Techniques for Dynamic State Estimation of Machines in Power Systems

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Abstract: - The knowledge of dynamic states of electrical machine, especially the relative rotor position and velocity, are very important for us to understand the machine performance and to possibly design advanced control systems. This paper addresses the state estimation problem of synchronous machines in power systems, both in deterministic and stochastic cases during small transients. The paper examines Extended Kalman Filters (EKF) and Particle Filter (PF) approaches. With real-time data collected by phasor measurement unit (PMU) and sufficiently known machine model, the simulation results show that the states dynamics can be successfully and accurately estimated. The method proposed in this paper can be easily applied to other type machines or extended to include parameter estimation.

Key-Words: - State Estimation, Synchronous Machine, Particle Filter, PMU

1 Introduction

Electrical machines are one of the most important components in power systems, and their control is one of the greatest challenges in maintaining stable operation. The use of machine dynamic states in advanced controls requires either direct measurement or model-based estimation. With the emerging use of Phasor Measurement Units (PMUs), there is an opportunity to combine these two methods for full dynamic state computation using both direct data application and model prediction. The Extended Kalman Filter (EKF) has been examined for use with the advanced control of smaller electric machines [1]-[2]. However, the EKF uses local linearization of the nonlinear functions and assumes a Gaussian distribution for the model and measurement errors. It is not a sufficient description of the nonlinear and non-Gaussian system. In this paper, a novel method for state estimation of electric machines is proposed. It utilizes the theory of nonlinear filtering, uses the Sequential Importance Sampling (SIS) algorithm, also known as the Particle Filter (PF) method to handle highly nonlinear dynamics in power systems [3]. The PF method is applied to a large synchronous machine model with voltage and speed controls. Simulation results show that the states can be successfully and accurately estimated, even in transients. Comparisons are made between the EKF and PF methods. Future applications will also examine the potential for estimating model parameters using real-time PMU measurements.

The remainder of this paper provides the system model in section 2, followed by a description of the nonlinear filtering algorithm in section 3 and the results in section 4. Conclusion and other considerations are given in section 5.

2 System Model

The system considered is a synchronous machine in power system. This paper uses the conventions and notations of [4], which are consistent with standard industry modeling and simulation software. The nonlinear model is a two-axis machine model with IEEE-Type I exciter/AVR and turbine/governor. The differential-algebraic equations are summarized as:

$$\begin{aligned} \dot{x} &= f(x,y,u) \\ 0 &= g(x,y) \end{aligned} \quad \text{or} \quad \begin{aligned} x_{k+1} &= f(x_k, y_k, u_k) \\ 0 &= g_k(x_k, y_k) \end{aligned} \tag{1}$$

where x is the system state, y is the output, and u is the control vector.

The detailed n-bus-m-machine system model is in the following general form:

Differential Equations (2)

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s$$

$$\frac{d\omega_i}{dt} = \frac{T_{Mi}}{M_i} - \frac{\left[E'_{qi} - X'_{di} I_{di}\right] I_{qi}}{M_i} - \frac{\left[E'_{di} - X'_{qi} I_{qi}\right] I_{di}}{M_i} - \frac{D_i\left(\omega_i - \omega_s\right)}{M_i}$$

$$\frac{dE'_{qi}}{dt} = -\frac{E'_{qi}}{T'_{doi}} - \frac{\left(X_{di} - X'_{di}\right) I_{di}}{T'_{doi}} + \frac{E_{fdi}}{T'_{doi}}$$

$$\frac{dE'_{di}}{dt} = -\frac{E'_{di}}{T'_{qoi}} + \frac{\left(X_{qi} - X'_{qi}\right) I_{qi}}{T'_{qoi}}$$

$$\frac{dE_{fdi}}{dt} = -\frac{K_{Ei} + S_E(E_{fdi})}{T_{ei}} E_{fdi} + \frac{V_{Ri}}{T_{Ei}}$$

$$\frac{dV_{Ri}}{dt} = -\frac{V_{Ri}}{T_{Ai}} + \frac{K_{Ai}}{T_{Ai}} R_{fi} - \frac{K_{Ai} K_{Fi}}{T_{Ai} T_{Fi}} E_{fdi} + \frac{K_{Ai}}{T_{Ai}}\left(V_{refi} - V_i\right)$$

$$\frac{dR_{fi}}{dt} = -\frac{R_{Fi}}{T_{Fi}} + \frac{K_{Fi}}{\left(T_{Fi}\right)^2} E_{fdi}$$

$$\frac{dT_{Mi}}{dt} = -\frac{T_{Mi}}{T_{CHi}} + \frac{P_{SVi}}{T_{CHi}}$$

$$\frac{dP_{SVi}}{dt} = -\frac{P_{SVi}}{T_{SVi}} + \frac{P_{Ci}}{T_{SVi}} - \frac{\left(\omega_i - \omega_s\right)}{R_D \omega_s T_{SVi}}$$

for $i = 1, 2, \ldots, m$

Stator Algebraic Equations (3)

$$E'_{di} - V_i \sin\left(\delta_i - \theta_i\right) - R_{si} I_{di} + X'_{qi} I_{qi} = 0$$

$$E'_{qi} - V_i \cos\left(\delta_i - \theta_i\right) - R_{si} I_{qi} - X'_{di} I_{di} = 0$$

for $i = 1, 2, \ldots, m$

Network Algebraic Equations (4)

$$I_{di} V_i \sin\left(\delta_i - \theta_i\right) + I_{qi} V_i \cos\left(\delta_i - \theta_i\right) + P_{Li}(V_i) - \sum_{k=1}^{n} V_i V_k Y_{ik} \cos\left(\theta_i - \theta_k - \alpha_{ik}\right) = 0$$

$$I_{di} V_i \cos\left(\delta_i - \theta_i\right) - I_{qi} V_i \sin\left(\delta_i - \theta_i\right) + Q_{Li}(V_i) - \sum_{k=1}^{n} V_i V_k Y_{ik} \sin\left(\theta_i - \theta_k - \alpha_{ik}\right) = 0$$

for $i = 1, 2, \ldots, m$

$$P_{Li}(V_i) - \sum_{k=1}^{n} V_i V_k Y_{ik} \cos\left(\theta_i - \theta_k - \alpha_{ik}\right) = 0$$

$$Q_{Li}(V_i) - \sum_{k=1}^{n} V_i V_k Y_{ik} \sin\left(\theta_i - \theta_k - \alpha_{ik}\right) = 0$$

for $i = m+1, \ldots, n$

With reference to (1), this model has:

$$x_i = [\delta_i, \omega_i, E'_{qi}, E'_{di}, E_{fdi}, V_{Ri}, R_{Fi}, T_{Mi}, P_{SVi}]^t$$

$$y_{mi} = [I_{di}, I_{qi}]^t$$

$$u_i = \left[V_{refi}, P_{Ci}\right]^t$$

$$i = 1, 2, \ldots, m \qquad (5)$$

$$y_{nj} = [V_j, \theta_j]^t$$

$$j = 1, 2, \ldots, n$$

and $y = [y_m^t, y_n^t]^t$

Now, considering the system and measurement noises, and modeling uncertainties of the system, the dynamic model is rewritten as:

$$\begin{aligned} \dot{x} &= f(x, y, u) + v \\ 0 &= g(x, y) + \lambda \end{aligned} \quad \text{or} \quad \begin{aligned} x_{k+1} &= f(x_k, y_k, u_k) + v_k \\ 0 &= g_k(x_k, y_k) + \lambda_k \end{aligned} \qquad (6)$$

where v and λ are noise or model uncertainties.

Equation (6) is a realistic description of the real system, because of the included noise and uncertainties.

3 Nonlinear Filtering

The model of a synchronous machine has numerous nonlinearities. The nonlinear filtering is based on the following factors:

- A knowledge of the system model
- The statistical characteristic of the system and measurement noise and uncertainties.
- The initial probability distribution function (*pdf*) $p\left(x_0 \mid y_0\right) \equiv p\left(x_0\right)$, also known as apriori.

From the information, the posterior *pdf* $p\left(x_k \mid y_{1:k}\right)$ may be obtained recursively in two stages: prediction and updates.

3.1 Extended Kalman Filter (EKF)

The EKF is a sub-optimal algorithm for nonlinear filtering. It assumes that v and λ are drawn from Gaussian distributions of known parameters. The EKF utilizes the first term in a Taylor expansion of the nonlinear function described in equation (1), assuming the local linearization of the equations maybe a sufficient description of the nonlinearity.

$$\begin{aligned} x_k &= \hat{F}_k x_{k-1} + D_k y_{k-1} + B_k u_k + v_{k-1} \\ y_k &= \hat{H}_k x_k + \lambda_k \end{aligned} \qquad (7)$$

where $\hat{F}_k$ and $\hat{H}_k$ are the local linearizations of the nonlinear functions f and h. v_{k-1} and λ_k have zero mean and covariances Q_{k-1} and R_k respectively. A higher order EKF that retains further terms in the Taylor expansion exists, but the additional complexity has prohibited its use in this problem. Based on this approximation, the posterior *pdf* $p(x_k \mid y_{1:k})$ is approximated by a Gaussian distribution.

$$
\begin{aligned}
p(x_{k-1} \mid y_{1:k-1}) &\approx N(x_{k-1}; m_{k-1|k-1}; P_{k-1|k-1}) \\
p(x_k \mid y_{1:k-1}) &\approx N(x_k; m_{k|k-1}; P_{k|k-1}) \\
p(x_k \mid y_{1:k}) &\approx N(x_k; m_{k|k}; P_{k|k})
\end{aligned}
\tag{8}
$$

where,

$$
\begin{aligned}
m_{k|k-1} &= f_k(m_{k-1|k-1}, y_k, u_k) \\
P_{k|k-1} &= Q_{k-1} + \hat{F}_k P_{k-1|k-1} \hat{F}_k^T \\
m_{k|k} &= m_{k|k-1} + K_k(y_k - h_k(m_{k|k-1})) \\
P_{k|k} &= P_{k|k-1} - K_k \hat{H}_k P_{k|k-1}
\end{aligned}
$$

and (9)

$$
\begin{aligned}
S_k &= \hat{H}_k P_k \hat{H}_k^T + R_k \\
K_k &= P_{k|k-1} \hat{H}_k^T S_k^{-1}
\end{aligned}
$$

The linearization approximation of EKF will lose some information. Moreover, it always approximates $p(x_k \mid y_{1:k})$ to be Gaussian. If the true density is non-Gaussian, then a Gaussian assumption will contain additional error. The particle filtering methods introduced in the next section will not require this Gaussian assumption.

3.2 Particle Filter (PF)

Compared with EKF, the assumption of the Gaussian distribution has been removed, and there is no information lost due to the linearization of the nonlinear system. Hence, PF is more suitable for systems with nonlinearities and non-Gaussian noise and uncertainties. PF can process all measurements regardless of their precision, to provide a quick and accurate estimate of the variables of interests.

The basic idea of PF methods is to represent the required posterior *pdf* by a set of random samples with associated weights; and compute estimates based on the samples & weights.

The basic algorithm of the PF method is:

Step 0: Sample the system $\{x_0^i, \omega_0^i\}_{i=1}^{N_s}$

Step 1: Predict the system evolution using $x_k^i = f_k(x_{k-1}^i, y_k, u_k, v_k)$

Step 2: Update the associated weights based on observation y_k, $\omega_k^i \propto \omega_{k-1}^i \dfrac{p(y_k \mid x_k^i)\, p(x_k^i \mid x_{k-1}^i)}{q(x_k^i \mid x_{k-1}^i, y_k)}$.

Step 3: Resample if necessary.

$$p(x_k \mid y_{1:k}) \approx \sum_{i=1}^{N_s} \omega_k^i \delta(x_k - x_k^i).$$

Step 4 (if necessary): Go back to step 1.

This algorithm is explained through Figure 1. as follows:

We sample from the distribution at time 1, with only 5 samples shown here as dots. The size of each dot represents its weight. When moving to time 2, the position of each dot is changing according to the formula in Step 1, the size of each dot, which means the associate weight, is changing according to the formula in Step 2. Same happens as moving to time 3.

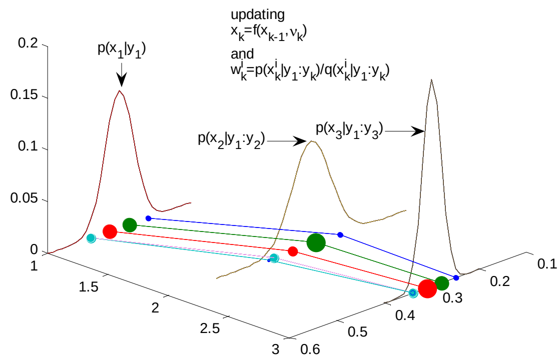


Fig.1 The illustration of Particle Filter Algorithm

The quasi-code of PF adopted from [3]:

Algorithm: General PF with Resampling

$$\left[\{x_k^i, \omega_k^i\}_{i=1}^{N_s}\right] = PF\left[\{x_{k-1}^i, \omega_{k-1}^i\}_{i=1}^{N_s}, y_k\right]$$

- FOR $i = 1 : N_s$
  - Draw $x_k^i \sim q(x_k \mid x_{k-1}^i, y_k)$
  - Assign the particle a weight, ω_k^i, according to $\omega_k^i \propto \omega_{k-1}^i \dfrac{p(y_k \mid x_k^i)\, p(x_k^i \mid x_{k-1}^i)}{q(x_k^i \mid x_{k-1}^i, y_k)}$
- END FOR
- Calculate $\hat{N}_{eff} = \dfrac{1}{\sum_{i=1}^{N_s} (\omega_k^i)^2}$

- IF $\hat{N}_{eff} < N_T$
  - Resample using $\left[\left\{x_k^i, \omega_k^i, -\right\}_{i=1}^{N_s}\right] = \text{RESAMPLE}\left[\left\{x_k^i, \omega_k^i\right\}_{i=1}^{N_s}\right]$
- END IF

.

Algorithm: Resampling

$$\left[\left\{x_k^{j*}, \omega_k^j, i^j\right\}_{j=1}^{N_s}\right] = \text{RESAMPLE}\left[\left\{x_k^i, \omega_k^i\right\}_{i=1}^{N_s}\right]$$

- Initialize the CDF: $c_1 = 0$
- FOR $i = 2 : N_s$
  - Construct CDF: $c_i = c_{i-1} + \omega_k^i$
- END FOR
- Start at the bottom of the CDF: $i = 1$
- Draw starting point: $u_1 \sim \mathbf{U}\left[0, N_s^{-1}\right]$
- FOR $j = 1 : N_s$
  - Move along the CDF: $u_j = u_1 + N_s^{-1}(j-1)$
  - WHILE $u_j > c_i$; $i = i + 1$
  - END WHILE
  - Assign sample: $x_k^{j*} = x_k^i$
  - Assign weight: $\omega_k^j = N_s^{-1}$
  - Assign parent: $i^j = i$
- END FOR

The algorithm presented above forms the basis for most particle filters that have been developed so far. The various versions of PF proposed in the literature can be regarded as special cases of the general PF. This general PF algorithm was applied to the state estimation problem of the synchronous machine.

4 Simulation Results

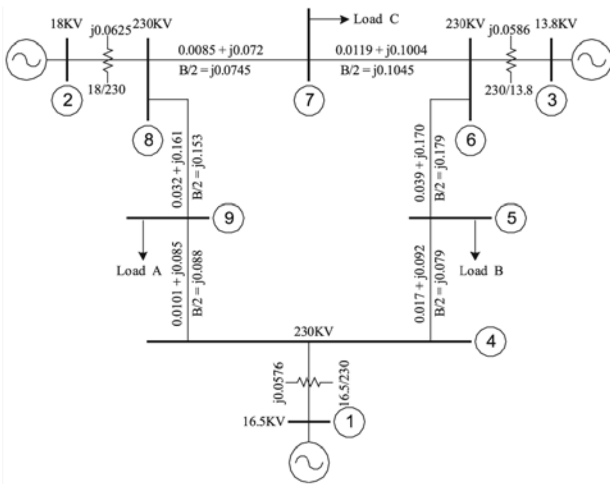


Fig. 2 WSCC 3-machine, 9-bus system

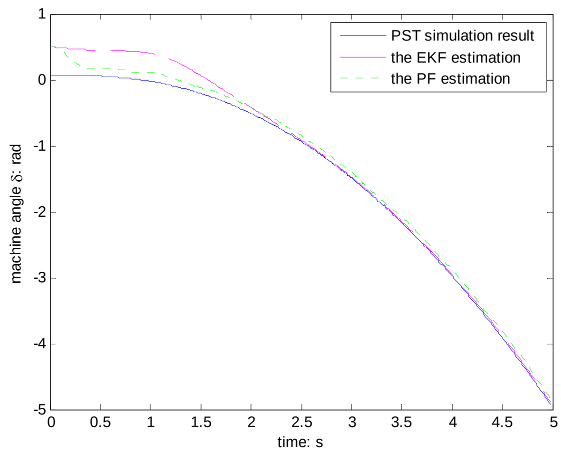
The popular Western System Coordinating Council (WSCC) 3-machine, 9-bus system is used in the case study [4].

The Power System Toolbox (PST 2.0 – [5]) was used to generate the system dynamics and collect the data as a realistic simulation. The output of this simulation was then used as the measurements for the nonlinear filtering algorithm, to obtain the estimates of the system states[1].

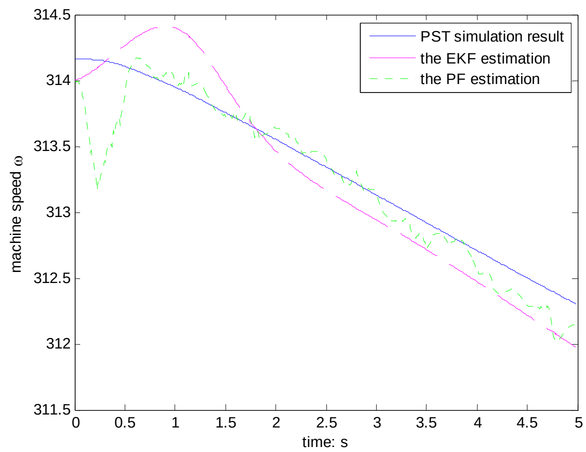
4.1 WSCC Case 1: Deterministic

In this case, there are no system uncertainties or measurement noises. The whole system is fully deterministic. The changes of real power load on bus 5 cause the system transients.

With no knowledge of the initial value, we applied both EKF and PF on state estimation of the synchronous machine as bus 1.

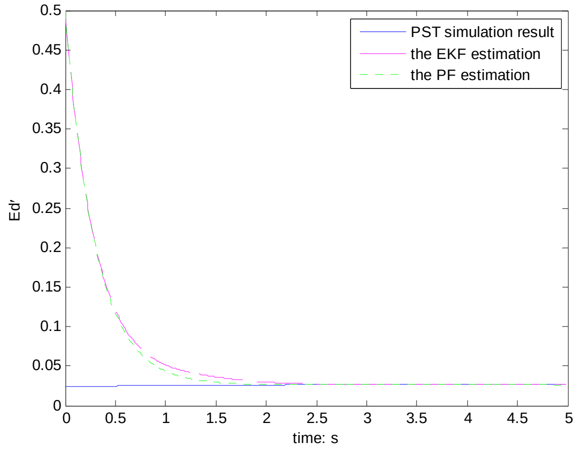


(a)



(b)

[1] The dynamic machine model used in PST is slightly different from the one described in equation 2. The difference will be taken care of as the unknown system uncertainties is the algorithm.



(c)

Fig. 3 The PST simulation data and estimation results from EKF and PF on WSCC system. The assumed noise variance is small in EKF/PF. Machine angle, velocity and $E_d^{'}$ are shown in figure (a)-(c) respectively. Other states are not shown here.

Table 1: RMSE of EKF and PF methods

RMSE	δ	ω	$E_q^{'}$	$E_d^{'}$	E_{fd}	R_f	V_R
EKF	0.22	0.24	0.11	0.082	0.030	0.036	0.026
PF	0.11	0.22	0.20	0.082	0.033	0.037	0.027

From figure 3 and table 1, we can see that:

(1) The estimation from both EKF and PF can converge to the true value.

(2) For the states that both PF and EKF converge, PF also converges to the true value faster than EKF.

(3) Take a further look at the results, PF shows that the posterior is not necessarily a Gaussian distribution, as in figure 4, which also gives us a more accurate description of the system.

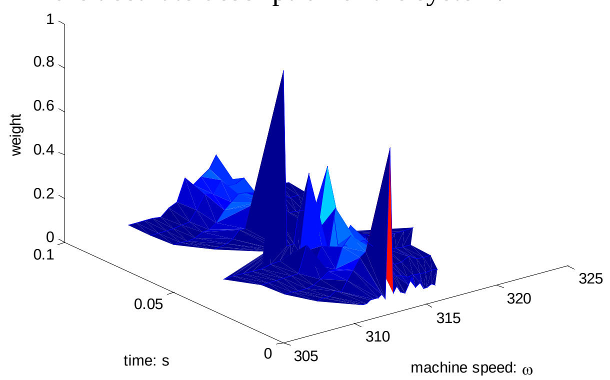
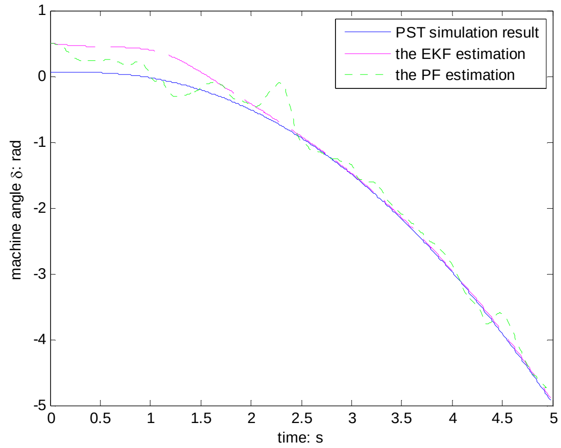
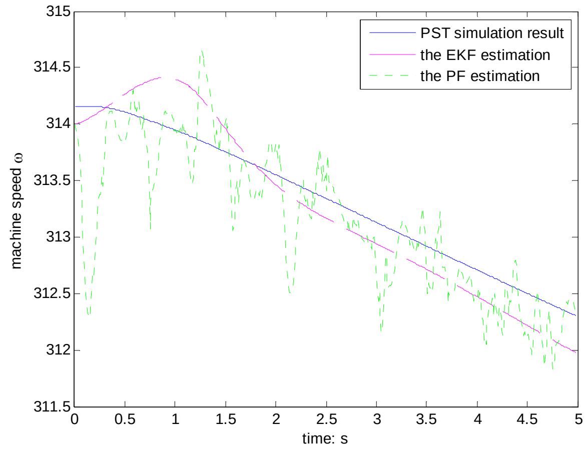


Fig. 4 the posterior of machine speed from PF

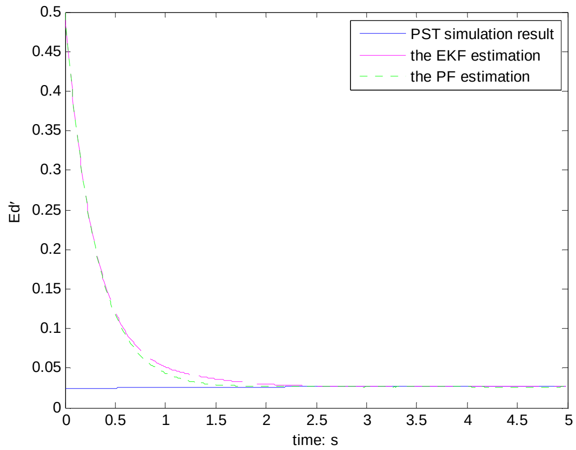
It should be mentioned that, if the guessed initial value of system states are way off the true value, the estimation might not converge, for both EKF and PF. Also, such results are based on the knowledge of system stochastic characters. If such information is not available wrongly assumed, PF estimation might not have good performance, as shown in figure 5.



(a)



(b)



(c)

Fig. 5 The PST simulation data and estimation results from EKF and PF on WSCC system. The assumed noise variance is large in EKF/PF.

4.2 WSCC Case2: Dynamic with noises

In this case, the changes of real power load on bus 5 are exactly the same as the changes of the case in section 4.1. But there are measurement noise, which is Gaussian distribution $n \sim N(0,1)$, and the system noise, which is Uniform distributed, $v \sim \mathbf{U}(0,1\%)$.

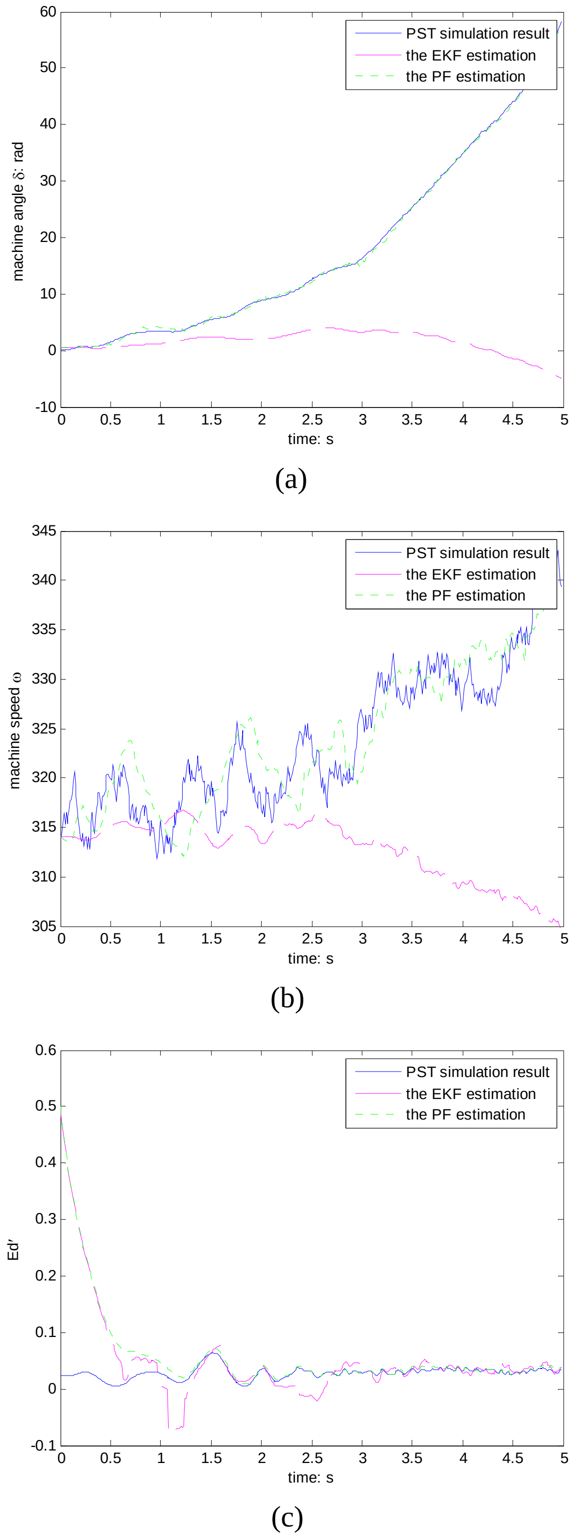


Fig. 6 The PST simulation data and estimation results from EKF and PF on WSCC system with noises. Machine angle, velocity and $E_d^{'}$ are shown in figure (a)-(c) respectively.

The simulation results in this case show that PF method can follow the simulation result fairly well, while EKF method fails to do so. It is not saying that EKF doesn't work for all such cases, but generally PF works better in cases when there are noises or uncertainties involved.

5 Conclusion

In this paper, the particle filter algorithm has been introduced to estimate the system states of synchronous machine in power systems. Compared with conventional sensorless estimation methods, PF does not have any limitation, and can get all system states as needed. PF lifts the assumption of Gaussian distribution from the EKF method. And PF is easier to implement since there is no local linearization. The simulation results meet the expectation, and PF also has better convergence than EKF. With few modifications, PF can be applied to parameter estimation and extended to other types of machines.

Acknowledgements

This research was supported by the National Science Foundation under grant number CNS-0540216, and the Grainger Endowments to the University of Illinois at Urbana-Champaign.

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