Gravitational Wave Signal identification and transformations in time-frequency domain

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Abstract: A gravitational wave signal carries information about an astrophysical source, a time varying quantity that has to be analyzed in the time frequency domain. There are varieties of transforms that can be applied to understand the complex evolution time-varying frequencies and chirps. This paper discusses various techniques of transforms that can be applied to various categories of this problem to identify and analyze signal, and to assess their efficacy.

Key–Words: Time frequency, Chirp wave analysis, Gravitational Wave, Spectral methods

1 Introduction

The general tool for signal analysis is the Fourier transform, that decomposes a signal into its frequency components [1]. The power spectrum provides information about frequency not temporal localization of the data. A time frequency distribution (TFD) is a transform that maps a 1-D signal into a 2-D time-frequency map, which describes the evolution of spectra over time. More well-known TFD's are short time fourier transform (STFT) [2], the Gabor representation and wavelet transform [3]. The quadratic time frequency distributions are used to analyze time-varying power spectra. Well-known methods are Spectrogram, Wigner-Ville distribution (WD) and Choi-Williams distributions [4, 5]. The WD is very useful due to its capability to analyze phase modulated signals. This paper discusses in detail the various ways of identifying signals in time-frequency domain and the data sets which have been adopted for chirp waveform analysis. The following sections will introduce Laser Interferometer Space Antenna (LISA) data sets and provide details of extraction of gravitational wave form and sources of gravitational waves.

2 LISA Data

The Laser Interferometer Space Antenna [LISA] is jointly sponsored by the European Space Agency (ESA), as a Cornerstone mission in ESA's Cosmic Vision Programme, and by NASA's Astronomy and Astrophysics Division, as part of the Structure and Evolution of the Universe 2003 roadmap, "Beyond Einstein: From the Big Bang to Black Holes." It is intended to look for Gravitational Radiation [gravity waves] from intense astrophysical sources, from merging black hole binaries, from extreme mass ratio inspirals [EMRIs], and from cosmic stochastic background sources from the early universe (http://lisa.nasa.gov/WHATIS/intro.html). The LISA consists of three spacecraft, floating and "cartwheeling" in a semi-rigid formation, each separated by 5 million km. Each spacecraft will send data to ground at 15 seconds intervals (ie, roughly 2 million data points per year or orbit from each spacecraft). The frequency of signals is in the millihertz range.

The task is to analyze data from Massive Black Hole Binaries (MBH) and EMRIs for stationary and/or 'chirp' signal detection in time - frequency space. The "center" frequencies for such sources lie below 0.01 milliHz. The power is in order of 10^{-20}. Fast fourier transform (FFT) method is used to extract single hidden sources (for example approx. 20 verification binaries). The problem of solving/identfying gravitational waves can be divided into two parts viz., (i) Identifying/extracting frequency of signal from raw data source (ii) Based on identified frequency, search for location of the source in the sky as well as other parameters. Once the frequency or chirp characteristics have been extracted, then extensive search methods or Monte Carlo modeling is done to extract all the other characteristics of the source [location, polarization, inclination, initial phase, distance, separation in the binary, etc.] Thus, the problem ends up operating across a 17-dimension data search, and year-long or multiyear data [still at 15second cadence]. The wave-forms may range from various chirp forms to interlaced chirps. Various types of transforms (similar to FFT) have to be applied to identify and extract signals, and other characteristics. The signals sensitivity at millihertz frequencies controls to extract its source location parameters. Typical 'simulation' data sets and descriptions can be viewed and downloaded from http://astrogravs.nasa.gov/docs/mldc/. The following section explains about various methods of estimating spectra for LISA data sets. The LISA spacecraft data sets are referred to X, Y, Z Time Delay Interferometer variables. These values are signals from each of the three space craft separated by 5 million kilometers.

3 Spectral Estimation Method

There are several methods available for spectral estimation [6, 7] that can be classified as:

3.1 Nonparametric Methods

The power spectral density (PSD) is estimated directly from the signal. The simplest method is a periodogram, and the most advanced are Welch and multitaper methods [8]. In periodograms, a discrete-time fourier transform is applied to the samples and one computes the magnitude squared of the result. The performance of periodogram with respect to leakage, resolution, bias and variance is a critical issue. The spectral leakage is solely based on the length of the record. Resolution refers to the ability to discriminate spectral features. The periodogram is asymptotically biased even when the data length is long. In statistical terms, the periodogram is not a consistent estimator of the PSD. Nevertheless, it is a useful tool where Signal Noise Ratio is high, and especially if the data record is long.

3.2 Parametric Methods

The PSD is estimated from a signal that is assumed to be an output of a linear system driven by white noise. This works very well when data length of the available signal is relatively short. Yule-Walker Auto-regressive and Burg methods are examples of parametric methods.

3.3 Subspace Methods

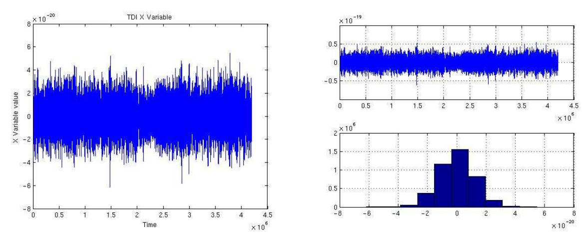
In subspace methods, frequency components are estimated based on eigen analysis or eigen decomposition of the correlation matrix. These methods are effective in detection of sinusoidal signals buried in white noise when signal-noise-ratio is low. Multiple signal classification (MUSIC) or Eigen vector method are examples of these methods.

Table 1: Summary Statistics of TDI - X variable

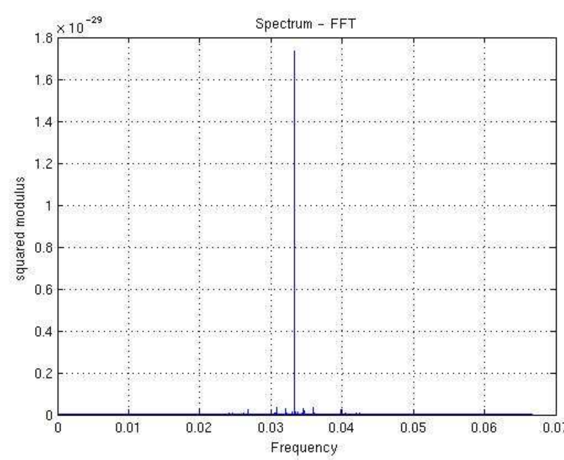
Statistics	Values
Mean	-3.55144e-24
Variance	1.4322e-40
Skewness (Normalized)	-0.0119729
Kurtosis (Normalized)	0.0245682

4 Exploratory Data Analysis

A simulated LISA one year time series data set for X TDI variable is plotted and shown in figure 1. Initially a histogram plot is adopted for the given data set, indicative of gaussian distribution.



(a) TDI X Variable (b) Histogram of TDI X Variable



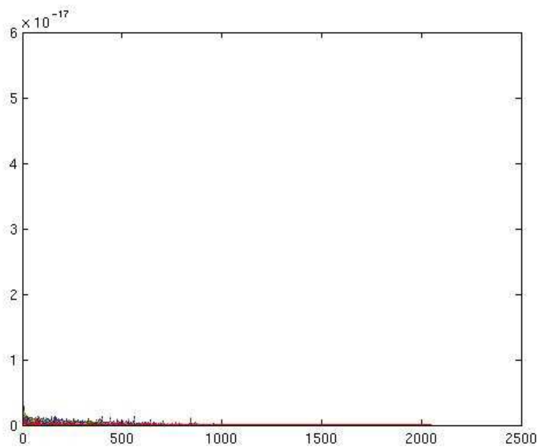
(c) FFT of TDI X Variable

Figure 1: FFT Analysis of TDI - X Variable

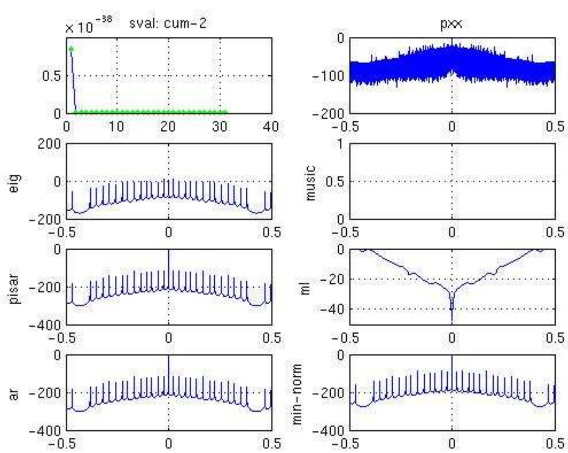
The key statistics of the data like mean, variance, skewness and kurtosis are computed to understand data characteristics, as provided in Table 1.

Harmonic models and spectral analysis are applied to data to estimate peaks in the data. The spectral estimates based on the Eigenvector, Music, Pisarenko, ML, AR, periodogram methods, and the minimum-norm method are computed and shown in figure 2(b).

Gabor representation of time frequency plane of signal is analyzed. For upto 2 Million data points, the Gabor representations and coefficients can be computed with available computer resources. The chirp rate from a global measure is computed to estimate chirp rate with averaged time center and frequency. In figure 1(c), certain strong signals are shown in FFT around 0.03 Hz, but FFT cannot yield chirp wave forms and evolution rates. Using harmonic models, the frequency of signals can be estimated and focussed for further analysis.



(a) STFT of TDI-X

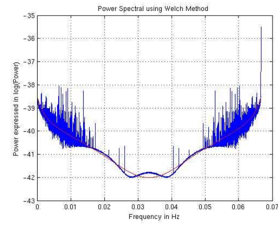


(b) Harmonic models of TDI-X

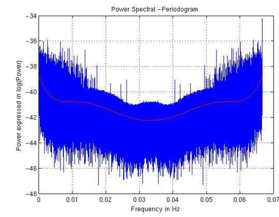
Figure 2: Harmonic models of TDI - X Variable.

In figure 3 and 4, various spectral methods are shown for the same TDI X variable. Using spectrograms, the overall chirp and evolution of structure can be estimated. The start of chirp frequency can be computed using Welch, Periodogram and Yule methods. Covariance and modified covariance methods do not yield any information regarding chirps.

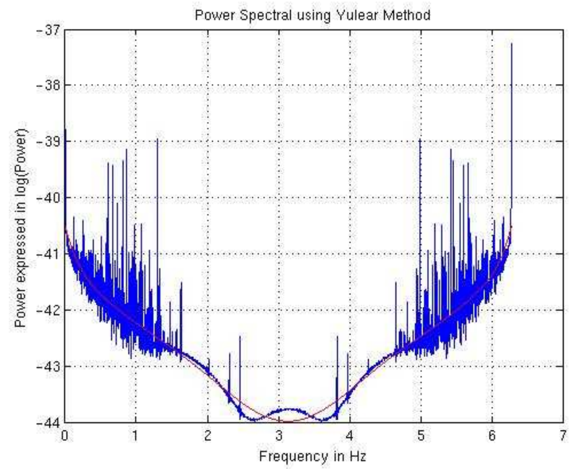
In figure 5(a), Wigner distribution (WD) is computed based on a derived signal from X TDI variable. The derived signal is constructed based on every 10000^{th} data point from the raw signal and the data set is reduced. From 4 Million data points, derived data points length is 4000. The Wigner distribution plots time frequency contour plot and the strong signal which can be considered from time frequency plot is based on contour structure. Figure 5 (b) shows a zoomed version of WD time frequency contour plot. Various methods to analyze chirp waves, Hilbert transform [9] and Wigner distribution method yield the most informations regarding signal and its structure.



(a) Welch Method



(b) Periodogram



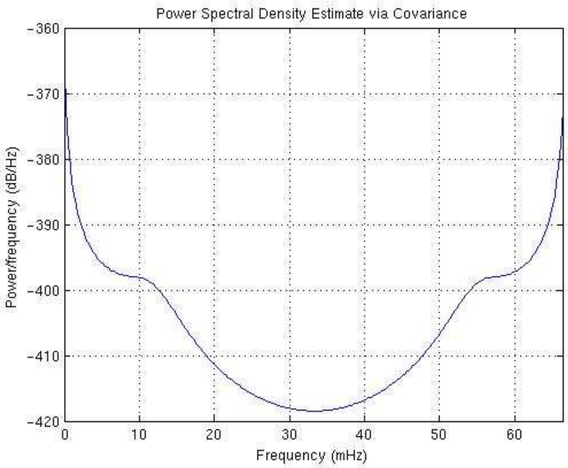
(c) Yulear Method

Figure 3: Spectral Analysis Methods - A

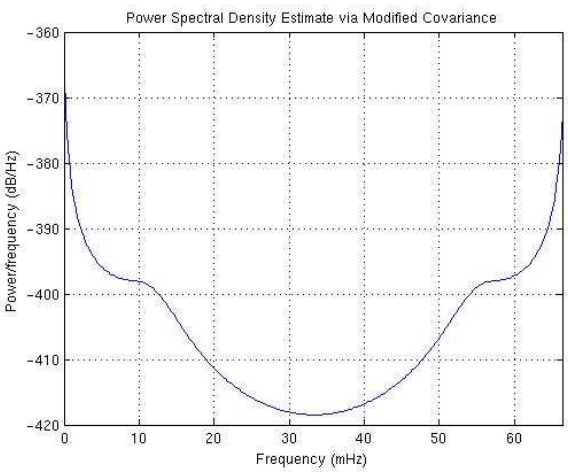
5 Problems in Gravitational Wave Analysis

There are various critical issues from signal analysis aspects:

- 15 seconds cadence data which is integrated over a year corresponds to approximately 2 Million data points
- Complex transforms like Wigner-Ville transforms can not be applied to this huge data set because of complex conjugate computation of mirror image of huge data sets.

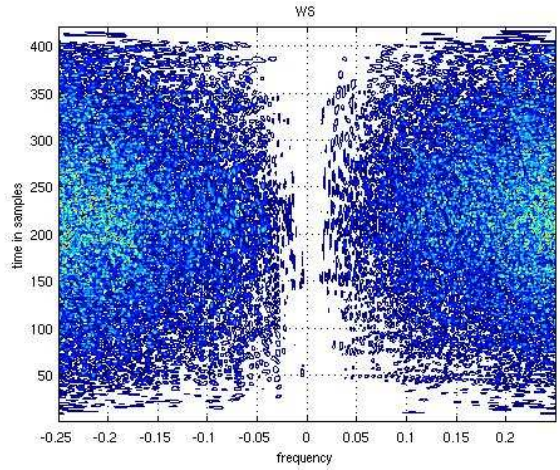


(a) Covariance Method

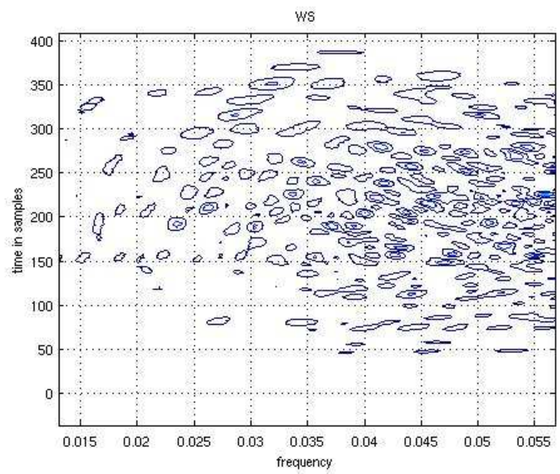


(b) Modified covariance

Figure 4: Spectral Analysis Methods - B



(a)WD -Sampled TDI-X



(b)WD in low frequency

Figure 5: Wigner Distribution

- Amplitude of data is extremely tiny
- Normalisation causes loss of information due to sensitivity of the signal
- Millions of sources, all with low frequency (Millihertz range)
- Variety of chirp evolution rates
- Source parameter search is very time consuming, even after settled with source frequency characteristics
- Multiple iteration/separation of source parameters makes the problem harder to solve with limitations of computational resources
- No single method is available to extract and identify signals

Multiple transforms and spectral methods implemented in hardware needed to identify and extract gravitational signal in near real time. Empirical mode decomposition and computation of instantaneous frequency for larger data set for extreme low frequency with high sensitivity should be adopted to extract gravitational wave characteristics.

6 Conclusion

Resolving close, complex signals in a given time-series is a significant challenge at low frequency. In addition, aliasing creates a similar power-spectrum at higher frequencies in the FFT; and if one does not know in advance what frequency band to look across, a major ambiguity will need to be resolved. A better process characterizing the frequency sources is needed to achieve rapid signal extraction and identification; we will implement our best methods on field programmable gate arrays (FPGA) to lock in the faster methods[10].

In another approach, the signals can be classified based on certain energy level using a neural network. Once a signal is identified and processed using the neural network, the data corresponding to the signal-region could be further processed 'locally' to determine the requisite source parameters rather than dealing with the entire time-series. Such data reduction would enable much faster convergence. This method can be applied as a validating method for FPGA analysis.

Computational cost and resolution capabilities have to be extended for complex signal structure in low frequency. Symmetric Multi-Processing (SMP) using cluster-computing technology is an alternative for rapid and faster convergence with embedded signal extraction methods. Also, implementing Parallel Virtual Machine (PVM) techniques to the signal search process would lead to a robust and more scalable real-time signal identification and search process.

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