Multi-Expert Opinions Combination Based on Evidence *Theory*

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Abstract: - An approach is presented to multi-expert's opinions combination based on the Dempster-Shafer evidence theory. In the method, we use multi-expert's knowledge as evidence, the possible value of weight as frame of discernment, expert's evaluation to a weight on frame of discernment as basic probability assignment, and use D-S rule combining to give fusion basic probability assignment m. Finally, the weight is given according to fusion basic probability assignment. The result is shown that the method can keep exactitude information, reduce conflict factor, strong degree opinion and improve knowledge quality..

Key-Words: - Fuzzy cognitive map, D-S evidence theory, expert knowledge Combination

1 Introduction

Multi-Expert Opinions Combination is a fundamental problem in many decision processes. Generally, in order to avoid individual expert's knowledge subjectivity, one-sidedness and limitations, the constructing fuzzy cognitive map (FCM) process that each expert builds individual FCM, and then combines them by weight average. However, the method cannot effectively keep exactitude information, reduce conflict factor, strong degree opinion and improve knowledge quality. There is an urgent need to develop methods for multi-expert knowledge combination. Dempster -Shafer evidence theory provides solving method for the problem. In this paper, we study the solving strategy of multi-expert system based on D-S evidence theory.

The paper is organized as follows. Section 2 presents the basic concepts of evidence theory. Section 3 presents the formalization representation of FCM. Section 3 presents a brief overview of the immune algorithm. Section 4 introduces how to use FCM for goal-oriented decision support. Section 5 applies the proposed methodology to goal-oriented analysis. Section 6 is the conclusion and suggestions for future works.

2 Dempster-Shafer evidence theory

Dempster-Shafer evidence theory provides a powerfully intelligent tool for multi-expert Opinions combination. It is introduced by Dempster[1] and extended later by Shafer[2]. D-S theory is concerned with the question of belief in a proposition and systems of propositions. Evidence can be considered in a similar way when forming propositions, and it is concerned with evidence, weights of evidence and belief in evidence. The theory does not make any assumption concerning the way human imagination works. Simply, it describes decision-makers receiving information from different sources and evaluating to what extent the evidence that they provide is compatible or contradictory. Once conflicts have been evaluated, a decision-maker may hold beliefs on whatever possibility he may envisage. The role of evidence theory is that of telling the decision-maker which evidence supports the possibility that he is considering.

In this section we briefly review basic notions of DS theory of evidence.

2.1 Frame of discernment:

In D-S theory, a problem domain is represented by a finite set Θ of elements; An element can be a mutually hypothesis, an object or our case a fault. we called Θ as the frame of discernment [29]. In the standard probability framework, all elements in Θ are assigned a probability. And when the degree of support for an event is known, the remainder of the support is automatically assigned to the negation of the event.

2.2 Mass functions, focal elements, and kernel elements:

When the frame of discernment is determined, the mass function m is defined as a mapping of the power set m：$2^{\Omega}\rightarrow[0,1]$

1、$m(\phi)=0$ (1)

2、$\sum_{A\subset\Omega} m(A)=1$ (2)

the mass function m is also called a basic probability assignment function. m(A) expresses the proportion of all relevant and available evidence that supports the claim that a particular element of H belongs to the set A but to no particular subset of A. In engine diagnostics, m(A) can be considered as a degree of belief held by an observer regarding a certain fault; different evidence can produce different degrees of belief with respect to a given fault. Any subset A of Θ such that m(A) > 0 is called a focal element; the union of all focal element $C=\cup_{m(A)\neq 0}A$ is called a kernel element of mass function m in the frame of discernment.

2.3 Belief and plausibility functions

The belief function Bel is defined as:

$$Bel: 2^{\Omega}\rightarrow[0,1] \quad \forall A\subset\Omega$$

$$PI(A)=1-Bel(\overline{A})=\sum_{B\subseteq\Omega} m(B)-\sum_{B\subset\overline{A}} m(B)=\sum_{B\cap A\neq\Phi} m(B) \quad (3)$$

The belief function Bel(A) measures the total amount of probability that must be distributed among the elements of A; it reflects inevitability and signifies the total degree of belief of A and constitutes a lower limit function on the probability of A.

The plausibility function Pls and double function Dou are defined as:

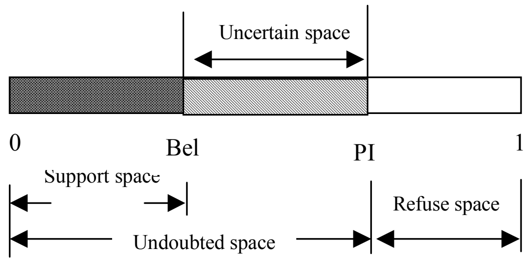
$$Pl: 2^{\Omega}\rightarrow[0,1]$$

$$PI(A)=1-Bel(\overline{A})$$
$$Dou(A)=Bel(\overline{A}) \quad (4)$$

The plausibility function Pl(A) measures the maximal amount of probability that can be distributed among the elements in A; it describes the total belief degree related to A and constitutes an upper limit function on the probability of A. it describes the total belief degree related to A and constitutes an upper limit function on the probability of A.

2.4 Belief interval [Bel(A), Pl(A)]:

The belief interval reflects uncertainty. The



interval-span Pl(A)– Bel (A) describes the unknown with respect to A. Different belief intervals represent different meanings. See Fig

Fig 1 the uncertain representation of informatio

2.4 Evidence combination

Let Bel_1 and Bel_2 be two belief functions in the same frame of discernment, then the corresponding basic belief assignment are m_1 and m_2 based on information obtained from two different information sources in the same frame of discernment Θ, focus elements are X_1，X_2，…，X_k and Y_1，Y_2，…，Y_k，see figure 1:

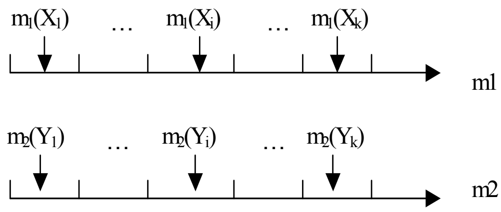


Fig 1 the base belief assignment

if $X_i\cap Y_j=A$, $X\subset\Omega$, then $m_1(X_i)m_2(Y_j)$ is the belief assignment to A, The total belief of A is:

$$\sum_{X_i\cap Y_j=A} m_1(X_i)m_2(Y_j),\ A\neq\phi$$

when $A=\phi$, $\sum_{X_i\cap Y_j=\phi} m_1(X_i)m_2(Y_j)$ is on the belief of void set φ. . We have the rule of evidence combination[10].

$$m(A)= m_1 \oplus m_2 = \begin{cases} 0 & A=\Phi \\ \frac{1}{1-k}\sum_{X\cap Y=A} m_1(X)m_2(Y) & A\neq\Phi \end{cases} \quad (5)$$

Where $k = \sum_{X\cap Y=\phi} m_1(X)m_2(Y)$, K represents a basic probability mass associated with conflicts among the sources of evidence. It is determined by summing the products of mass functions of all sets where the intersection is null. K is often interpreted as a measure of conflict between the sources. The larger the value of K is, the more conflicting are the sources, and the less informative is their combination.

The produced function m =m1 $\oplus$ m2 is also a mass function in the same frame of discernment Θ, it represents the combination of m1 and m2 and carries the joint information from the two sources.

In the case of n mass functions $m_1, m_2, \ldots, m_n$ in Θ, according to rule of evidence combination:

$$m(A)= m_1 \oplus m_2 \oplus \ldots \oplus m_n = \begin{cases} 0 & A=\Phi \\ \frac{\sum_{A_1\cap A_2\cdots\cap A_n=A} \prod_{i=1}^{n} m_i(A_i)}{1-k} & A\neq\Phi \end{cases} \quad (6)$$

Where $k= \sum_{A_1\cap A_2\cdots\cap A_n=\Phi} \prod_{i=1}^{n} m_i(A_i)$

3 Fuzzy Cognitive Map

Fuzzy cognitive map (FCM) [3], [4] is an approach to knowledge representation and inference that are essential to any intelligent system. It emphasizes the connections as basic units for storing knowledge and the structure represents the significance of system. Because FCM can be easily built and represent knowledge directly. and form mapped relations with the knowledge structures in the brains of the experts of this area, FCM models are always built directly by experts in practice. At the same time, in order to overcome the unilateralism of the personal evaluation, we usually have many experts build systemic FCM, then, combine them. To combine individual expert's incertitude opinion is a matter of combining incertitude information. Nowadays, there is a lack of the research aimed at combining FCM of different experts', method which uses simplified arithmetical average is widely used to combine experts' FCM .is a soft computing method for simulation and analysis of complex system, which combines the fuzzy logic and theories of neural networks. It offers a more flexible and powerful framework for representing human knowledge [1,5] and for reasoning. Unlike traditional expert systems that explicitly implement "IF/THEN" rules. FCM encodes rules in its structure in which all concepts are causally connected. Rules are fired based on a given set of initial conditions and on the underlying dynamics in the FCM. The result of firing the concepts represents the causal inference pattern of FCM; its inference can be computed by numeric matrix operation [6]. One of the most useful aspects of the FCM is its prediction capability as a prediction tool. Little research has been done on the goal-oriented analysis with FCM. Fuzzy cognitive maps have been used for representing knowledge and artificial inference and have found many applications, for instance, geographic information systems [7], [8], fault detection [9],, policy analysis [10], etc.

The graphical illustration of FCM is a signed directed graph with feedback, which is consisted of nodes and weighted arcs. Nodes of the graph stand for the concepts that are used to describe the behavior of the system and they are connected by signed and weighted interconnections representing the causal relationships. A FCM is a method to draw a graphical representation of a system, and consists of nodes-concepts, each node-concept represents one of the key-factors of the system, and it is characterized by a value C $\in$ (0,1), and a causal relationship between two concepts is represented as an edge wij. wij indicates whether the relation between the two concepts is direct or inverse. The direction of causality indicates whether the concept Ci causes the concept Cj. There are three types of weights:

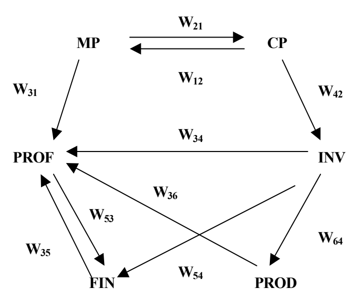
wij>0 indicates direct causality between concepts Ci and Cj. That is, the increase (decrease) in the value of Ci leads to the increase (decrease) on the value of Cj.

w_{ij}<0 indicates inverse (negative) causality between concepts C_i and C_j. That is, the increase (decrease) in the value of C_i leads to the decrease (increase) on the value of C_j.

w_{ij}=0 indicates no relationship between C_i and C_j.

A model of FCM can be equivalently defined by a square matrix, called connection matrix, which stores all weight values for edges between corresponding concepts represented by rows and columns. The system of n nodes can be represented by nXn connection matrix. An example FCM model and its connection matrix are shown as follow:

fig .3. A fuzzy cognitive map



the connectivity of the FCM (Fig. 3) can be conveniently represented by a connection matrix W.

$$W = \begin{bmatrix} 0 & W_{12} & 0 & 0 & 0 & 0 \\ W_{21} & 0 & 0 & 0 & 0 & 0 \\ W_{31} & 0 & 0 & W_{34} & W_{35} & W_{36} \\ 0 & W_{42} & 0 & 0 & 0 & 0 \\ 0 & 0 & W_{53} & W_{54} & 0 & 0 \\ 0 & 0 & 0 & W_{64} & 0 & 0 \end{bmatrix}$$

Where w_{ij} specifies the value of a weight for an edge from ith to jth concept node.

In order to discuss conveniently, we presented the formal definition FCM as follows:

A fuzzy cognitive map ***F*** is a 4-tuple (***V, E, C, f***) where --***V***={$v_1, v_2, \ldots, v_n$} is the set of n concepts forming the nodes of a graph.

--***E:***(v_i, v_j)$\rightarrow$ $\boldsymbol{w_{ij}}$ is a function $\boldsymbol{w_{ij}} \in \boldsymbol{E}$, $v_i, v_j \in \boldsymbol{V}$, with $\boldsymbol{w_{ij}}$ denoting a weight of directed edge from v_i to v_j. Thus ***E (V× V)=(w_ij)*** is a connection matrix.

--***C:*** $v_i \rightarrow \boldsymbol{C_i}$ is a function that at each concept v_i associates the sequence of its activation degrees, such as $\boldsymbol{C_i(t)}$ given its activation degree at the moment t. ***C(0)*** indicates the initial vector and specifies initial values of all concept nodes and ***C(t)*** is a state vector at iteration t.

--***f*** is a transformation function, which includes recurring relationship between ***C(t+1)*** and ***C(t)***.

$$C_i(t+1) = f\left(\left(\sum_{\substack{i=1 \\ j\neq i}}^{n} w_{ij} C_j(t)\right)\right) \quad (7)$$

the transformation function is used to confine the weighted um to a certain range, which is usually set to [0, 1].

$$o_i(t+1) = \frac{1}{1+e^{-C(t)}} \quad (8)$$

Eq. (1) describes a functional model of FCM. An FCM represents a dynamic system that evolves over time, it describes that the value of each concept is calculated by the computation of the influence of other concepts to the specific concept.

4 Multi-expert's opinions combination based on FCM

According to the formulized definition of FCM, experts' opinions are reflected on the estimate of the degree of the cause that is between nodes in the referred concept set, namely weight estimate. In the construction of FCM, multi-experts' opinions combination is represented as the combination of the corresponding elements in the connection matrix provided by experts. Then each expert's estimate of some cause relation can be regarded as evidence. The possible value of the affection degree of the cause relation between concepts forms a frame of discernment. The combined belief assignment function is regarded as the evidence of last weight integration.

A FCM equals the code of experts' knowledge, In general, because of experts' different preferences and knowledge structures, the understandings about the problem may be different. Such as, different experts differ in how they assign causal strengths to edges and in which concepts they deem causally relevant. There is a requirement to build a selection rule of concept set and to enact a standard of cause effect degree before FCM combination.

Definition 1:Connection Matrix Standardization

Suppose there are n experts, the FCM of each expert's is established according to their own experiences and knowledge. The connection matrices of n experts' are F_1, F_2, …, F_n. The union (m) of all experts' concepts is regarded as a set of concept. The connection matrices of experts' are expanded to m × m, and we fill the row or column absent of concept nodes with 0. The process is called the standardization of connection matrix.

The general process of combining multi-experts' FCM with evidence theory is as follows:

1) A frame of discernment is firstly defined, it translates the research of proposition into the research of a set.

2) Basic probability assignments are established (BPA) according to evidence.

3) the basic probability assignment functions are combined according to the combination rule of evidence theory, then the target type is determined by the rule of belief evaluation.

4) Applying weighted average on all elements of the frame according to integrative probability.

4.1 The Building of Frame of Discernment

The selection of frame of discernment depends upon our knowledge and cognition, and we known and want. In application of FCM, the expert estimates the weight using linguistic weight. Their values are usually nothing, very weak, weak, medium, strong, very strong.

Example:

(none, very weak, weak, strong, very strong, extremely strong) →{0, 0.2, 0.4, 0.6, 0.8, 1}

or:

(none, weak, strong, extremely strong) →{0, 0.4, 0.6, 1}

The possible values of weight form a frame of discernment, which is defined by the demand of accuracy.

We can define a frame of discernment according to the example above.

Ω={0，0.2，0.4，0.6，0.8， 1}
Or: Ω={0，0.4，0.6，1}

4.2 The Mass function of Evidence

According to the experience and knowledge, each expert makes a basic belief assignment function m(also called the mass function m) to every element of the connection matrix in a frame of discernment. Suppose there are n experts, we can be gained n basic probability assignment functions: $m_1, m_2, \cdots, m_n$.

The n experts evaluate a weight in a frame of discernment can be captured in a matrix form: The matrix M is as follows:

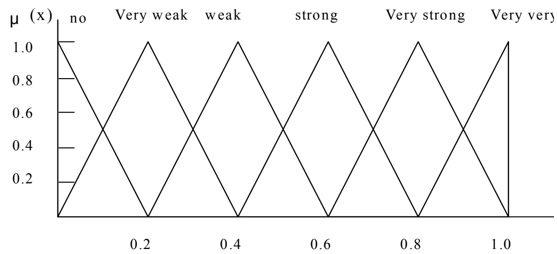
$$M = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1k} \\ m_{21} & m_{22} & \cdots & m_{2k} \\ & & \vdots & \\ m_{n1} & m_{n2} & \cdots & m_{nk} \end{bmatrix}$$

Where each row in matrix M represents the evaluating result of i^{th} expert; Each column of M represents the evaluating result of n experts to j^{th} element of the frame of dscernment. m_{ij} denote the belief assignment of the j^{th} element of the frame of discernment Ω of i^{th} expert.

The result that the i^{th} expert estimates a weight in the frame of discernment is a fuzzy value. A basic belief assignment function m_i is produced by solving membership of the fuzzy value.

For example, solveing m, when a fuzzy value is 0.48, the membership function is defined as follows:

Fig.4. the six membership functions corresponding to each one of the six linguistic



variables the fuzzy number

$$\mu_0(x) = \begin{cases} (0.2 - x)/0.2 & 0 < x \le 0.2 \\ 0 & other \end{cases}$$

$$\mu_{0.2}(x) = \begin{cases} (0.2 - x)/0.2 & 0 < x \le 0.2 \\ (0.4 - x)/0.2 & 0.2 < x \le 0.4 \\ 0 & other \end{cases}$$

$$\mu_{0.4}(x) = \begin{cases} (x - 0.2)/0.2 & 0.2 < x \le 0.4 \\ (0.6 - x)/0.2 & 0.4 < x \le 0.6 \\ 0 & other \end{cases}$$

$$\mu_{0.6}(x) = \begin{cases} (x - 0.4)/0.2 & 0.4 < x \le 0.6 \\ (0.8 - x)/0.2 & 0.6 < x \le 0.8 \\ 0 & other \end{cases}$$

$$\mu_{0.8}(x) = \begin{cases} (x - 0.6)/0.2 & 0.6 < x \le 0.8 \\ (1.0 - x)/0.2 & 0.8 < x \le 1.0 \\ 0 & other \end{cases}$$

$$\mu_{1.0}(x) = \begin{cases} (1.0 - x)/0.2 & 0.8 < x \le 1.0 \\ 0 & other \end{cases}$$

According to formula above:

$$\mu_{0.4}(0.48)=0.6$$

$$\mu_{0.6}(0.48)=0.4$$

We can obtain the base belief assignment m=[0,0,0.6,0.4,0,0]

4.3 Evidence Combination

Firstly, the conflict between the expert's opinions is calculated with $k = \sum_{X \cap Y = \phi} m_1(X) m_2(Y)$.

If combination condition is satisfied, then the combinative belief is calculated according to formula as bellow:

$$\text{m(A)} = m_1 \oplus \text{m}_2 = \begin{cases} 0 & A = \Phi \\ \dfrac{1}{1 - \text{k}} \sum_{X \cap Y = A} m_1(X) m_2(Y) & A \ne \Phi \end{cases}$$

4.4 Calculate Integrated Weight

The integrated weight *w* is defined as:

$$w = \sum_{j=1}^{m} \frac{a_j}{\sum_{j=1}^{m} a_j} \theta_j \qquad (9)$$

Where a_j is the probability of the *j*th state, θ_j is the *j*th state value of the frame of discernment.

5 Combination Calculate complexity Analysis

The frame of discernment is set as: Ω= { Θ_1, Θ_2,… Θ_n}, there are k evidences that k experts offer. In the extreme, each group of evidences has 2^n-1 mass function values: m({ Θ_1})， m({ Θ_2})， …， m({ Θ_1 ， Θ_2})， …， m({ Θ }). On this condition, the complexity degree of the information is O(k*2^n). Then we'll discuss the complexity degree of using the combination formula of 2 evidences and the combination formula of k evidences to combine k experts' knowledge. For using the combination formula of two evidences, the main calculation is the multiplication of two mass functions, so the complexity degree is (2^n-1)*(2^n-1)=O(2^{2n}).And for the knowledge combination of k experts , the complexity degree of information is k*O(2^{2n})=O(k*2^{2n}). For the combination formula of multi-evidences, the main calculation is the multiplication of k mass functions. So the complexity degree is $(2^n-1)^k$, namely $O(2^{kn})$.

Based on the analysis above, when the knowledge of k experts' are being combined in the same frame of discernment, the complexity degree of two evidences combination relates linearly to the number of evidences, and may form a exponential relation with the number of possible results in the frame of discernment. For the multi-evidences combinations, the complexity degree has an exponential relationship with the number of evidences and the number of possible results in the frame of discernment.

For the problem of combining many experts' knowledge on FCM, when there are n values of mass function in each evidence, the complexity degree of the information is O(kn). Using the combination formula of two evidences to calculate k evidences, the complexity degree of the information is $O(n^2)$. And for the knowledge combination of k experts', the complexity degree of information is $k*O(n^2)=O(k*n^2)$. For the combination formula of multi-evidences, the complexity degree is $O(n^k)$.

6 Application

To demonstrate the feasibility of the proposed method, we applied the proposed method to the combination of three experts' opinions

For example, there are three experts giving judgment to the cause affection degree of Ci and Cj in concept set$\{C_1, C_2, \ldots C_n\}$.see table 1.

table1 expert knowledge

state / expert	0	0.2	0.4	0.6	0.8	1
Expert 1	0	0	0.7	0.3	0	0
Expert 2	0	0.1	0.8	0.	0	0
Expert 3	0	0	0.9	0.1	0	0

According to formula $k = \sum_{X \cap Y = \phi} m_1(X) m_2(Y)$, we get k=0.41.

The combinative result of Expert 1 and expert 2 according to Eq (5) is shown in table 2.

Table 2 experts knowledge combination (1)

state / expert	0	0.2	0.4	0.6	0.8	1
Expert 1	0	0	0.7	0.3	0	0
Expert 2	0	0.1	0.8	0.1	0	0
Combination result 1	0	0	0.95	0.05	0	0

Again, according to $k = \sum_{X \cap Y = \phi} m_1(X) m_2(Y)$, we get k=0.14

The combinative result of expert 1, expert 2 and expert 3 according to Eq (5) is shown in table 3.

table 3 experts knowledge combination (2)

State / Expert	0	0.2	0.4	0.6	0.8	1
Result 1	0	0	0.95	0.05	0	0
Expert 3	0	0	0.9	0.1	0	0
Combination result	0	0	0.994	0.0058	0	0

The result of three experts' conbination can be seen from table 3. The belief assignment function is 0.994 when state value is 0.4 and m(0.6) is 0.0058.

Using (9) to solve the integrated weight According to the combined belief assignment function m.

Based on the example Above,we get wij=0.4*0.994+0.6*0.0058=0.40108

7 Conclusion

We have developed a method for Multi-Expert Opinions Combination Based on Evidence Theory. . In the method, we use multi-expert's knowledge as evidence, the possible value of weight as frame of discernment, expert's evaluation to a weight on frame of discernment as basic probability assignment, and use D-S rule combining to give fusion basic probability assignment m. Finally, the weight is given according to fusion basic probability assignment. The feasibility and effectiveness of the method were illustrated. This strategy can gradually reduce the hypothesis sets and approach the truth with the accumulation of evidences, which make the result of decision more all-around and more scientific. Consummating the proposed method and exploring the applying area are the direction of our future work.

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