Harmonic Analysis of Power System Waveforms Based on Chaari Complex Mother Wavelet

DAN EL MONTOYA
Universidad Central de Venezuela
Electrical Engineering School
University City Caracas
VENEZUELA

Abstract: This work shows an approach based on Chaari Complex Mother Wavelet for the evaluation of harmonic content of power system waveforms. This paper is based on the work developed by Pham and Wong [1] and it is an improvement of the algorithm that they made. Their algorithm can simultaneously identify all harmonics including integer, noninteger and subharmonics based on Gaussian Wavelet. In this work the Wavelet Continuous Transform is applied via a sum based on a modification of the Chaari Wavelet to get harmonic content of a signal. If this modification is used in the calculus of Pham and Wong [1] then better results can be obtained in the determination of harmonic content.

Key–Words: Harmonic, Wavelet Continuous Transform, Power System Waveforms.

1 Introduction

Wavelets are oscillating waveforms of short duration which decay quickly to zero. This waveforms are dilated and shifted to form wavelets families. The Wavelet Continuous Transform is based on the wavelets families to obtain better information about spectral analysis of a waveform.

The wavelet transform has been implemented in the last decade on many issues on power electric systems. For example the spline, Daubechies and Morlet's wavelets have been used to detect different faults. The applications have been focused on the classification of the disturbances in measurement on power quality [2]-[5].

The wavelet analysis has been used on steady state signal analysis. In this way the wavelet transform has been used to identify harmonics, subharmonics and noninteger harmonics [1].

Wavelet analysis has been used to calculate the flicker voltage. The Morlet and the Gaussian wavelets have been used specifically in the determination of the flicker signal spectral [6]-[9].

This paper shows an approach based on Chaari Complex Mother Wavelet for the evaluation of harmonic content of power system waveforms. It is based on the work developed by Pham and Wong [1] and it is an improvement of the algorithm that they made. Their algorithm can simultaneously identify all harmonics including integer, noninteger and subharmonics based on Gaussian Wavelet.

In this work the Wavelet Continuous Transform is applied via a sum based on a modification of the Chaari Wavelet to get harmonic content of a signal. If this modification is used in the calculus of Pham and Wong [1] then better results can be obtained in the determination of harmonic content.

2 Wavelet Transform

The Wavelet Transform has been used in the last years as common tool in signals analysis. In this section the aspects more important on this theory are shown .

Although the Fourier Transform is widely used to analyze signals in frequency domain it only gives useful information when the signal is stationary or periodic. If infinite longitude base is used, the sudden changes are spreaded in frequency domain and that does not allow the information extraction.

In these cases the Short Time Fourier Transform is better, and it is defined in the following equation :

$$STFT_x(\omega,\lambda) = \int_{-\infty}^{\infty} x(t)v(t-\lambda)e^{-j\omega t}dt \qquad (1)$$

The Short Time Transform Fourier works with the same time-frequency resolution through the analysis. For this reason its application might not give reliable results.

The Continuous Wavelet Transform might be used to resolve the time-frequency resolution problem.

The Wavelet Transform makes a signal approximation through wavelet families. These families are obtained by dilating and shifting a mother wavelet. This procedure allows analyze data at different scales or resolutions. That is, the Wavelet Transform allows to analyze as thick information as detailed information.

A Wavelet is a function $\psi(t) \in L^2(\mathbb{R})$ with zero average (bandpass signal):

$$\int_{-\infty}^{\infty} \psi(t)dt = 0 \tag{2}$$

The space $Ł^2(\mathbb{R})$ contains the energy functions, this is:

$$\int_{-\infty}^{\infty} \psi^2(t)dt < \infty \tag{3}$$

A wavelet is normalized to $||\psi(t)|| = 1$. The wavelet center is near of t=0. The signal $\psi(t)$ is called mother wavelet. A wavelet family is obtained by scaling and shifting the mother wavelet $\psi(t)$ through ”a” and ”b”.

$$\psi(t) = \frac{1}{\sqrt{a}}\psi(\frac{t-b}{a}) \tag{4}$$

An example of wavelet is the modulated Gaussian function:

$$\psi(t) = e^{j\omega_0 t - 0.5t^2} \tag{5}$$

The Gaussian Wavelet above is used by Pham and Wong [1]in their development.

The Chaari Wavelet [10] is used in this work to improve the results obtained by Pham and Wong [1]. This Wavelet is defined as follows:

$$\psi(t) = (1 + s|t| + \frac{s^2}{2}t^2)e^{-|s|t}e^{j\omega_0 t} \tag{6}$$

And ”s” is:

$$s = \frac{2\pi}{\sqrt{3}} \tag{7}$$

Also is used in this work the following Chaari Wavelet modification :

$$\psi(t) = (1 + s|t| + \frac{s^2}{2}t^2 + \frac{|s|^3}{6}t^3)e^{-|s|t}e^{j\omega_0 t} \tag{8}$$

The Continuous Wavelet Transform is defined as:

$$CWT_f(a,b) = \langle f(t), \psi_{a,b}(t)\rangle \tag{9}$$

$$CWT_f(a,b) = \int_{-\infty}^{\infty} f(t)\frac{1}{\sqrt{a}}\overline{\psi}(\frac{t-b}{a})dt \tag{10}$$

As was commented before, the resolution does not change in the Short Time Fourier Transform. If the Wavelet Transform is used, the resolution changes with the value ”a”. This change allows to improve as frequency as time resolution, this is, both detailed and thick information can be analyzed.

The original signal x(t) can be reconstructed from the Wavelet Transform coefficients by the Inverse Continuous Wavelet Transform:

$$x(t) = \frac{1}{C_\psi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} CWT_f(a,b)\psi_{a,b}(t)\frac{dadb}{a^2} \tag{11}$$

3 Harmonic identification using Continuous Wavelet Transform

The calculus of the Continuous Wavelet Transform is approximated through a Sum. If time variable is sampled with a Ts period then the Continuous Wavelet Transform can be approximated as follows:

$$CWT_f(a,b) = T_s\sum_{n=-\infty}^{\infty} f(t)\frac{1}{\sqrt{a}}\overline{\psi}(\frac{nT_s-b}{a}) \tag{12}$$

The equation number (12) can be reduced if the wavelet analysis is done without wavelet shifting. However This is possible only if the wavelet is dilated from the mother wavelet center instead from zero. If the duration of the signal f(t) is T_{ds} and the duration of the signal is T_{dw} then $T_{ds} = N_{ds}T_s$ and $T_{dw} = N_{dw}T_s$. The values N_{ds} and N_{dw} are the number of points of the analyzed signal and mother wavelet respectively. If this changes are done then the Wavelet Transform can be approximated as follows:

$$CWT_f(a) = T_s\sum_{n=1}^{N_{ds}} f(t)\frac{1}{\sqrt{a}}\overline{\psi}(\frac{nT_s}{a} - \frac{N_{dw}T_s}{2}) \tag{13}$$

If F_0 is the wavelet center used in frequency domain and F_{Min} and F_{Max} are the values that define the frequency band where the signal is analyzed, then the values of the scale ”a” will be between:

$$a_{Min} = \frac{F_0}{F_{Max}} \tag{14}$$

and

$$a_{Max} = \frac{F_0}{F_{Min}} \tag{15}$$

If F_s is the sampled frequency then the frequency maximum value will be $F_s/2$ then:

$$a_{Min} = \frac{2F_0}{F_s} \tag{16}$$

Harmonic Amplitudes can be obtained from the Continuous Wavelet Transform coefficients in a similar way they are obtained from Fourier Transform. The amplitude of the nth harmonic is given by [1]:

$$A_n = \frac{C\sqrt{a}|CWT(a)_n|}{T_{dw}} \quad (17)$$

where C is a constant. The value of C can be found by transforming a sinusoidal waveform. The phase of the nth harmonic is given by [1]:

$$\theta_n = \arctan(\frac{Im(CWT(a)_n)}{Re(CWT(a)_n)}) \quad (18)$$

4 Examples of Harmonic identification of synthesized Waveforms

These examples shows the harmonic identification of synthesized waveforms based on Wavelet Transform and Fourier Transform. In the first example is used the mother wavelets of Gaussian, Chaari and modified Chaari. The analized waveform has been taken from Pham and Wong's [1] work and is:

$$\begin{aligned} x(t) &= 0.3cos(2\pi f_1 t + 70^o) + 0.2cos(2\pi f_2 t + 60^o) \\ &+ 0.7cos(2\pi f_3 t + 80^o) + 1.0cos(2\pi f_4 t) \\ &+ 0.5cos(2\pi f_5 t + 90^o) + 0.4cos(2\pi f_6 t + 40^o) \end{aligned} \quad (19)$$

Where f_1,f_2,f_3,f_4,f_5 and f_6 are 25 Hz, 35.35 Hz, 35.85Hz, 50 Hz, 86.6 Hz and 150 Hz respectively.The fundamental component is 50 Hz. The 25 Hz, 35.35 Hz and 35.85 Hz are subharmonics. The last two ones are separated by 0.5 Hz. The 86.6 Hz and the 150 Hz components are noninteger and integer harmonic respectively.

In this application the sampled frequency F_s is 440 Hz and the number of points is $N_{ds} = 4000$. Then the analyzed signal duration is $T_{ds} = 9.09s$. approximately. If the sampled frequency is 440 Hz then the highest frequency analyzed $F_{Max}$is 220 Hz. F_{Min} is chosen in 20 Hz. In the Pham and Wong Work is suggested the F_0 value in 38 Hz and this value will be assumed. The extremes values of the scaling are calculated with the equations (15) and (16). These values are $a_{Min} = 0.1725$ and $a_{Max} = 1.9$.

The results obtained using the Fourier Transform and the Wavelet Transform with the Gaussian, Chaari and modified Chaari mother wavelets are shown in Figure 1.

The Harmonic amplitudes calculated by the Fourier Transform and Wavelet Transform are shown in Table 1.

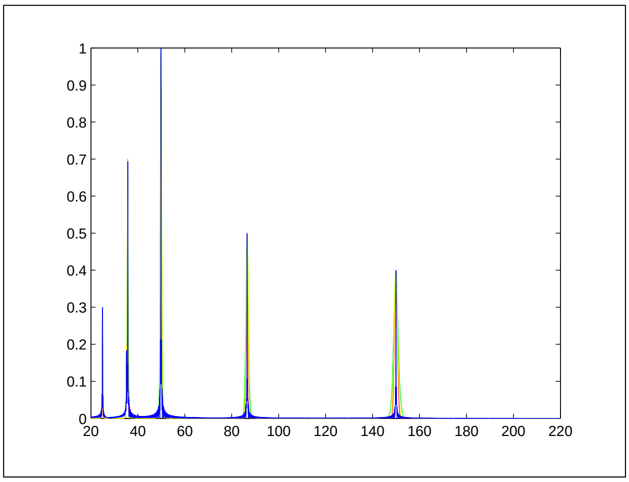

Figure 1: DFT and CWT of the signal

Amplitude				
True	**Fourier**	**Gaussian**	**Chaari**	**Mod.Chaari**
0.3	0.29936	0.299555	0.299915	0.299718
0.2	0.18309	0.198247	0.183571	0.195983
0.7	0.6933	0.69936	0.68618	0.698246
1	1	1	1	1
0.5	0.49931	0.499936	0.49999	0.499965
0.4	0.39953	0.399954	0.399992	0.399975

Table 1: Harmonic Amplitudes of the First Example

Based on the results obtained in Table 1, if Fourier Transform and Gaussian Wavelet Transform are applied, the results are similar to those found by Pham and Wong [1]. They found that the new method (Gaussian Wavelet Transform) calculates better amplitudes than Fourier Transform. The analysis made with the Chaari Mother Wavelet shows that the calculus of the amplitude is more accurate this way, excepting the harmonics separated by 0.5 Hz. This ones give better results by the Wavelet Transform using Gaussian Mother Wavelet. If the mother wavelet is changed for Modified Chaari Mother Wavelet the results improve respect the results found by Chaari Wavelet Mother without modification, but other time the subharmonics separated by 0.5 Hz are better calculated by the Gaussian Mother Wavelet than the other ones.

The next table shows the amplitudes results for the second example. This is setting f_1,f_2,f_3,f_4,f_5 and f_6 in 25 Hz, 35.35 Hz, 36.35Hz, 50 Hz, 86.6 Hz and 150 Hz respectively. In this example the second and third harmonic are separated by 1 Hz.

The results found on this cases show that the best analysis in frequency domain is achieved by Wavelet Transform with Chaari Mother Wavelet .

Amplitude				
True	**Fourier**	**Gaussian**	**Chaari**	**Mod.Chaari**
0.3	0.302016	0.299623	0.299927	0,299755
0.2	0.207110	0.199713	0.200185	0.200716
0.7	0.702050	0.699929	0.699935	0.699477
1	1	1	1	1
0.5	0.499623	0.499935	0.49999	0.499964
0.4	0.399911	0.399953	0.399992	0.399974

Table 2: Harmonic Amplitudes of the Second Example

The Harmonic phases calculated by the Fourier Transform and Wavelet Transform for the first example are shown in Table 3.

Phases (grad)				
True	**Fourier**	**Gaussian**	**Chaari**	**Mod.Chaari**
70	70.4533	70.0121	70.0121	70.0121
60	73.5158	59.5063	170.7771	58.1054
80	79.1283	80.046	81.5201	80.1741
0	-0.0562	-0.0017	-0.0003	-0.0009
90	90.0832	90.0029	90.0006	90.0016
40	39.9482	39.9992	39.9999	39.9995

Table 3: Harmonic Phases of the First Example

The results showed in the previous table indicate that better phases results are obtained by applying Wavelet Transform instead Fourier Transform. The best results are obtained through the Chaari Mother Wavelet. The analysis made with the Chaari Wavelet Mother shows that the calculus of the phase is more accurate by this way but the harmonics separated 0.5 Hz are excepted

The phases results obtained for the second example are shown in the following table.

Phases (grad)				
True	**Fourier**	**Gaussian**	**Chaari**	**Mod.Chaari**
70	70.3376	70.0087	70.0087	70.0087
60	61.1767	60.0954	59.6739	58.8376
80	79.8436	79.995	80.0172	80.1012
0	0.0219	-0.0017	-0.0003	-0.0009
90	90.1227	90.0029	90.0006	90.0016
40	39.9345	39.9991	39.9999	39.9995

Table 4: Harmonic Phases of the Second Example

The previous results show that the best tool to analyze these signals in frequency domain is the Wavelet Transform. The Chaari Mother Wavelet gives better results than the Gaussian Mother Wavelet without the harmonics separated by 1 Hz.

5 Conclusion

The results explained in the previous sections show that the Wavelet Transform is a better tool to analyze power systems waveforms in frequency domain than the Fourier Transform. Better results are obtained through the Chaari Mother Wavelet, comparatively with the Gaussian Mother Wavelet if the harmonic in the frequency domain are sufficiently separated. This development complete the previous works in this area and can be used to improve the harmonic analysis of Power Systems Waveforms.

References:

[1] V.L. Pham, K.P. Wong, Wavelet-Transform-Based Algorithm for Harmonic Analysis of Power Waveforms, *IEE Proc.-Gener. Transm. Distrib.* Vol.16, No. 3, 1999, pp. 249–254.

[2] L. Angrisani, M. Apuzzo, and A. Testa, A Measurement Method on the Wavelet Transform for Power Quality Analysis, *IEEE Trans. Power Delivery* Vol.13, No. 10, 1998, pp. 990–998.

[3] A. Gaouda, M. Sultan, and A. Chikhani, Power Quality Detection and Classification Using Wavelet-Multiresolution Signal Decomposition, *IEEE Trans. Power Delivery* Vol.14, No. 10, 1999, pp. 1469–1476.

[4] O. Poisson, R. Pascal, and M. Meunier, Detection and Measurement of Power Quality Disturbances Using Wavelet Transform, *IEEE Trans. Power Delivery* Vol.15, No. 7, 2000, pp. 1039–1044.

[5] M. Karimi, M. Hossein, and M. Reza, Wavelet Based on-line Disturbance Detection for Power Quality Applications, *IEEE Trans. Power Delivery* Vol.15, No. 10, 2000, pp. 1212–1220.

[6] S. Huang and Ch. Hsieh, Applications of Continuous Wavelet Transform for Study of Voltage Flicker-Generated Signals, *IEEE Trans. Aerosp. Elect. Syst* Vol.36, No. 7, 2000, pp. 952–932.

[7] M. Chen and P.Meliopoulos, Wavelet-based Algorithm for Voltage Flicker Analysis, *Proc. IEEE 9th Int. Conf. Harmonics Qual. Powe* 2000, pp. 732–738.

[8] M. Chen and P. Meliopoulos, Hybrid digital algorithm for harmonic and flicker measurements, *Proc. IEEE PES Winter Meeting* 2002, pp. 1488–1493.

[9] S. Huang and Ch. Wen, Enhancement of digital equivalent voltage flicker measurement via continuous wavelet transform, *IEEE Trans. Power Delivery* Vol.19, No. 4, 2004, pp. 663–670.

[10] O. Chaari, M. Meunier and F. Brouaye, Wavelets: A new tools for the resonant grounded power distributions systems relaying, *IEEE Trans. Power Delivery* Vol.11, No. 3, 1996, pp. 1301–1308.