# A Model for Subjective Contour by Anisotropic Potential Field Maximization

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*Abstract:* The subjective contour generation in the human brain depends on the interaction of local and comprehensive processes realize this mechanism. The purpose of this paper is to propose a model that outputs subjective contours from line figures. This model, for a local process, detects endpoints and generates potential fields that represent probability of an occluding contour's existence. Each field is anisotropic and spreads perpendicularly to the line figure. Then, for a comprehensive process, the direction of potential fields are corrected to the degree their intersections are maximized in the image. Finally, it fixes subjective contours by tracking potential ridgelines and outputs a result image. The generated subjective contour is smoothly curved and the shape is appropriate compared to what we perceive.

*Key–Words:* Subjective contour, Illusory contour, Occluding contour, Visual perception

## 1 Introduction

The visual information that we get is not only the information received optically and what physically exists [1][2]. For example, although a circle is not drawn in Fig.1(a), we clearly perceive a circle at the center of the image. Moreover, the circle is perceived as being closer to us, rather than being in the same plane as the physically existing lines, and is more brightly perceived than the background. The contour which is perceived in this phenomenon is called subjective contour (illusory contour), and many hypotheses of this mechanism in a brain have been suggested.

In physiological research, cells that react to subjective contours were discovered in the visual cortex area V2 [3]. This function suggests that fundamental function and local processing, such as partial edge detection, are important for the mechanism. On the other hand, a psychological study has concluded that our brain aggressively creates information to interpolate the defective region to be consistent in the entire image from the viewpoint of depth perception and others [4][5]. Then, the inducing figure's shape affects the perception [1][2]. Those reports suggest that a comprehensive visual information process in the higher levels of the brain is also important. Although a decisive theory has not yet proven, it is assumed that the interaction realizes the phenomenon.

The purpose of this paper is to propose a model that simulates the subjective contour from line figures on digital images. We tried to reproduce occluding contours by paying attention to depth perception. As a local process, this model computes the probability of an occluding contour's existence (potential) around the line figure's endpoints, without concluding the subjective contour's shape. The generated potential field anisotropically spreads around the endpoint.

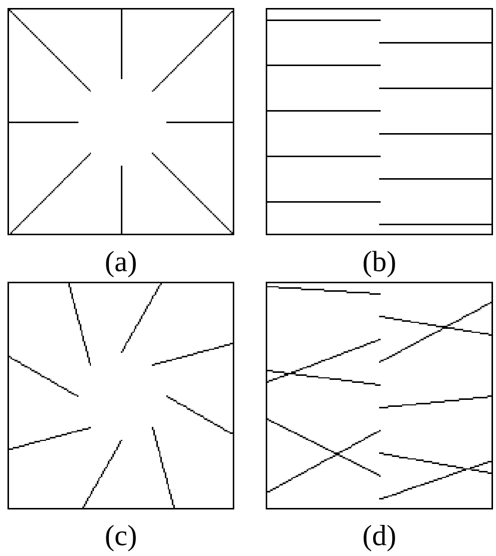


Fig. 1: Figures that induce a subjective contour.

As a comprehensive process, the model corrects the direction of the field expanse in order that the output contour's shape is simple. Finally, when the potential ridgeline connects two endpoints smoothly and continuously, it fixes the ridgeline as a subjective contour. The comprehensive process makes it possible to create the appropriate contour's shape. After explanation of this algorithm, we show several simulation results and discuss the generated shape.

## 2 Prior studies

Ullman investigated subjective contour shapes and suggested the shape will consist of two circular arcs, and these curvature radii should be minimal [6]. Then, he suggested that such a shape will be reproduced by a simple neural network model. Nevertheless, he did not show experimental results.

Kass and Witkin proposed a model that segments the image boundary with smooth curved contours [7]. The model is called Snakes. The model makes a contour line with a spline-curve, and the curve changes the shape and fits along the object's edges. This model well reproduces a subjective contour in a specific case, but the contour must close.

Williams and Jacobs and Thornber proposed stochastic completion fields model which is impressive [8][9]. These models use particles that can move and decay. The particles start from edges, spread around a specific direction and interconnect. As a result, the model outputs a graded gray level image. The brightness represents probability of subjective contour. However, boundaries that divide a region are not drawn clearly, and showed only the subjective contour which is perpendicular to the inducing line figures. Thus, the model differs from the model we propose in that they would not take the comprehensive process of an entire image into consideration.

Besides, there are models that reproduce subjective contours by high-pass filtering [10] or a neural network system [11]. These were appropriate to relatively simple images, for example, in case that the models output only straight lines.

## 3 Fundamental assumption

When we visually detect an endpoint of a line figure in an image, it is rational if thinking that our brain feels that the extension line is covered by an invisible figure, as in Fig.2(a). Given this rule, it is consequential that the subjective contour is perceived to be nearer than the image. We can therefore define that an occluding contour always passes at endpoints first. Then, the line figure and the occluding contour have

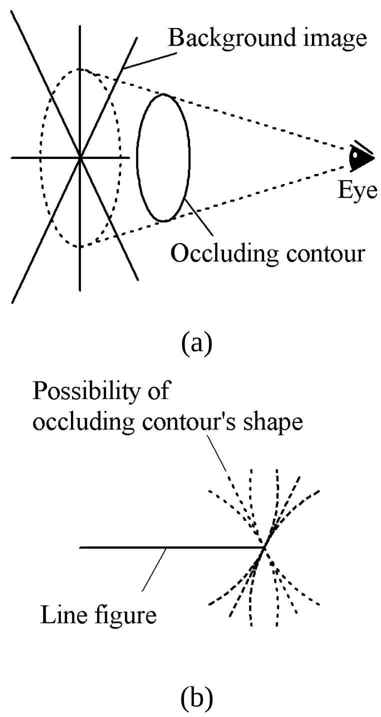


Fig. 2: Relationship of a background image and an occluding contour.

the possibility to intersect with various angles. Additionally, the occluding contour can become straight and curved, and has the possibility to make various shapes, as indicated in Fig.2(b). However, in Fig.1, the intensity of perception from Fig.1(c) is lower than Fig.1(a), or does not induce. Therefore, it seemed reasonable to think that the probability of an occluding contour's existence will be intensified in the neighborhood in the direction which is perpendicular to a line figure. Nevertheless, in the case of perception of the circle in (c), it has a simple shape similar to the circle from (a). This issue suggests that the direction of potential is not always generated perpendicular to a line figure. In such a case, it seems that a comprehensive process in the brain works in such a way that the perceived contour's shape becomes a relatively simple shape. The procedure of this model follows the above hypothesis.

## 4 Algorithm

In this section, we explain the details of this model. The images, which were used for the explanation, are $256$ grayscale and the size is $160 \times 160$ pixels. We drew the inducing figures in black $(0)$, and the background is white $(255)$.

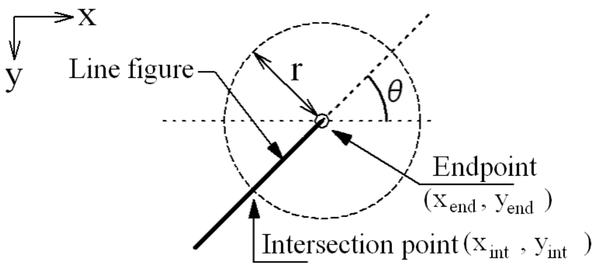


Fig. 3: Angle of line figure.

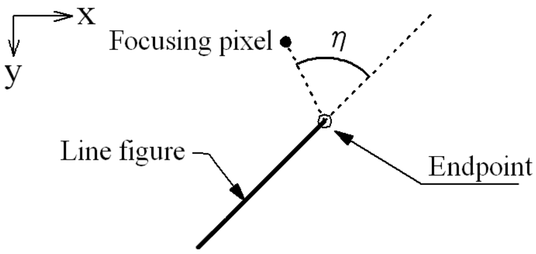


Fig. 4: Angle of focusing pixel from endpoint.

### 4.1 Endpoint and direction detection

In this paper, we define inducing figures as line figures with a width of a single pixel. Therefore, if the state of a figure's pixel satisfies the following condition when investigating eight neighborhoods, it detects the coordinates as an endpoint.

- The number of black pixels is one.
- The number of times of change into black from white or into white from black is twice .

At the same time, it takes the direction $\theta$ of the line figure to decide the direction of potential field expanse. We computed the $\theta$ by detection at the intersection point of an extension of the line figure and a circle, where the radius is r and the center is the endpoint, according to Fig.3 and Equation (1).

$$\theta = \arctan\left(\frac{y_{\mathrm{int}} - y_{\mathrm{end}}}{x_{\mathrm{end}} - x_{\mathrm{int}}}\right) \qquad (1)$$

### 4.2 Initial potential field generation

The potential field is a function that has anisotropic expanse around an endpoint. Initial potential $P$ at a pixel $(x, y)$ is computed by Equation (2) - (4).

$$G_n(x,y) = \frac{1}{\sqrt{2}\sigma}\exp\left(\frac{-\sqrt{(x-X_n)^2+(y-Y_n)^2}}{2\sigma^2}\right) \qquad (2)$$

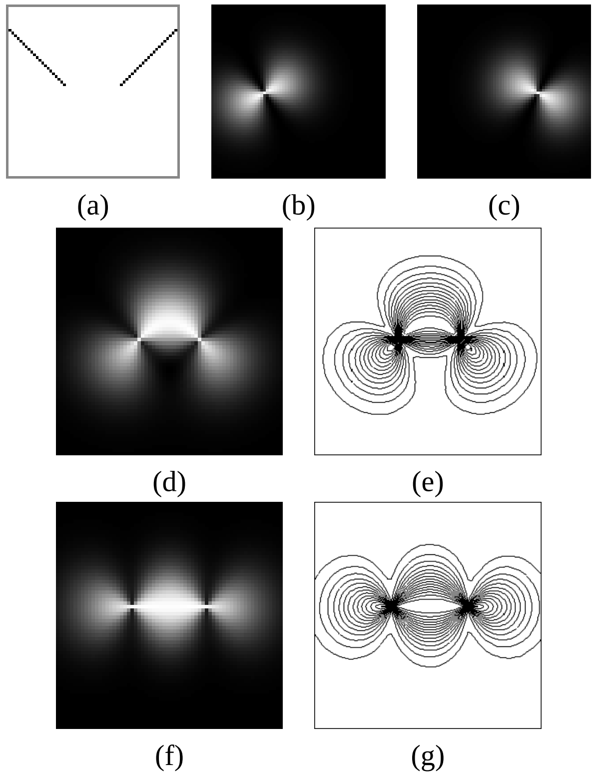


Fig. 5: Multiple potential fields and their composite.

(a) Line figures (b) Potential field from left figure (c) Potential field from right figure (d) Composed potential field (e) Level curve of (d) (f) Maximized potential field (g) Level curve of (f)

$$\eta_n = \arctan\left(\frac{y - Y_n}{X_n - x}\right) - \theta_n \qquad (3)$$

$$P(x,y) = \sum_{n=1}^{N} G_n(x,y) \mid \sin\eta_n \mid^{\alpha} \qquad (4)$$

Where suffix $n (1 \leq n \leq N)$ is the endpoint number; $X$ and $Y$ are coordinates of endpoints; $\eta$ is the angle between an extension of the line figure, and a line from the endpoint to a segment connection focusing pixel, as shown in Fig.4. The generated potential field has Gauss distribution with standard deviation $\sigma$ perpendicular to the line figure, and circumferential distribution is proportional to $|\sin\eta|^{\alpha}$, where $\alpha$ is a parameter to adjust attenuation speed of the potential.

We show an image of a potential field in Fig.5(a) - (e). At this stage, the potential field has an expanse perpendicular to the line figure. The parameters were $\sigma = 10.0$ and $\alpha = 2.0$. We normalized potential so that the maximum value is 255. The level curve of Fig.5(e), which is composed from Fig.5(b),(c), shows that the ridgeline draws a smooth curve.

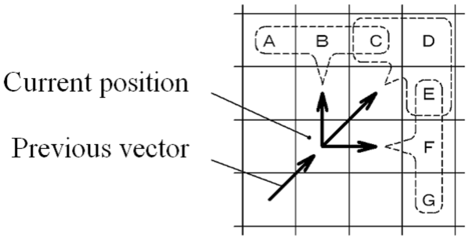


Fig. 6: Potential ridgeline tracking.

### 4.3 Potential field maximization

This step corrects the direction of the expanse of potential fields to ensure that the overlap of potential fields in the image is maximized. This operation is important to make the conclusive subjective contour's shape simple. We compute the degree of overlap $P_{\text{sum}}$, according to Equation (5).

$$P_{\text{sum}} = \sum_{y} \sum_{x} P(x, y)^2 \tag{5}$$

Next, we show the specific procedures required to maximize $P_{\text{sum}}$.

1. Vary $\eta$ for an endpoint from $+\delta$ to $-\delta$. Then compute $P^{+}_{\text{sum}}$ and $P^{-}_{\text{sum}}$ according to Equation (5).
2. Compare $P^{+}_{\text{sum}}$ and $P^{-}_{\text{sum}}$ and present value $P_{\text{sum}}$. Then, set $\eta$ to the angle which producesto the largest value.
3. Repeat 1. - 2. for all endpoints.
4. Repeat 1. - 3. until $P_{\text{sum}}$ does not increase.

Where $\delta$ is the resolution to correct of potential field direction. In this paper, we fixed $\delta = 1.0$ degree. Fig.5(f) and (g) show a corrected potential field image. The simplest shape, which connects both endpoints, will be straight from the viewpoint of distance and curvature. According to the assumption, we see that the ridgeline between two endpoints is corrected to be straight.

### 4.4 Ridgeline tracking and fixing the subjective contour

This step fixes a subjective contour from a potential field. Fundamentally, in brief, when tracking the ridgeline of a potential field and detecting connection continuously from an endpoint to another one, it defines the track as a subjective contour. The concrete procedure is below.

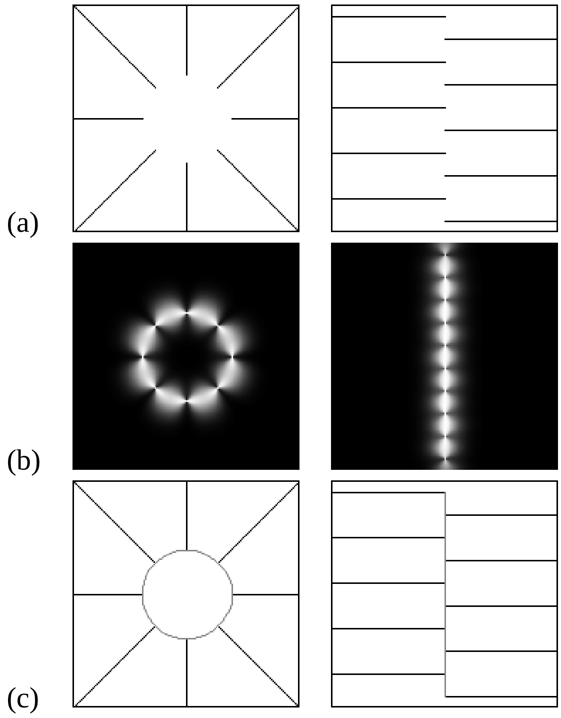


Fig. 7: Simulation of cases where maximization is not necessary.

(a) Input image (b) Initial potential field (c) Output image

1. Set current pixel to an endpoint as a start position.
2. Select the next pixel as the one with the maximum potential value among the eight neighbor pixels.
3. After second movement, decide the next pixel to be a change of movement vector within $\pm\pi/4$, as shown in Fig.6.
4. Fix the track as the subjective contour if the pixel reaches another endpoint. If potential becomes below a threshold, returns an already passed pixel or collides with a line figure during tracking, stop tracking.
5. Repeat 1. through 4. from all endpoints and draw a fixed subjective contour on the output image.

The reason for limiting the bend range of vector is that the subjective contour is quite unlikely to bend rapidly. Moreover, when choosing the next pixel, the model does not compare the surrounding pixels' potentials directly, but the combined potentials of the three pixels following on from the surrounding pixels. For example, in Fig.6, when comparing A+B+C,

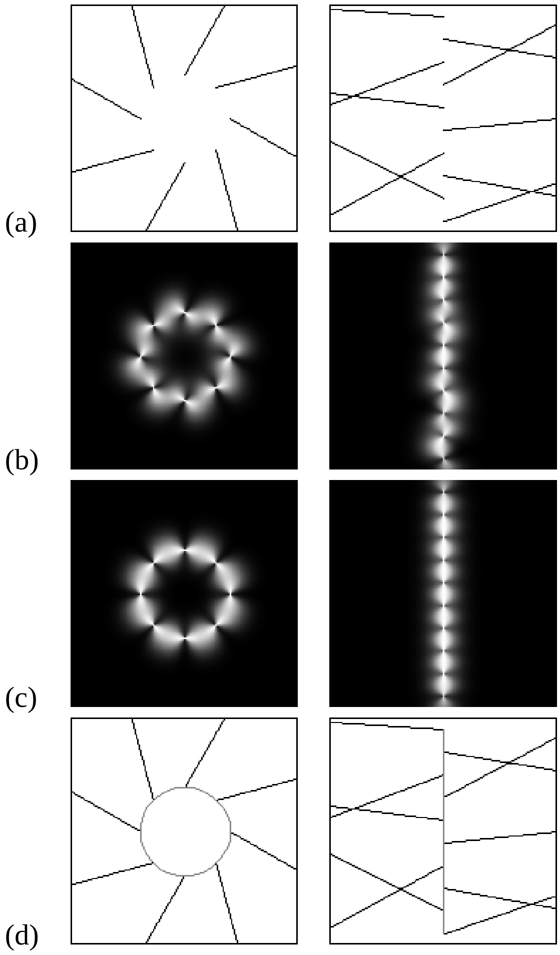


Fig. 8: Simulation of cases where maximization is necessary.

(a) Input image (b) Initial potential field (c) Maxmized potential field (d) Output image

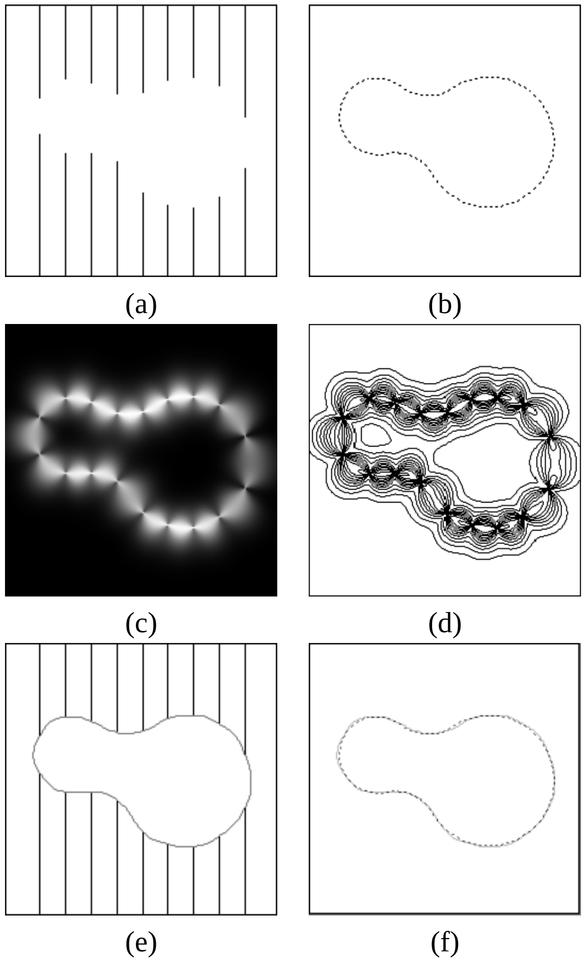


Fig. 9: Simulation of an image which outputs a closed contour.

(a) Input image (b) Occluding contour (c) Maximized potential field (d) Level curve of potential field (e) Output image (f) Overlay of (b) and the geneted contour

C+D+E and E+F+G, and if the largest value was E+F+G, the next pixel would be right (pixel F). This operation makes it possible to make a smoothly curved shape without rapidly bending. Incidentally, we set the threshold at $0.1$ for all experiments in this paper. This is equivalent to $1/10$ of the potential at the endpoints.

# 5 Experimental results

Fig.7 and Fig.8 show the simulation results for Fig.1. The parameters in this experiment are $\sigma = 10.0$ and $\alpha = 2.0$. Fig.7 contains images where the subjective contour crosses perpendicularly to the line figures. In these cases, this model outputs appropriate subjective contours by only local process. On the other hand, in Fig.8, the model does not output an appropriate result after local processing. By applying potential field maximization, the shape of the subjective contour becomes proper.

Moreover, to consider the generated contour's shape, we show another simulation results in Fig.9 and Fig.10. These inducing figures make us perceive more complicated subjective contours than a former example. These image's size are $200 \times 200$ pixels and the parameters are $\sigma = 16.0$ and $\alpha = 2.0$. We made the input image (a) by putting (b) over a grating image. We drew (b) to have both a direction curve and variable curvature. We made image (f) to compare the occluding contour's shape and the output shape. The shapes correspond well, which indicates that the model can reproduce occluding contours correctly. In

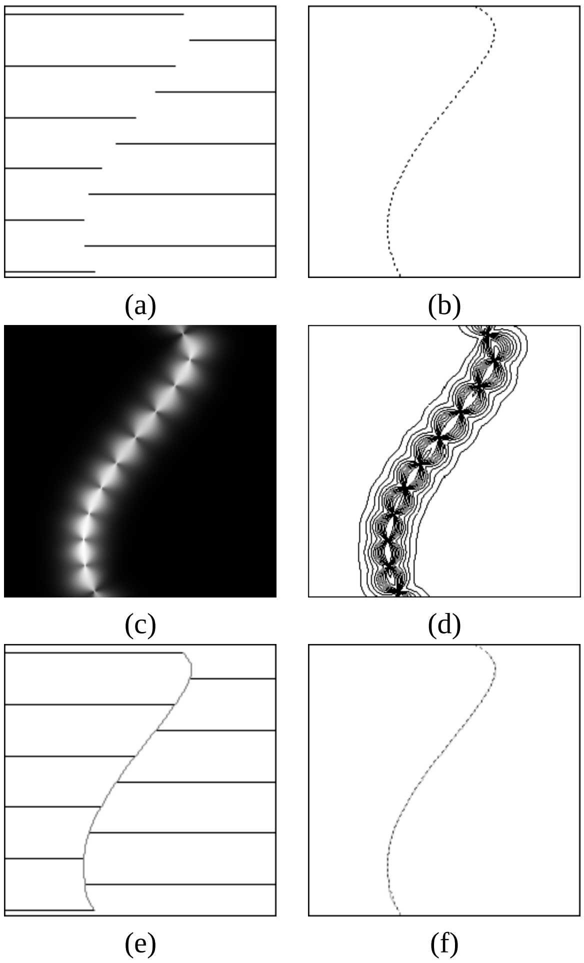

Fig. 10: Simulation of an image which outputs a segmented contour.

(a) Input image (b) Occluding contour (c) Maximized potential field (d) Level curve of potential field (e) Output image (f) Overlay of (b) and the geneted contour

both of examples, the maximum deviation of the occluding contour and the generated subjective contour was 2 pixels.

## 6 Conclusion

We proposed a subjective contour model that is applicable to line figures. The model's fundamental aim is to compute the probability of an occluding contour's existence. This model outputs not only smoothly curved contours, but also shapes which correspond well to what we perceive. Furthermore, the model can both output closed contours and segments. We expect that we can apply this study to complement the incomplete area of various types of images.

It is, however, necessary to adjust parameters for an expected result. For example, we have to adjust $\sigma$ according to the gap distance of endpoints. If the expanse of a potential field is too small, a ridgeline cannot connect endpoints; if too broad, the output shape will not draw a smooth curve. This issue may be the next theme we have to solve.

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