The Minimum Dissipated Active and Reactive Power: an Equilibrium State of the Circuits Under Nonsinusoidal Conditions

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Abstract: **-** The use of the power and energy functional in the analysis of the electric circuits makes it possible to appreciate the energetic equilibrium state attained in the circuit at a certain moment. This paper is concerned with a demonstration of the principle of minimum dissipated active power for the steady state circuits under nonsinusoidal conditions. It is shown that the equilibrium state is one of a minimum energetic state. A Multisim application is used**.**

Key-Words: **-** power functionals, minimum dissipated active power, nonsinusoidal conditions, Multisim application.

1 Introduction

Conservative systems accept the definition of functionals expressed in terms of power or energy. Calculating the limits of these functionals represents an important breakthrough in formulating and solving optimization problems.

Numerous valuable contributions have appeared in the literature over the years. In the theory of the steady state circuits, the results obtained by Millar [1] and Stern [2] related to the “co-content” function for nonlinear resistive and reciprocal network have a special theoretic importance due to their generality. C. A. Desoer and E. Kuh, [3] (pp. 770-772), had proved the same generally properties of the minimum dissipated power for the linear and resistive networks. As well, the Romanian professors V. Ionescu [4] and C.I. Mocanu [5] (pp. 350-353), had important contributions at the theoretical development of the electrical circuits minimax theorems. All these results are basically consequences of Maxwell's principles of minimum-heat [6] (pp. 407-408).

Research works published by authors such as [7-14] have thoroughly demonstrated the minimum energetic principle for the electric circuits in stationary and quasi-stationary regime. Thus, for the d.c. circuits, this takes the form of the first principle of minimum absorbed power: "*The minimum of the absorbed power by the branches of a linear and resistive circuit in a stationary regime is verified by the solutions of the currents and voltages in the circuit, and these are the currents and voltages that verifies the first and the second of Kirchhoff's theorems*" or, in other words: "*In a reciprocal and resistive d.c. circuit the currents and voltages get distributed such as the absorbed power by the branches of the circuit should be minimal*".

In the case of a.c. circuits, the description is the second principle of the minimum active and reactive absorbed power: "*The minimum of active and reactive absorbed (generated) power by the branches of a linear circuit in quasi-stationary a.c. is verified by the solutions in currents and voltages in the circuit, and these are the currents and voltages that verify the first and second Kirchhoff's theorems*". And here is just another description of the same principle: "*In the linear and reciprocal circuit in quasi-stationary a.c. regime the currents and voltages are distributed such as the active and reactive absorbed (generated) power by the branches of the circuit should be minimal*".

Starting from these principles, this paper presents an original contribution concerning the principle of the minimum dissipated active power applied for the steady state circuits under nonsinusoidal conditions.

2 Power Functional for Determining the Minimum of the Active and Reactive Dissipated Power for Steady State Circuits Under Nonsinusoidal Conditions

Let's consider an arbitrary linear circuit under periodic nonsinusoidal conditions with N nodes. Writing, for each branch of the circuit, the voltage and current as Fourier series, we may state that

$$u_k(t) = \sum_{p=0}^{m} U_k^{(p)} \sqrt{2} \sin(p\omega t + \varphi_k^{(p)}) \tag{1}$$

$$i_k(t) = \sum_{p=0}^{m} I_k^{(p)} \sqrt{2} \sin(p\omega t + \varphi_k^{(p)} - \gamma_k^{(p)}) \tag{2}$$

where $k = 1,...,L$ is the number of branches, m is the finite number of harmonics, $U_k^{(p)}$ and $I_k^{(p)}$ represents the rms values of p-harmonic of voltage respectively current, $\varphi_k^{(p)}$ and $\varphi_k^{(p)} - \gamma_k^{(p)}$ represents the phase angle of p-harmonic of voltage respectively current. For k-branch of the circuit, shown in figure 1, by using the complex representation for p-harmonic of voltage and current, we get

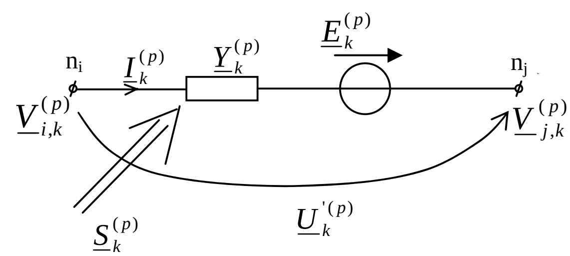

Fig. 1. A branch of circuit under nonsinusoidal conditions

$$\underline{U}_k^{'(p)} + \underline{E}_k^{(p)} = \frac{\underline{I}_k^{(p)}}{\underline{Y}_k^{(p)}} = \underline{U}_k^{(p)} \tag{3}$$

We note, for each p-harmonic: the complex potentials of nodes, the admittance and the voltage source of k-branch, with respectively

$\underline{V}_{i,k}^{(p)} = x_{i,k}^{(p)} + jy_{i,k}^{(p)}; \underline{V}_{j,k}^{(p)} = x_{j,k}^{(p)} + jy_{j,k}^{(p)};$

$\underline{Y}_k^{(p)} = G_k^{(p)} - jB_k^{(p)}; \underline{E}_k^{(p)} = a_{E,k}^{(p)} + jb_{E,k}^{(p)},$

where, for clarity of presentation and generality, $G_k^{(p)}, B_k^{(p)}, a_{E,k}^{(p)}, b_{E,k}^{(p)}$ are constants, and $x_{i,k}^{(p)}, y_{i,k}^{(p)}, x_{j,k}^{(p)}, y_{j,k}^{(p)}$ are variables. Then the p-harmonic of complex power dissipated by all the L branches is

$$\begin{aligned} \underline{S}^{(p)} &= \sum_{k=1}^{L} \underline{S}_k^{(p)} = \sum_{k=1}^{L} \underline{U}_k^{(p)} \underline{I}_k^{(p)*} = \\ &= \sum_{k=1}^{L} (G_k^{(p)} + jB_k^{(p)})[(x_{i,k}^{(p)} - x_{j,k}^{(p)} + a_{E,k}^{(p)})^2 + \\ &+ (y_{i,k}^{(p)} - y_{j,k}^{(p)} + b_{E,k}^{(p)})^2] \end{aligned} \tag{5}$$

The real and imaginary parts of the p-harmonic of complex power dissipated can be defined as the functionals in Hilbert space

$$
\begin{aligned}
&F_R^{(p)} \equiv \frac{1}{2}\mathrm{Re}[\underline{S}^{(p)}]: R^{2N} \to R \\
&F_R^{(p)}(x_1^{(p)}, x_2^{(p)},...,x_N^{(p)}, y_1^{(p)}, y_2^{(p)},...,y_N^{(p)}) \equiv \frac{1}{2}\mathrm{Re}[\underline{S}^{(p)}] = \\
&= \frac{1}{2}\sum_{k=1}^{L} G_k^{(p)}[(x_{i,k}^{(p)} - x_{j,k}^{(p)} + a_{E,k}^{(p)})^2 + (y_{i,k}^{(p)} - y_{j,k}^{(p)} + b_{E,k}^{(p)})^2]; \\
&F_I^{(p)} \equiv \frac{1}{2}\mathrm{Im}[\underline{S}^{(p)}]: R^{2N} \to R \\
&F_I^{(p)}(x_1^{(p)}, x_2^{(p)},...,x_N^{(p)}, y_1^{(p)}, y_2^{(p)},...,y_N^{(p)}) = \frac{1}{2}\mathrm{Im}[\underline{S}^{(p)}] = \\
&= \frac{1}{2}\sum_{k=1}^{L} B_K^{(p)}[(x_{i,k}^{(p)} - x_{j,k}^{(p)} + a_{E,k}^{(p)})^2 + (y_{i,k}^{(p)} - y_{j,k}^{(p)} + b_{E,k}^{(p)})^2]
\end{aligned}
\tag{6}
$$

and they are quite obviously a function class C^2 in R^{2N}.

Always, the real functional of the complex power is positively defined, $F_R^{(p)}(x_1^{(p)}, x_2^{(p)},...,x_N^{(p)}, y_1^{(p)}, y_2^{(p)},...,y_N^{(p)})\rangle 0$, for all the pair $(x_i^{(p)}, y_i^{(p)}), i = 1,..., N; p = 1,..., m$ because $G_k^{(p)}\rangle 0$, and then *the active dissipated power has a minimum*.

The imaginary functional of the complex power can be: 1) positively defined, for all the pair $(x_i^{(p)}, y_i^{(p)}), i = 1,..., N; p = 1,..., m$ and for $B_k^{(p)}\rangle 0$, i.e. a resistive-inductive circuit, then a, and we can define that the *reactive absorbed power has a minimum;* 2) negatively defined, for all the pair $(x_i^{(p)}, y_i^{(p)}), i = 1,..., N; p = 1,..., m$ and for $B_k \langle 0$, i.e. a capacitive-inductive circuit, then $F_I^{(p)}(x_1^{(p)}, x_2^{(p)},..., x_N^{(p)}, y_1^{(p)}, y_1^{(p)},..., y_1^{(p)})\langle 0$ and we can define that the *reactive generated power has a minimum*.

Consequently, the minimum points of the functionals (6) are the solutions of the system which contains 4*N* equations:

$$
\begin{aligned}
&\frac{\partial F_R^{(p)}}{\partial x_1^{(p)}} = 0,\ \frac{\partial F_R^{(p)}}{\partial y_1^{(p)}} = 0,...,\ \frac{\partial F_R^{(p)}}{\partial x_N^{(p)}} = 0, \frac{\partial F_R^{(p)}}{\partial y_R^{(p)}} = 0, \\
&\frac{\partial F_I^{(p)}}{\partial x_1^{(p)}} = 0,\ \frac{\partial F_I^{(p)}}{\partial y_1^{(p)}} = 0,...,\ \frac{\partial F_I^{(p)}}{\partial x_N^{(p)}} = 0, \frac{\partial F_I^{(p)}}{\partial y_M^{(p)}} = 0
\end{aligned}
\tag{7}
$$

If we calculate the algebrical sum of the solutions, with one of them multiplied with (-1) or ±j , we obtain the expressions:

$$
\sum_{l_k \in n_1} \underline{I}_k^{(p)*} = 0,\ \sum_{l_k \in n_2} \underline{I}_k^{(p)*} = 0,...,\ \sum_{l_k \in n_N} \underline{I}_k^{(p)*} = 0 \tag{8}
$$

which are identical with the Kirchhoff's equations for the p-harmonic of currents (1st Kirchhoff theorem), expressed in all the *N* nodes of the circuit.

Consequently, the following principle can be issued (the generalization of the Principle of Minimum Active and Reactive dissipated Power in linear circuits under nonsinusoidal conditions): *in the circuits under nonsinusoidal conditions, for each p-harmonic, the voltages and currents of the branches verify the Kirchhoff's theorems and correspond to the minimum dissipated active and reactive power of the circuit.*

3 Example

Let's consider a linear circuit "splitter" which is used in TV cable for power amplifier of signals, shown in figure 2.

The instantaneous values of nonsinusoidal voltage source have the practical form:

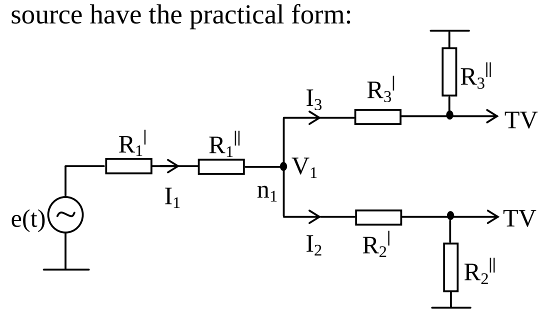

Fig.2. The "splitter"

$$
\begin{aligned}
&e(t) = 0.27386 + 0.27386\sqrt{2}\sin(2\pi 40\cdot 10^6) + \\
&+ 0.27386\sqrt{2}\sin(2\pi 900\cdot 10^6)[V]
\end{aligned}
\tag{9}
$$

If we consider the potential V_1 variable, for each harmonic of voltage source we can calculate the minimum of the power dissipated functionals.

a) The power dissipated functional for d.c. regime of the splitter can be expressed by

$$
F_R^{(0)} = \frac{1}{2}\left[G_1(-V_1^{(0)} + E^{(0)})^2 + G_2 V_1^{(0)^2} + G_3 V_1^{(0)^2}\right]\rangle 0, \forall V_1^{(0)} \in R \tag{10}
$$

where $G_1 = \frac{1}{R_1^{'} + R_1^{''}}, G_2 = \frac{1}{R_2^{'} + R_2^{''}}, G_3 = \frac{1}{R_3^{'} + R_3^{''}}$ not depend of the frequency. Then, the minimum point of functional are obtained when

$$
\frac{\partial F_R^{(0)}}{\partial V_1^{(0)}} = -G_1(-V_1^{(0)} + E^{(0)}) + G_2 V_1^{(0)} + G_3 V_1^{(0)}) = 0 \tag{11}
$$

relation who is identical with the 1st Kirchhoff's theorem expressed in node 1.

b) If we consider the potential $\underline{V}_1^{(p)} = x + jy^{(p)}$, for the two harmonics, $p = 8\cdot 10^5$, and $p = 18\cdot 10^6$, then for a.c. regime of the splitter the functional of the complex dissipated power is

$$F_R^{(p)} = \frac{1}{2}\left\{ \begin{aligned} & G_1\left[(-x^{(p)} + E^{(p)})^2 + y^{(p)^2}\right] + G_2(x^{(p)^2} + y^{(p)^2}) + \\ & + G_3(x^{(p)^2} + y^{(p)^2}) \end{aligned} \right\} > 0, \quad (12)$$

$\forall (x^{(p)}, y^{(p)}) \in R$

Then, the minimum of the power functional are the solutions of the system

$$\begin{aligned} \frac{\partial F_R^{(p)}}{\partial x^{(p)}} &= -G_1(-x^{(p)} + E^{(p)}) + G_2 x^{(p)} + G_3 x^{(p)} = 0 \\ \frac{\partial F_R^{(p)}}{\partial y^{(p)}} &= G_1 y^{(p)} + G_2 y^{(p)} + G_3 y^{(p)} = 0 \end{aligned} \quad (13)$$

and, if we calculate the algebrical sum of the solutions of system (13), with the second equation multiplied with $-j$, we obtain the 1st Kirchhoff's theorem expressed in node 1

$$-\underline{I}_1^{(p)^*} + \underline{I}_2^{(p)^*} + \underline{I}_3^{(p)^*} = 0 \quad (14)$$

The imaginary part of power functional is zero, $F_I^{(p)} = 0$.

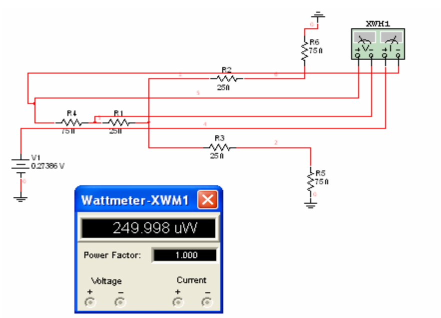

Fig.3, a. The power dissipated by $R_1^{'}$ in d.c.

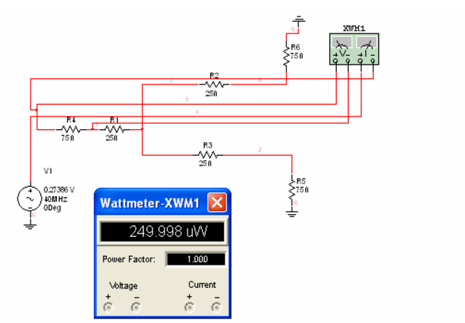

Fig.3, b. The active power dissipated by $R_1^{''}$ in a.c.

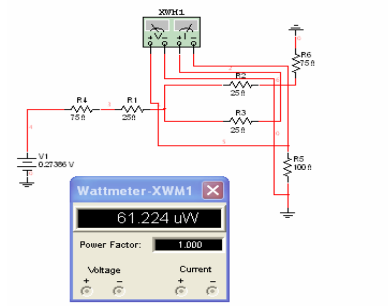

Fig.3, c. The power dissipated by $R_2^{''}$ in d.c.

We calculate using Multisim, figure 3, a, b, and c, the dissipated active power for different values of the resistances. The results are shown in table 1. The first line contains the nominal values of conductance, and the last line of table contains the adapted values of all the resistances, i.e. each resistance absorbed by the circuit the maximum active power. In this case, the value of the dissipated active power is the big one compared with the other values.

Table 1

Values of Conductances (S)	Total Dissipated Power in d.c. (mW)	Total Dissipated ActivePower in a.c. (mW)
$G_1 = G_2 = G_3 = 0.01$	499.593	499.593
$G_1 = 0.01$, $G_2 = 0.013$ $G_3 = 0.01$	524.818	524.818
$G_1 = 0.01$, $G_2 = 0.02$, $G_3 = 0.025$	1.662	1.662
$G_1 = 0.01$, $G_2 = 0$, $G_3 = 0.01$	374.832	374.832
$G_1 = 0.01$, $G_2 = 0.04$, $G_3 = 0.01$	624.810	624.810
$G_1 = 0.02$, $G_2 = 0.01$, $G_3 = 0.01$	749.664	749.664

5.Conclusions

To determine the extreme of the power functional in case of the linear circuits is a problem of utmost importance, with quite useful didactic, theoretical and practical applications.

It has been established that the solutions of the linear electric circuit, under nonsinusoidal conditions, represent a minimum of the dissipated power in the circuit.

The energetic problem under debate in the present work has a wide range of practical applications and it aims at cutting down the wastes in the energetically systems.

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