Study and Simulation of Nonlinear effects in Bire-Fringent Fibers Using Efficient Numerical Algorithm

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Abstract: Pulse propagation in a birefringent medium carries due weightage and importance in passive modelocking of fiber lasers. We have carried out study of nonlinear effects in birefringent medium by simulating pulse propagation using adaptive step size method, which is globally an efficient algorithm for solving Schrodinger type equations using split step Fourier method. Nonlinear optical loop mirror was realized and it was revealed that the results can be used in forming a figure of eight fiber laser using NOLM, where NOLM behaves like a fast saturable absorber. Novelty lies in producing accurate results using adaptive step size method.

Index Terms: Non-linear Schrödinger equation (NLSE), Self Phase modulation (SPM), cross phase modulation (XPM), kerr non-linearity, Symmetrized split step Fourier method (SSSFM), Nonlinear Optical Loop Mirror (NOLM).

1. Introduction

Linear birefringence causes the components of light travel with different group velocities [1,5]. Even a single mode fiber supports two orthogonally polarized modes with the same spatial distribution. In an idea fiber the effective refractive indices n_x and n_y of both the modes are identical [1]. It is pertinent to mention here that all fibers exhibit some modal birefringence $(n_x \neq n_y)$; this is due to unintentional variations in the core shape and anisotropic stresses along the fiber length. However the degree of modal birefringence, $B_m = |n_x - n_y|$ and the orientation of x and y axes changes randomly over a length scale $\approx 10m$ unless it is catered for [1,2].

A variety of linear and nonlinear fiber properties enable new applications in photonic devices as elements of a fiber laser cavity. One of the most interesting fiber element is the loop interferometer, also termed as Sangnac interferometer; it has transmission characteristics that can be controlled by outside influences, such as strain birefringence, current birefringence and temperature variations [3]. The nonlinear optical loop mirror (NOLM) [4] which is an example of Sangnac interferometer, has been used in a number of applications such as optical switching, passive modelocking of fiber lasers, logic gates, fiber sensors and all optical demultiplexing of data streams [5, 6, 7, 8, 9]. The basic concept of the conventional NOLM is based on the differential nonlinear phase phase shift between the counter propagating, linearly polarized light beams in the loop interferometer. Such a NOLM has high/low power reflection coefficient and provides nonlinear switching by an assymetrical coupler at the loop's input, i.e. one in which coupling coefficient differs from 50/50.

The transmission properties of the NOLM can be modified by using birefringent fiber and different polarization orientations between the counter propagating beams.

The difference in mode propagation constant for different polarizations is defined as the modal birefringence [1] or phase birefringence B.

$$B = |n_x - n_y| = \frac{|\beta_x - \beta_y|}{2\pi/\lambda_0} = \frac{|\beta_x - \beta_y|}{k_0} \text{ ----- (1)}$$

n_x, n_y, β_x and β_y are the effective mode indices and mode propagation constants of the two orthogonal principle axes respectively. Since the effective mode index of the two polarizations are not the same, by injecting a linearly polarized light at 45^o to one principle axis, the polarization will evolve periodically from linear to elliptical, elliptical to circular and vice versa along the length of the fibre. The length of fibre for the light to exit with the exact same polarization as at the input is called the beat length LB.

$$L_B = \frac{2\pi}{|\beta_x - \beta_y|} = \frac{\lambda}{B} \text{ ---- (2)}$$

Figure 1 shows a schematic diagram of how the polarization varies along a birefringent fibre.

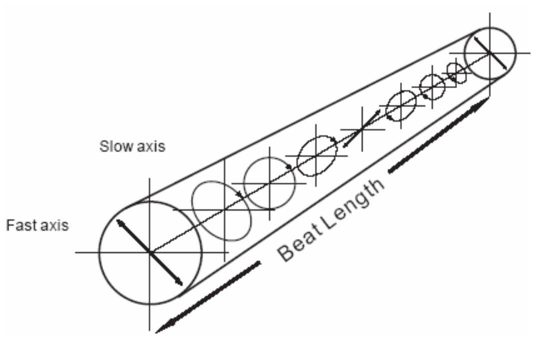

Fig. 1: A schematic diagram of how the polarization vary along a birefringent fibre when the input was polarized at 45^o to one axis.

Two pulses propagating in two orthogonal axes of a birefringent fibre at different group velocities (due to the difference in the propagation constants); they give rise to group velocity mismatch (GVM), and can be quantified by the group birefringent parameter. Group birefringence is proportional to the difference of the group index δn of the two orthogonal modes. δn is related to the difference in propagation constant $\delta\beta$ and hence by knowing $\delta\beta$ we can determine the group birefringence, and $\delta\beta$ is given by.

$$\delta\beta = k_0 B \text{ ----- (3)}$$

Applying Taylor expansion on equation 3 results into following expression:

$$\delta\beta(\omega) = \delta\beta_0 + \delta\beta_1(\omega - \omega_o) + \frac{\delta\beta_2}{2!}(\omega-\omega_0)^2 + \frac{\delta\beta_3}{3!}(\omega-\omega_0)^3 + \dots \quad \text{- (4)}$$

$$\delta\beta_m = \left.\frac{d^m\delta\beta}{d\omega^m}\right|_{\omega=\omega_0}, (m = 0,1,2,3...) \text{ --- (5)}$$

The parameter $\delta\beta_1$ is called the group birefringent parameter, and is proportional to the difference in group index of the axes. Differentiating $\delta\beta_1$ with respect to ω gives the difference in the GVD parameters $\delta\beta_2$. Rest of the paper is organized as follows: part two describes pulse propagation birefringent medium along with introducing some equations which are to be used in further work, part three discusses simulation process and results, part four shows the comparison of results between constant and adaptive step size method along with error analysis whereas part five comprises of conclusion.

2. Pulse Propagation in Bire-fringent medium

The coupled propagation equation governing evolution of two polarization components, where A_x and A_y is the respective slowly varying envelope approximation, are as follows:

$$\frac{\partial A_x}{\partial z} = -i\frac{\beta_2}{2}\frac{\partial^2 A_x}{\partial t^2} + \frac{\delta\beta}{2}\frac{\partial A_y}{\partial t} + i\frac{k}{2}A_y + i\gamma\left(|A_x|^2 + \frac{2}{3}|A_y|^2\right)A_x + i\frac{\gamma}{3}A_x^*\left(A_y\right)^2\exp(-2ikz) \quad \text{- (6A)}$$

$$\frac{\partial A_y}{\partial z} = -i\frac{\beta_2}{2}\frac{\partial^2 A_y}{\partial t^2} + \frac{\delta\beta}{2}\frac{\partial A_x}{\partial t} + i\frac{k}{2}A_x + i\gamma\left(|A_y|^2 + \frac{2}{3}|A_x|^2\right)A_y + i\frac{\gamma}{3}A_y^*\left(A_y\right)^2\exp(-2ikz) \quad \text{- (6B)}$$

$\delta\beta$ = Difference between the inverse group velocities of the two polarizations.

and $\delta\beta = \left|\frac{1}{\upsilon_g^x} - \frac{1}{\upsilon_g^y}\right| = \frac{\Delta n}{c}$

$k = |\beta_{1x} - \beta_{1y}| = \frac{2\pi}{L_B}$ is the wave vector mismatch due to a linear or modal bire-fringence.

β_2 = Group velocity dispersion coefficient.

γ = cross phase modulation coefficient representing the non-linear interaction between the two components and it is dependent on kerr coefficient & fiber parameters.

If FWM terms in coupled mode equations is neglected then it can have a vector soliton solution [10]; which is as follows:

$$u_{x,y}(\xi,\tau) = \sec h\left[\left(1+\frac{2}{3}\right)^{1/2}\tau\right]\exp\left[i\left(1+\frac{2}{3}+\delta^2\right)\frac{\xi}{2} \mp i\delta\tau\right] \text{- (7)}$$

$$u_{x,y} = \frac{A_{x,y}}{\sqrt{P_o}}; \xi = \frac{z}{L_D}; \tau = \frac{t}{T_o}; \delta = \frac{\delta\beta L_D}{2T_o}.$$

The sign of last phase term involving the product of $\delta\tau$ reflects the shift of carrier of the soliton components in opposite directions.

Equations 13A & 13B can be simplified for optical fibers with large bire-fringence. For such fibers the beat length L_B is much smaller than typical propagation distances. The simplified equations governing propagation of optical pulses in an elliptically birefringent fiber are governed by the following set of coupled mode equations [2]:

$$\frac{\partial A_x}{\partial z} + \beta_{1x}\frac{\partial A_x}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A_x}{\partial t^2} + \frac{\alpha}{2}A_x = i\gamma\left(|A_x|^2 + B|A_y|^2\right)A_x \quad \text{--- (8A)}$$

$$\frac{\partial A_y}{\partial z} + \beta_{1y}\frac{\partial A_y}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A_y}{\partial t^2} + \frac{\alpha}{2}A_y = i\gamma\left(|A_y|^2 + B|A_x|^2\right)A_y \quad \text{--- (8B)}$$

Equations 15A & 15B are the coupled NLS equations. The coupling parameter B depends on the ellipticity angle θ and can vary from $\frac{2}{3}$ to 2 for values of θ in the range $0 \to \frac{\pi}{2}$. For a linearly Birefringent fiber $\theta = 0$

and $B=\frac{2}{3}$. For a circularly birefringent fiber $\theta=\frac{\pi}{2}$ and $B=2$.

Wave number difference between two modes $=2\beta=\frac{2\pi\Delta n}{\lambda}$.

Based on the birefringent fiber nonlinear optical loop mirror can be developed just by joining the two ends of a coupler using birefringent fiber [11]. (see figure 2 below)

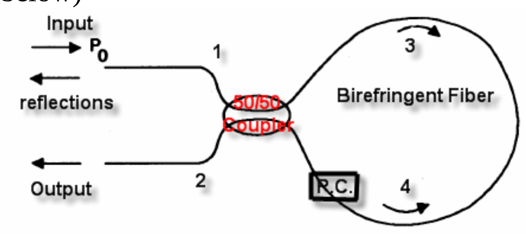

Fig. 2: Schematic of a Nonlinear Optical Loop Mirror.

The input is provided from port number 1 of coupler and after the pulse is propagated in the loop from two opposite ends the output is achieved at the port number 2. In order to complete the loop to make a laser cavity, port number 1 and port number 2 of the coupler can also be joined, (see figure 3) [11].

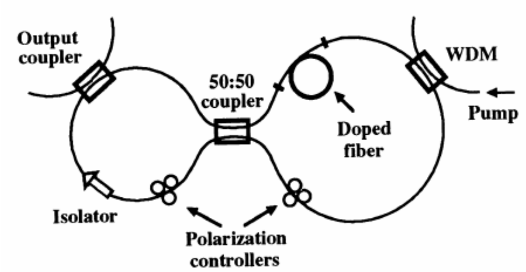

Fig.3: Schematic of a figure-8 cavity for mode-locked fiber laser. [11]

However in this paper, we will restrict our results to pulse propagation in a nonlinear optical loop mirror and it will be shown that these results can be used for building up a passively modelocked fiber laser by adding gain medium.

3. Simulation and Results

The schematic of NOLM has been shown in figure 2. the input to the loop is labeled 1; the signal is split by a 50/50 coupler into two counter propagating sections of the loop. The polarization controller rotates the polarization of the counter propagating pulses by 90 degrees to provide inverted switching characteristics. The evolution of the two counter propagating pulses in the NOLM is governed by equation 6(a&b).

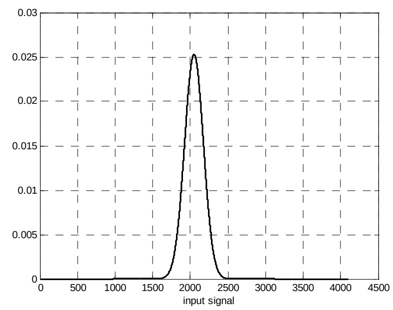

Fig.4: Input Signal (Chirped Gaussian Pulse)

The input pulse is linearly polarized at 45 degrees to the principal axis, its phase is as follows:

$$ux=u.*\exp(i\pi/4);uy=u.*\exp(-i\pi/4) \text{ -- } (9)$$

The input pulse power is kept at a value so as to get the accumulated power of signal P=1; it was kept so as to have 0.5W power still available after the signal is being splitted into two equal parts at the coupler. Input from point labeled 2 was assumed as zero by defining a zero vector equal in length to signal at input port 1. Input coupling in 50/50 splitter is as follows $(\alpha=0.5)$:

$$\begin{aligned} u_3^{xy} &= \sqrt{\alpha}*u_1^{xy}+i(1-\alpha)*u_2^{xy} \\ u_4^{xy} &= i\sqrt{(1-\alpha)}*u_1^{xy}+\sqrt{\alpha}*u_2^{xy} \end{aligned} \text{ ---- } (10)$$

As the pulse propagating counter clockwise from point 4 passes through a polarization controller that rotates the polarization by 90^o before being launched in the birefringent fiber, hence the phase of signal u_4^{xy} will be changed as follows:

$$u_4^x=u_4^x.*\exp(i\pi/2);u_4^y=u_4^y.*\exp(-i\pi/2) \text{ - } (11)$$

Coupled non-linear Schrödinger equations responsible for propagation in loop were solved in Matlab using Symmetrized split step Fourier algorithm. In case of coupled mode equations output signal from one equation is to be fed in the other equation. This has been catered for very carefully. In Symmetrized split step Fourier method (SSSFM) a fiber is divided into small parts (not necessarily of equal length) and then non-linearity is considered at the center of the segment, while dispersion is considered at the two halves. Fig.4 below describes the process.

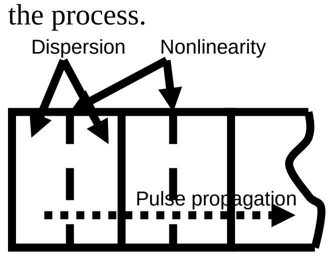

Fig.5: Symmetrized Split Step Fourier Method (SSSFM).

Schematic of solving coupled equations is shown in Fig.5 below. Coupled equations had been solved at various input values. The heart of calculations is the adaptive step size method. Adaptive step size method is explained in detail in [12] and has been implemented in [13, 14]. It has been observed that adaptive step size method is efficient and accurate at low nonlinearity values and provides accurate results at the expense of computational time in case of high nonlinearity values.

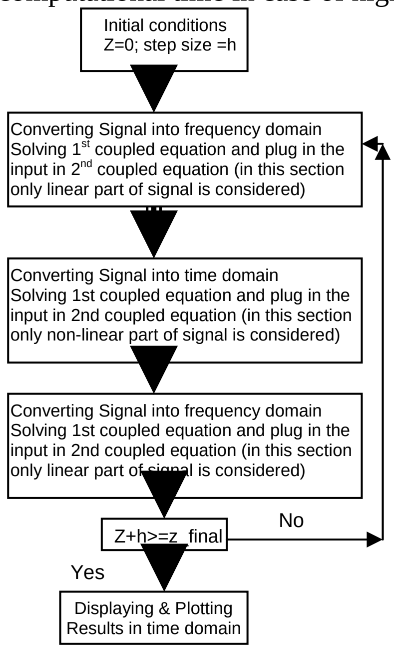

Fig.6: Schematic flow chart of solving coupled Schrödinger equations using SSSFM algorithm.

The fiber nonlinearity is .01/W/m, 2nd order dispersion is -20 e-27 s2/m. The clockwise rotating pulse will experience the birefringence bias rotation by 90^o after passing around the loop:

$u_3^x = u_3^x .* \exp(-i\pi/2); u_3^y = u_3^y .* \exp(i\pi/2)$ - (12)

Output coupling in 50/50 splitter is as follows:

$$u_{signal}^{xy} = \sqrt{\alpha} * u_3^{xy} + i\sqrt{1-\alpha} * u_4^{xy}$$
$$u_{reflections}^{xy} = i\sqrt{1-\alpha} * u_3^{xy} + \sqrt{\alpha} * u_4^{xy} \quad \text{-- (13)}$$

The nonlinear optical loop mirror length was kept comparable with beat length, the beat length used in our simulations is 150 meters. Below are results for 150 meters long NOLM:

Clockwise and counter clockwise propagating signal (both x & y components) at the output are shown in figure 7 & 8. Nonlinear phase shift is shown in figure 9. Peak amplitude of pulse at all intervals inside NOLM are shown in figure 10.

Nonlinear effects on birefringent fibers had been studied by observing various results, here we are producing some more results, where

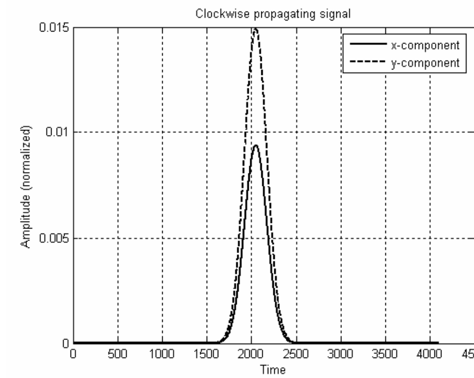

Fig.7: Clockwise propagating pulse in NOLM

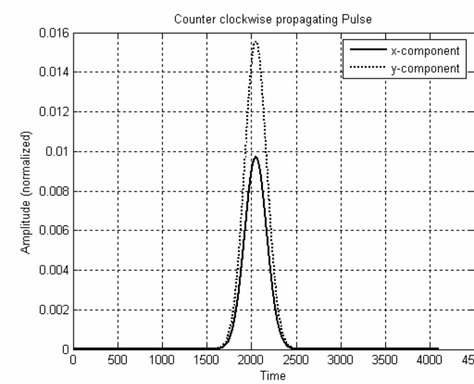

Fig.8: Counter clockwise propagating pulse in NOLM

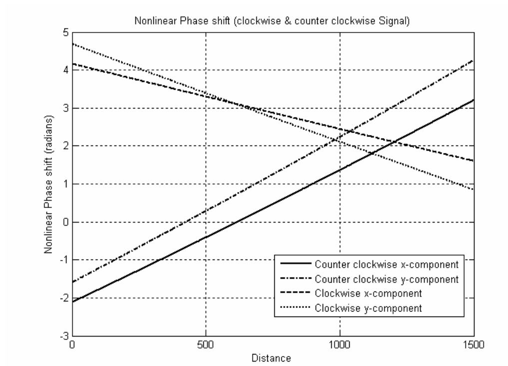

Fig.9: Nonlinear phase shift (NOLM length = Lb)

the length of NOLM is kept 2*Lb. (Lb is beat length). Nonlinear Phase shift for clockwise and counter clockwise propagating pulse is shown in figure 11, peak amplitudes of respective signals are shown in figure 12. Referring to figure 2, the signal at the output port of NOLM is shown in figure 13, whereas back reflections at port number 1 are shown in figure 14.

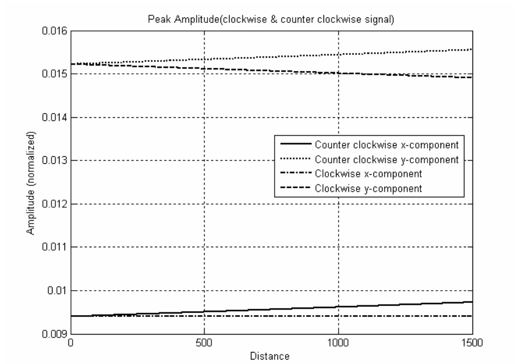

Fig.10: Peak amplitude (NOLM length = Lb)

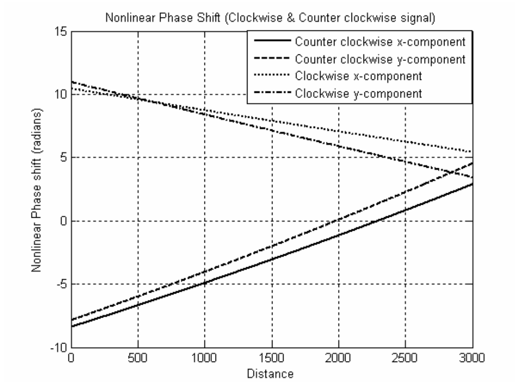

Fig.11: Nonlinear Phase shift (NOLM length = 2*Lb)

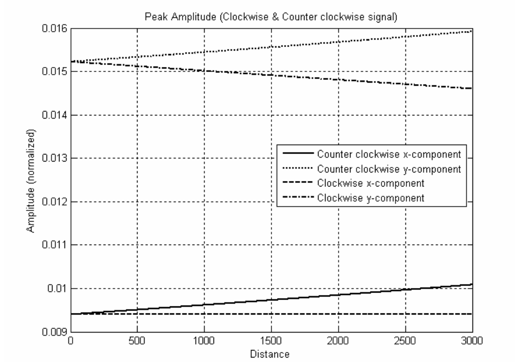

Fig.12: Peak Pulse Amplitude (NOLM length = 2*Lb)

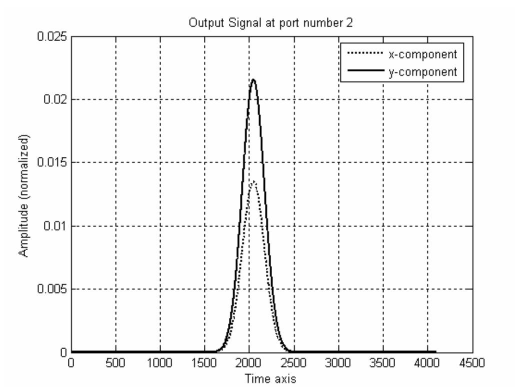

Fig.13: Output Signal at port number 2

4. Error analysis:

All the results were produced in parallel by Adaptive step size method, which was at the cost of computational time, which was increased by manifolds.

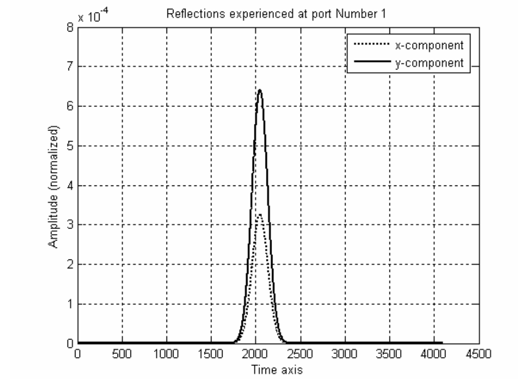

Fig.14: Reflections at port Number 1 of coupler

The adaptive step size method[12] was used and successfully implemented by us [13, 14] in solving schrodinger type coupled mode equations. Some of the error analysis results are produced as below (see figure 15, 16, 17 & 18):

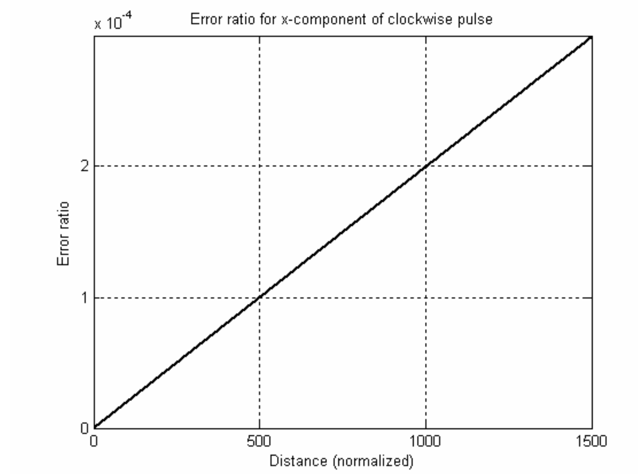

Fig. 15: Error ratio for x-component of clockwise Signal

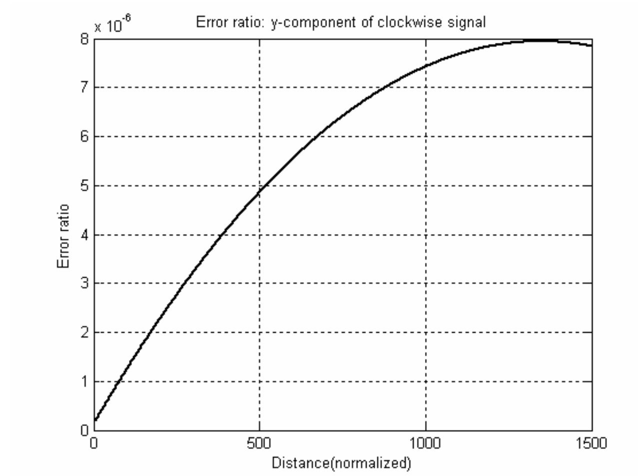

Fig.16: Error ratio for y-component of clockwise Signal

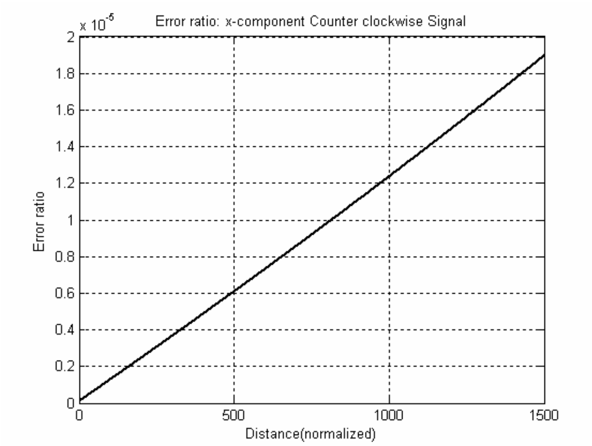

Fig.17: Error ratio for x-component of counter clockwise Signal

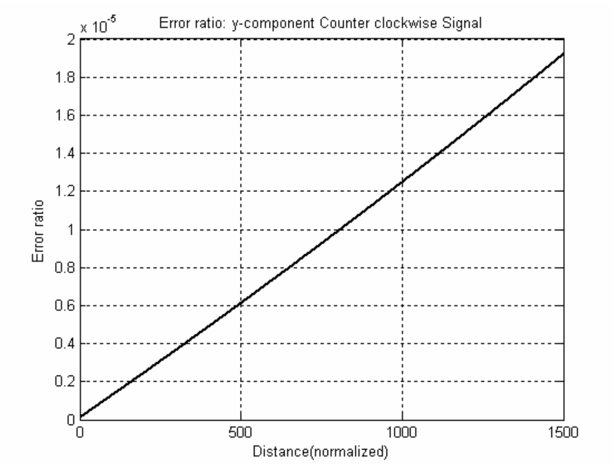

Fig.18: Error ratio for y-component of counter clockwise Signal

5. CONCLUSION

The study of pulse propagation in nonlinear optical loop mirror based upon birefringent fiber medium has shown us that pulse amplitude and phase is changing; which are occurring only due to application of nonlinear effects as the loss is assumed to be zero and gain medium is not realized in present work. It is pertinent to mention that pulse propagation through a non-linear medium (though it is single mode) cause the pulse under go various no linear phenomenon, like Self phase Modulation (S.P.M.), cross phase modulation (X.P.M.) and Four Wave Mixing (F.W.M.). The combined and managed effects of the above mentioned non-linear effects give rise to passive mode locking mechanism. The combined action of nonlinear effects inside NOLM acts similar to fast saturable absorber action. The pulse compression ratio after one round of NOLM between the output pulse (see figure 13) and input pulse (see figure 4) is 0.9932. Hence it shows that our numerical model of NOLM can be successfully used in forming modelocked fiber lasers. The present research is being extended forward to design and formulate a passively modelocked fiber laser based on NOLM. Hence it proves the importance of work carried out.

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