the hypothesis 2 and the relation of $E[|Y_N(k)|^2]=M\sigma_N^2$, the mean of the noise norm is given by

$$E\{Q_N\}=\sigma_N^2\sum_{k=0}^{M-1}\left|G(k)\right|^2\,. \tag{16}$$

From the relation of $S_T(k)=(1/M)E[|Y_T(k)|^2]$, the mean of the pulse signals norm is given by

$$E\{Q_T\}=\sum_{k=0}^{M-1}\left|G(k)\right|^2 S_T(k)\,. \tag{17}$$

For evaluating the method, it is necessary to compare SNRs in the cases of which the matched filter is used and not used. When the matched filter is used, SNR is given as

$$SNR=\frac{E\{Q_T\}}{E\{Q_N\}}=\frac{\sum_{k=0}^{M-1}\left|G(k)\right|^2 S_T(k)}{\sigma_N^2\sum_{k=0}^{M-1}\left|G(k)\right|^2}\,. \tag{18}$$

Substituting $G(k)=1$ into (18), when the matched filter is not used, SNR is given by

$$SNR_0=\frac{\sum_{k=0}^{M-1}S_T(k)}{M\sigma_N^2}\,. \tag{19}$$

We analyze the performance of the method by the improvement ratio of SNR of (18) to the SNR_0 defined by

$$R_{SNR}=\frac{SNR}{SNR_0}=\frac{M\sum_{k=0}^{M-1}\left|G(k)\right|^2 S_T(k)}{\sum_{k=0}^{M-1}\left|G(k)\right|^2\sum_{k=0}^{M-1}S_T(k)}\,. \tag{20}$$

This method works as long as the hypothesis 1 and 2 are satisfied, and does not require a specific statistical model for a pulse signal. Because the rectangular pulse shape, which is the simplest waveform, is not used in practice [2], we employ the Gaussian pulse waveform to analyze the performance in followings [2,8]. The Gaussian pulse waveform is given by

$$y_T(n)=\exp\left(-n^2/2d^2\right), \tag{21}$$

where d is a positive constant. The power spectrum of $y_T(n)$ is given by

$$S_T(k)=|Y_T(k)|^2. \tag{22}$$

DFT $Y_T(k)$ of $y_T(n)$ is shown in Fig. 4. As the constant d increases, the bandwidth of $Y_T(k)$ decreases. The constant d controls the shape of the power spectrum of the pulse signals.

When the power spectrum of the pulse signal is accurately estimated, the power spectrum of the matched filter $|G(k)|^2$ is identical to the power spectrum of the pulse signal $S_T(k)$. From the relation of (22), substituting the square of DFT $Y_T(k)$ of $y_T(n)$ in (21) to $|G(k)|^2$ and $S_T(k)$ in (20), the ratio R_{SNR} is calculated, and shown as a function of d in Fig. 5.

The ratio is compared with the integration processing as a conventional method. For the integration processing, the improvement ratio of SNR at 10 windows is computed. The improvement ratio of this conventional method is also exhibited in Fig. 5.

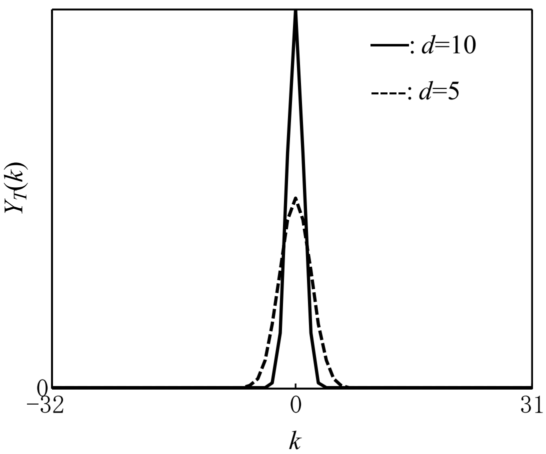
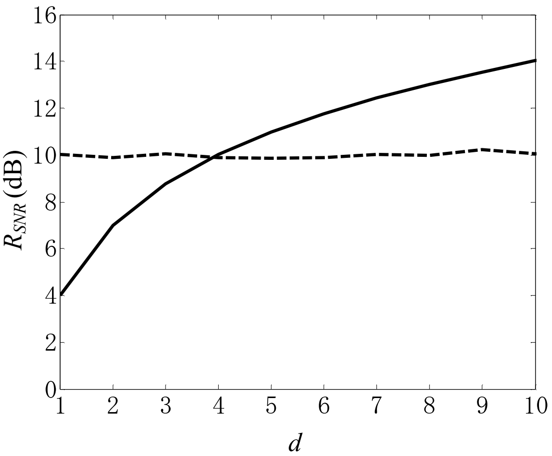


Fig. 4. DFT $Y_T(k)$ of the pulse signals.



—— : The proposed method
---- : The integration processing

Fig. 5. R_{SNR} as functions of d.

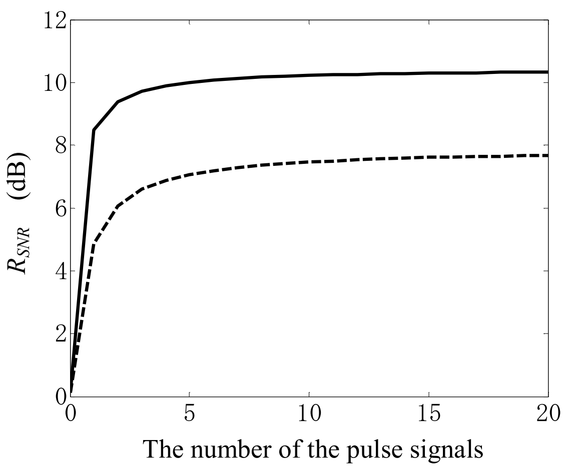
The ratio R_{SNR} increases as d increases, which implies the decrease of the spectrum bandwidth of the pulse signal. When the constant d is more than 4, the performance of the designed matched filter exceeds the conventional method, assuming that the power spectrum of the pulse signal of the pulse signal is perfectly estimated.

4.2 Simulation

When the number of pulse signals within L windows is large enough, the estimate of the power spectrum of the pulse signals becomes accurate. However the performance accordingly degrades when the number is small. Simulations are performed to see how many pulse signals are needed to acquire a desirable performance.

Table 1. Simulation parameters.

The value d of the pulse signal	5 and 10
Received signals, $L \times M$	100×64 matrices
The number of the pulse signals	0-20



—— : The proposed method when d = 10
---- : The proposed method when d = 5

Fig. 6. R_{SNR} by the proposed method.

Parameters used in the computer simulation are listed in Table 1, and also the Gaussian pulse waveform in (21) is used for the present simulations. The arriving signals are produced by the sum of $y_T(n)$ and the random number with zero-mean white Gaussian. The noise signals are created as the random number with zero-mean white Gaussian. Figure 6 shows the ratio R_{SNR} as a function of the number of the pulse signals in L windows.

As seen from the figure, R_{SNR} improves as the number of the pulse signals increases, and saturates after the certain number of pulse signals. The ratio R_{SNR} improves by increasing the constant d as expected from Fig. 5. When the pulse signals number is zero, the ratio is zero because such SNR is equal to the case when the matched filter is not used at all. When the number of pulse waveform signals is not large enough, the ratio does not reach its value given by (20) because the power spectrum of the pulse signals is not accurately estimated. If this is the case, the problem is eased by increasing the number L of the windows.

5 Conclusion

This paper has introduced the method of designing the matched filter for the passive radar sensor under the assumption that received signals are categorized into two types due to either the pulse signals or the noise. This method first estimates the autocorrelation of the received signals. The autocorrelation of the pulse signals is extracted by exploiting the difference between the statistics of the pulse signals and the noise. The estimate of the power spectrum of the pulse signals is obtained by taking DFT of its autocorrelation. The matched filter is designed from the knowledge of the power spectrum of the pulse signals. The conventional matched filter method has not been applied for the passive sensor because the target pulse signals are unknown, however the proposed method estimates the target pulse signals from the received signals and then designs the matched filter.

The performance of the proposed method was analyzed in terms of SNR using the pulse signals in (21). The simulations verified that the proposed method is useful for improving SNR for a passive radar sensor.

References:

[1] H. MANIWA, M. IWAMOTO, M. KOBAYASHI, T. KIRIMOTO, Deinterleaving of Radar Pulse Trains Using MHT Approach, *IEICE(B)*, Vol.J84-B, No.1, 2001, pp.116-128.

[2] M. I. Skolnik, *Introduction to RADAR systems, third edition*, McGraw-Hill Book Company, 2001.

[3] R. Nitzberg, *Radar Signal Processing and Adaptive Systems*, Artech House, 1999.

[4] J.G. Proakis, *Digital communications*, *third edition*, McGraw-Hill Book Company, 1995.

[5] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, *forth edition*, McGraw-Hill Book company, 2002.

[6] P.Z. Peebles, Jr., *Probability, Random Variables, and Random Signal Principles*, McGraw-Hill, Inc.,1987.

[7] K.E. Atkinson, *An Introduction to Numerical Analysis, second edition*, John Wiley & Sons Inc., 1989.

[8] M. I. Skolnik, *RADAR HANDBOOK, second edition*, McGraw-Hill Book Company, 1990.