Discrete Cosine Transform Domain Parallel LMS Equalizer

MOHAMMED H. WONDIMAGEGNEHU and TETSUYA SHIMAMURA
Graduate School of Science and Engineering
Saitama University
255 Shimo-Okubo, Sakura-ku, Saitama, 338-8570
JAPAN

Abstract: The least mean square (LMS) algorithm is the most popular adaptive filtering method due to its simplicity and predictable behavior. The convergence properties of the LMS algorithm can be improved by updating the filter coefficients in the transform domain. This paper presents a new transform domain LMS equalizer, called Discrete Cosine Transform Domain Parallel LMS (DCTPLMS) equalizer, which updates the filter coefficients in the discrete cosine transform domain for the purpose of equalization of time-varying channels. Computer simulation results obtained by using a second order Markov communication channel model show that the proposed DCTPLMS equalizer provides a significant performance improvement relative to the conventional DCT-domain LMS as well as other time-domain based LTEs and DFEs.

Key–Words: DCTPLMS equalizer, Time-varying channel, DCTLMS algorithm, LMS algorithm, Adaptive equalization.

1 Introduction

The most common problems which affect the reliability of modern communication systems such as mobile radio channels and high frequency (HF) channels are rapid time variation and multipath fading. Due to the phenomenon of time variation these channels suffer from intersymbol interference(ISI). To compensate for channel distortions which cause ISI in communication systems, adaptive equalization techniques can be used [1].

The least mean square (LMS) algorithm [2] which updates the filter coefficients by a gradient based method in the time-domain is, so far, the most popular adaptive equalization method due to its simplicity and predictable behavior.

Among many variants of adaptive equalizers, the linear transversal equalizer (LTE) and the decision feedback equalizers (DFE) are most frequently used. The DFE is a nonlinear device that attempts to remove ISI at the receiver of a digital communication system using both the received signal and previously detected symbols and, thus, has the potential to provide better performance than the corresponding LTE structure if the decision device involved in the DFE structure outputs correct sequences [3]. However, the feedback decisions will not be correct if the interference is severe and if a simple slicer is used as the decision device. If the feedback decisions are not correct, the DFE will suffer from performance degradation due to error-propagation [4].

Another alternative remedy studied by Shimamura [5] is to deploy a parallel structure of the conventional LTE and DFE, in which both the coefficients of the LTE and DFE are simultaneously adapted by the LMS algorithm. Moreover, a new LMS based nonlinear adaptive algorithm, called amplitude banded LMS (ABLMS) algorithm, which takes into account the amplitude information of a time-variant channel output in the coefficient adaptation process of the equalizer was discussed in [6] and [7].

However, the convergence speed of traditional time-domain LMS type adaptive equalizers also depends on the ratio of the maximum to the minimum eigenvalues of the autocorrelation matrix of the input. Adaptive filters having inputs with a wide eigenvalue spread often take longer to converge than filters with white noise inputs.

The recursive least squares (RLS) algorithm offers better performance and high convergence rate that is independent on the properties of the input signal, but its high computational complexity and numerical stability problems limit its use in real time applications [8].

On the other hand, it is known that by implementing adaptive filtering in the transform domain, significant improvements in the convergence rate over the conventional time domain approach are achieved [9]. Moreover in [10] various types of the transform domain LMS algorithm were used in tracking a class of time-varying plants.

In this paper, a parallel transform domain equalizer called discrete cosine transform domain parallel LMS (DCTPLMS) equalizer is proposed. Our motivation for considering such an adaptive equalizer is to retain the relative simplicity and convergence achieved in[5] over the conventional LMS based LTEs and DFEs as well as to incorporate some of the benefits of the computational efficiency and faster tracking performance acquired by adaptation of equalizer coefficients in the discrete cosine transform domain.

In the next section, the basic concepts of tran sform domain LTE and DFE adaptive equalizers are introduced. Section 3 discusses the configuration and formulation of the proposed DCTPLMS equalizer. Section 4 presents the experimental results obtained by computer simulations using a second order Markov communication channel model. Section 5 serves as the conclusion of the paper by summarizing the results of the proposed equalizer.

2 Transform Domain Equalizer

2.1 Transform Domain Adaptive Algorithm

Transform domain adaptive filters refer to LMS filters whose inputs are preprocessed with a unitary data-independent transformation followed by a power normalization stage. This preprocessing mitigates the eigenvalue distribution of the input autocorrelation matrix of the LMS filter and, as a consequence, ameliorates its convergence speed.

An LTE representation of the block diagram of the transform domain adaptive filter is shown in Figure 1.

The transform vector $\mathbf{Z}_n$

$$\mathbf{Z}_n = [z_{n0}, z_{n1}, ..., z_{n(N-1)}]^T \tag{1}$$

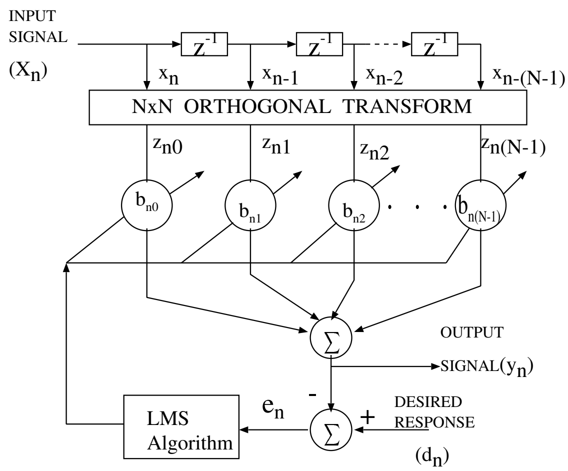

Figure 1: block diagram of the transform domain adaptive filter.

is related to the input vector

$$\mathbf{X}_n = [x_n, x_{n-1}, ..., x_{n-(N+1)}]^T \tag{2}$$

by the orthogonal transformation:

$$\mathbf{Z}_n = \mathbf{W}\mathbf{X}_n \tag{3}$$

where $\mathbf{W}$ is an $N \times N$ unitary matrix. The output and the corresponding error signal are

$$y_n = \mathbf{Z}_n^T \mathbf{B}_n \tag{4}$$

and

$$e_n = d_n - y_n \tag{5}$$

respectively, where

$$\mathbf{B}_n = [b_{n0}, b_{n1}, ..., b_{n(N-1)}]^T \tag{6}$$

is the transform domain weight vector. The LMS algorithm[2] is used to recursively update the weight vector $\mathbf{B}_n$. The weight vector update equation is

$$\mathbf{B}_{n+1} = \mathbf{B}_n + 2\mu \mathbf{D}_n e_n \bar{\mathbf{Z}}_n \tag{7}$$

where μ is the adaptive step size and $\mathbf{D}_n$ is $N \times N$ diagonal matrix whose(i,i)th element is equal to the power estimate of the ith transform domain output z_{ni}. The $(\bar{.})$ denotes a complex conjugate. Equations (4), (5) and (7) provide the adaptation concept of transform domain LMS (TDLMS) algorithm.

2.2 DCT Adaptive Filtering

In practical applications such as channel equalization where the input and desired signals are all real valued, it is appropriate to use real-valued orthogonal transforms such as the discrete cosine transform (DCT).

Moreover, by using statistical analysis, Kim and De Wilde [11] have shown that the DCTLMS algorithm provides faster convergence rate relative to the LMS, normalized LMS (NLMS) and variable step size LMS (VLMS) algorithms as well as the discrete Fourier transform LMS (DFTLMS) for a highly correlated input signals.

To implement the DCTLMS algorithm, the $N \times N$ unitary transform matrix $\mathbf{W}$ represents the discrete cosine transform. Each element of $\mathbf{W}$ is defined, for $i = 1, 2, ..., N$, $j = 1, 2, ..., N$ as [12]

$$W(i,j) = \sqrt{\frac{2}{N}} K_i cos(\frac{i(j+\frac{1}{2})\pi}{N}) \qquad (8)$$

where $K_i = \frac{1}{\sqrt{2}}$ for $i = 0$ and 1 otherwise.

3 The Proposed Method

3.1 Channel Model

The channel model assumed in this paper is given by

$$x_n = \sum_{i=0}^{L} h_i(n) u_{n-i} + v_n \qquad (9)$$

where $h_o(n), h_1(n), ..., h_L(n)$ are the channel coefficients, u_n is the transmitted sequence, and v_n is a white Gaussian noise uncorrelated with u_n. The channel output x_n becomes the input for the equalizer.

3.2 DCTPLMS Equalizer

The DCTPLMS equalizer proposed in this paper consists of two equalizer structures, an LTE and a DFE, connected in parallel. In general, transform domain DFE (TD-DFE)s [13] also exhibit superior performance than their LTE counterparts only when the input samples of the feedback filter are correct. However, in case of rapid time-varying channels, the feedback decisions will not always be correct and TD-DFEs will suffer from performance degradation due to the effect of error-propagation as conventional DFEs do.

Therefore, a parallel DCTLMS-LTE and DCT LMS-DFE structure whose adaptation scheme is described in the next section is proposed. The comparator in the proposed parallel structure ensures the preservation of, at least, the performance of the discrete cosine TD- LTE section, during intervals when incorrect decisions circulating in the feedback path cause performance degradation of the discrete cosine TD-DFE.

3.3 Adaptation Scheme

Figure 2 shows a block diagram of the DCTPLMS equalizer in the training mode. Both the coefficients of the LTE and DFE are individually and simultaneously updated based on the error sequences e_{ln} and e_{dn}, respectively. The DCTLMS algorithm is used as a common adaptation procedure for both the LTE and the DFE. The comparator provides $f_n = e_{ln}$ if $(e_{ln})^2 \leq (e_{dn})^2$ and $f_n = e_{dn}$ otherwise. Based on the comparator output, the DCTPLMS equalizer outputs y_{ln} when $f_n = e_{ln}$, and y_{dn} when $f_n = e_{dn}$.

The DCTLMS-DFE equalizer is implemented in such a way that the forward filtering is carried out in the transform domain, in the same manner as the transform domain LTE, while the feedback filtering is accomplished in the time domain through a time-varying finite impulse response (FIR) filter with uniform tap spacing.

Hence, by making some change of variables, the transform domain weight update equation (7) can be rewritten as the DCTLMS adaptation procedure of a TD-DFE as follows,

$$\mathbf{C}_n = \mathbf{C}_{n-1} + 2\mu e(n) * [\mathbf{D}_n, \mathbf{I}_{M_b}]^{-1} \mathbf{Z}'_n \qquad (10)$$

where

- $\mathbf{C}_n = [\mathbf{B}_n, \mathbf{G}_n]$ is the TD-DFE coefficient vector.
- $\mathbf{Z}'_n = [\mathbf{Z}_n, \mathbf{d}_{n-1}]$ is the TD-DFE input vector.
- $\mathbf{G}_n = [g_{n1}, g_{n2} ..., g_{n(M_b)}]^T$ is the feedback filter coefficient vector.
- $\mathbf{d}_{n-1} = [d_{n-1}, d_{n-2}, ..., d_{(n-M_b)}]^T$ is the decision device output vector.
- $\mathbf{I}_{M_b}$ is an $(M_b) \times (M_b)$ identity matrix.

The $M_f \times M_f$ diagonal power normalization matrix is calculated using

$$\mathbf{D}_n(i,i) = \beta \mathbf{D}_{n-1}(i,i) + (1-\beta) * z_{ni}^2, \qquad (11)$$
$$i = 0, 1, ..., M_f - 1$$

where β is a positive constant close to but less than one.

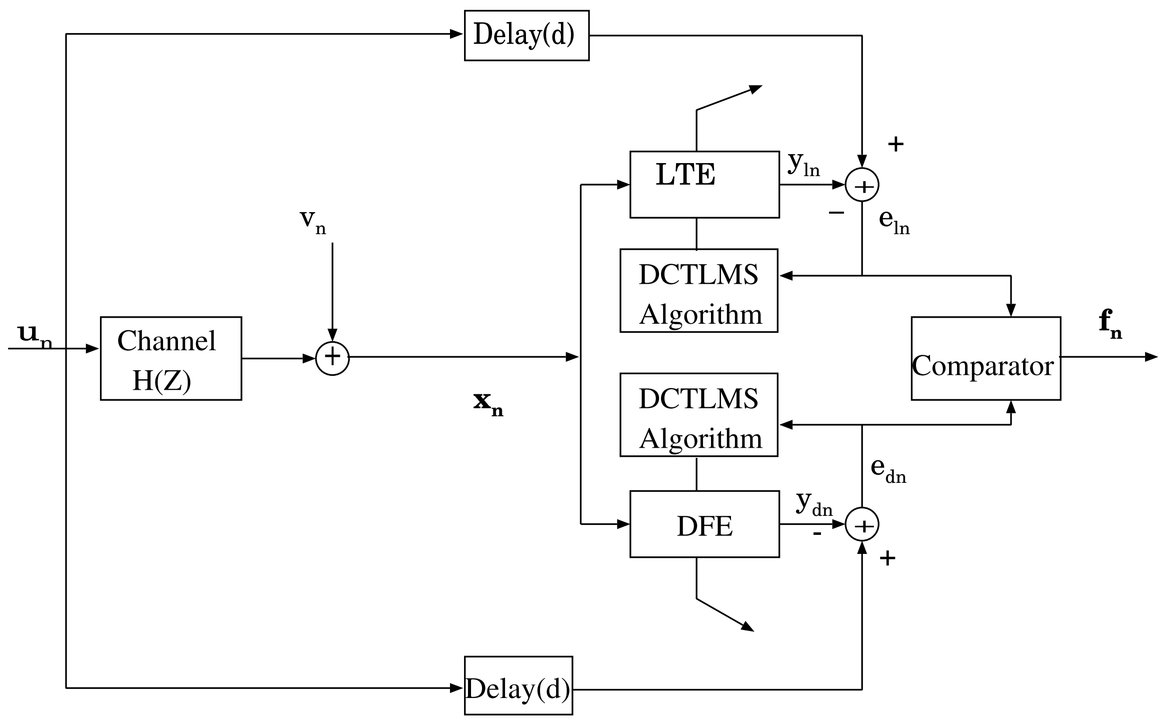

Figure 2: Configuration of the proposed DCTPLMS Equalizer.

Equation(10) can, thus, serve as the adaptation procedure for an $M_f + M_b$ length TD-DFE, where $M_f = N$ and M_b are the lengths of the feed forward and feedback filters, respectively.

In the proposed DCTPLMS equalizer, if the performance of the LTE is worse than the DFE, then the output of the DFE is selected. And if the performance of the DFE is worse than the LTE, then the output of the LTE is selected. Therefore, the parallel equalizer always provides better performance than either the standard DCTLMS-LTE or DCTLMS-DFE updated individually would provide.

However, since the proposed DCTPLMS structure requires a parallel adaptation of the DCTLMS-LTE algorithm with DCTLMS-DFE algorithm, the total computational complexity for implementing the proposed DCTPLMS is approximately twice of that required for implementing the DCTLMS-LTE as the computational complexity of the DCT LMS-DFE is nearly the same as that of the DCT LMS-LTE. This implies, that there is a trade-off between increased performance and mathematical complexity. However, with VLSI digital processors having increased computational resources becoming cheaper and are readily available, the benefit of the proposed parallel structure is far more advantageous than its drawbacks.

4 Simulation Results

The performance of the proposed DCTPLMS equalizer shown in Figure 2 was investigated using a second order Markov communication channel model. The channel equation is given by:

$$H(z) = h_0(n) + h_1(n)z^{-1} + h_2(n)z^{-2} \qquad (12)$$

where the time variant coefficients, $h_0(n)$, $h_1(n)$ and $h_2(n)$ are generated by passing Gaussian white noise at 2400 sample/s through second order Butterworth filters with 3 dB bandwidths on the order of the fade rate. The input sequence of this channel is an uncorrelated, pseudo-random sequence with values of +1 or −1. This channel is an HF channel model $H_3(z)$ used in [14].

Figures 3 shows mean square error (MSE) convergence plots for the normalized LMS-LTE (dashed line), normalized LMS-DFE (solid line), ABLMS-LTE (dased line with five pointed star) [7], parallel NLMS LTE-DFE (dash line with cross)[5], DCTLMS-LTE (solid line with triangle right), DCT LMS-DFE (dotted line with diamond) and proposed DCTPLMS (solid line with asterisk) equalizers at a channel fade rate $fd = 2$Hz and $SNR = 20$dB.

From Fig. 3 we can observe that the MSE performance of the proposed DCTPLMS equalizer is more convergent and also retains the lowest steady state MSE value. Thus, the proposed

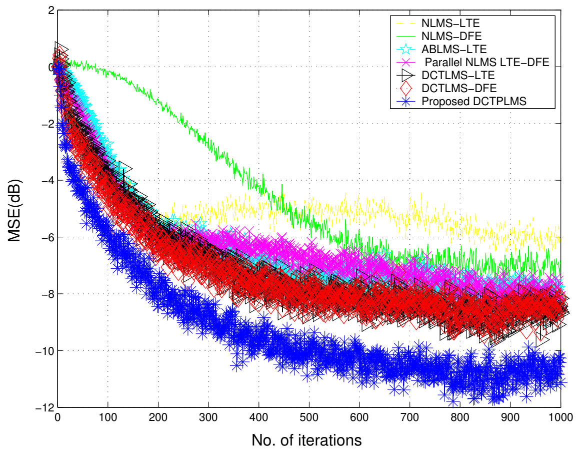

Figure 3: Comparison of MSE convergence at a channel fade rate $fd = 2Hz$.

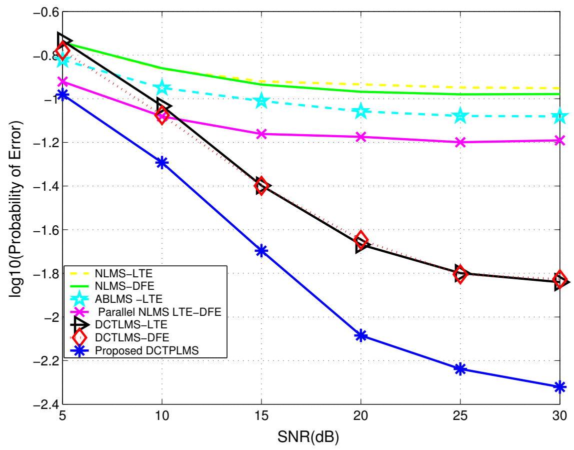

Figure 4: Comparison of BER performance against additive noise at a channel fade rate of $5Hz$.

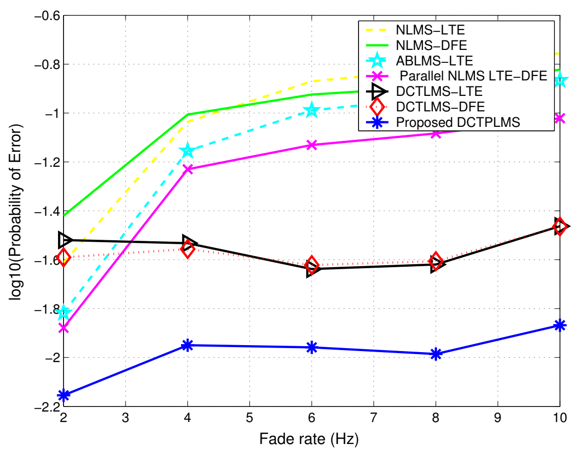

Figure 5: Comparison of BER performance against fade rate at $SNR = 20dB$.

DCTPLMS equalizer has shown significant improvement in tracking performance of rapidly time-varying channels.

Figure 4 illustrates the BER performances of the normalized LMS-LTE (dashed line), normalized LMS-DFE (solid line), ABLMS-LTE (dased line with five pointed star)[7], parallel NLMS LTE-DFE (dash line with cross)[5], DCTLMS-LTE (solid line with triangle right), DCTLMS-DFE (dotted line with diamond) and proposed DCTPLMS (solid line with asterisk) equalizers against additive noise on a channel model with a channel fade rate of $fd = 5$Hz.

Figure 5 illustrates the BER performances of the normalized LMS-LTE (dashed line), normalized LMS-DFE (solid line), ABLMS-LTE (dashed line with five pointed star)[7], parallel NLMS LTE-DFE (dash line with cross)[5], DCTLMS-LTE (solid line with triangle right) DCTLMS-DFE (dotted line with diamond) and the proposed DCTPLMS (solid line with asterisk) equalizers against channel fade rate on a channel model corrupted with an additive noise of $SNR = 20dB$.

The equalizers have a filter length of 9 for LTE structures while the DFE structures have a forward filter length of 7 and a backward filter length of 2. The step size parameter is $\mu = 0.5$ for the conventional NLMS, ABLMS and time domain parallel equalizers while it is set to $\mu = 0.08$ in case of the DCT domain LMS and proposed DCTPLMS equalizers. Those parameters have been optimized to provide the best performance for the filter structures used. The power normalization factor $\beta = 0.9$ and a delay $d = 4$ have been commonly used. 10,000 data samples were used for simulating Figs. 4 and 5.

From Figs 4 and 5, we observe that the proposed DCTPLMS exhibits a very significant improvement in BER performance. Moreover, from Fig. 5, we observe that the proposed equalizer is highly robust against increase in fade rate. This implies that the proposed DCTPLMS equalizer has the best capability of compensating for ISI under even severe fading channel conditions.

5 Conclusion

A new discrete cosine transform domain parallel LMS equalizer known as DCTPLMS equalizer has been proposed for the equalization of rapidly time-varying multipath channels. Simulations have demonstrated that the proposed equalizer shows better MSE and BER performances than the conventional LMS based equalizers as well as the standard DCTLMS-LTE and DCTLMS-DFE equalizers. Even though, the proposed DCTPLMS equalizer incurs some additional computational complexity, the performance gains achieved highly outweigh the extra mathematical cost and, thus, the trade-off may be acceptable.

References:

[1] S.U.H.Qureshi, "Adaptive equalization", *Proc.IEEE*, Vol-73, No.9, pp.1349-1387, 1982.

[2] B.Widrow and S.D.Stearns, *Adaptive Signal Processing*, Prentice-Hall, Englewood Cliffs, 1985.

[3] P.Monsen, "Feedback equalization for fading dispersive channels", *IEEE Trans. Information Theory*, Vol.IT-17, pp.56-64, Jan. 1971.

[4] J.J.O'Reilly and A.M.de Olivera Durate, "Error propagation in decision feedback receivers", *IEE Proc.*, Vol-32,Pt.F, No.7, pp.561-566, Dec. 1985.

[5] T.Shimamura , "A parallel equalizer with LMS adaptation for time variant multipath channels,"*Proc. IEEE Asia-Pacific conference on Circuits and Systems*, pp.439-442, November 1998.

[6] T.Shimamura and C.F.N.Cowan, "Equalization of time-variant multipath channels using amplitude banded techniques," *Proc. IEEE ICASSP*, Vol.3, pp.2497-2500, 1997.

[7] T.Shimamura and C.F.N.Cowan, "Equalization of time-variant multipath channels using amplitude banded LMS algorithms," *IEICE Trans. Fundamentals on Electronics, Communications and Computer Sciences*, Vol.E84-A, No.3, pp.802-812, March 2001.

[8] S.Haykin,*Adaptive Filter Theory*, Prentice-Hall, Englewood Cliffs, 1991.

[9] S. Shankar Narayan and A.M.Peterson, "Frequency domain LMS algorithm,"*Proc. IEEE*, Vol.69, No.1, pp.124-126, January 1981.

[10] Farhang-Boroujeny, B. and S.Gazor, "Performance of LMS-based adaptive filters in tracking a time varying plant," *IEEE Trans. on Signal Processing*, Vol.44-A, No.11, pp.2868-2871, November 1996.

[11] D.I.Kim and P.De Wilde, "Transform-domain adaptive filters: an analytical approach," *Elsevier Journal of Signal Processing*, Vol. 80, pp. 1629-1654, 2000.

[12] N. Ahmed, T.Natarajan, and K.R.Rao, "Discrete Cosine Transform," *IEEE Trans. Comput.*, pp.90-93, Jan.1974.

[13] A.Ghobrial and R.Adhami, "Discrete wavelet transform domain adaptive decision feedback equalization," *Proc. IEEE Thirty-Fourth Southeastern Symposium on System Theory*, pp.271-274, March 2002.

[14] F.Ling and J.G.Proakis, "Adaptive lattice decision-feedback equalizers - their performance and application to time-variant multipath channels," *IEEE Trans. Commun.*, vol.COM-33, pp.348-356, April 1985.