LMI Approach to Simultaneous Output-Feedback Stabilization for Interval Time-Delay Systems*

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Abstract: − This paper considers the problem of simultaneously stabilizing output feedback controller design for a collection of interval time-delay systems. It is shown that this problem is solvable if a matrix measure assignment problem is solvable. Thus, in this study, we proposed to solve a matrix measure assignment problem to get a solution of the considered problem. We also proved that the admissible solution set of the matrix measure assignment problem is convex. Then, the matrix measure assignment problem is shown to be equivalent to a linear matrix inequality (LMI) feasibility problem. In term of LMIs, a necessary and sufficient condition for the existence of static output feedback controllers is obtained. Finally, an example is provided to illustrate the proposed methodology.

Key-Words: - Matrix measure; Robust control; Output feedback; Linear Matrix Inequality.

1. Introduction

In this paper, the problem of simultaneous output feedback stabilization for a class of interval time delay systems is studied. It is known that time-delay often exists on linear and nonlinear systems such as chemical systems, electrical networks, hydraulic, and rolling mill systems. Since time delay is frequently a source of instability and poor performance, the stability problems of time delay systems have received considerable attention during recent decades.

In general, the explicit mathematical model of a real system is hard to obtain. Usually, we can only use an approximate model with some uncertainties to represent the real system. The robust stability analysis problem for time delay systems with uncertainties is quite complicated and recently, has been studied via several different techniques. The criteria for asymptotic stability of such systems can be classified as delay-independent, which are independent of the size of time-delay, for example [7,8], or delay-dependent, which include information on the size of delay, for example [9,10]. Meanwhile, some different stability criteria have also been proposed via the LMI approach [12,13,15,16].

In [7-16], only robust stability analysis problem is considered. However, for a time delay system with uncertainties, which does not satisfy the stability criteria, the problem of how to define a controller such that the closed-loop systems is stable is not considered. More recently, in [1-3], the problem of designing robust state feedback or output feedback controllers for time-delay systems with uncertainties have been studied. However, they proposed a

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feedback controller to stabilize only a single uncertain time delay system. The problem of simultaneous stabilization for a collection of uncertain time-delay systems via a single static output feedback controller is an important issue in the robust control theory. To the best of our knowledge, there are no general techniques for solving this problem in the literature.

In this paper, we focused on the problem of simultaneous stabilization for a collection of interval time-delay systems via a static output feedback. It will be shown that the considered problem is solvable if a corresponding matrix measure assignment problem is solvable. The matrix measure is widely applied in the analysis of stability properties of uncertain and/or time-delay systems [4,5,14]. Although it has been widely employed in the robustness analysis problem, nevertheless, few has investigated about the controller synthesis problem; that is, for given matrices (**A**, **B**, **C**), how to find a matrix **F** such that $\mu_2(\mathbf{A}+\mathbf{BFC})<\gamma$. As described in [11], there are no general techniques for solving this problem even in the case of **C**=**I**. Recently, linear matrix inequalities (LMI's) have emerged as a powerful formulation and design technique for a variety of linear control problems [6,17,18]. Software like Matlab's LMI Control Toolbox [18] is available to solve LMI's problems in a fast and user-friendly manner. In this paper, we shall show that the matrix measure assignment problem is equivalent to an LMI feasibility problem. Thus, a controller solving the matrix measure assignment problem and then solving the simultaneous stabilization problem for a collection of interval time delay systems can be obtained via solving an LMI problem.

Notations:

In what follows, **O** is a zero matrix with an appropriate dimension, **I** is an identity matrix with an appropriate dimension, $\mathbf{M}^T$ denotes the transpose of the matrix **M**, $\mathbf{M}^*$ denotes the conjugate transpose of the matrix **M**, **M** >**0** ($\mathbf{M}\ge\mathbf{0}$) means that the matrix **M** is positive definite (semidefinite), **M** <**0** ($\mathbf{M}\le\mathbf{0}$) means that the matrix **M** is negative definite (semidefinite), and $\|\mathbf{M}\|_s\equiv\sqrt{\lambda_{\max}(\mathbf{M}^*\mathbf{M})}$ is the spectral norm of the matrix **M**.

2. Problem Formulation and Preliminaries

Consider a collection of interval time-delay systems:

$$\dot{x}_i(t)=\hat{\mathbf{A}}_i x_i(t)+\hat{\mathbf{D}}_i x_i(t-h_i)+\mathbf{B}_i u_i(t)\quad,\quad i=1,2,\ldots,p \tag{1}$$

$$y_i(t)=\mathbf{C}_i x_i(t),\quad i=1,2,\ldots,p \tag{2}$$

where $x_i(t)\in\Re^n$ is the state, h_i is the time-delay of the system, $u_i(t)\in\Re^m$ is the control input, and $y_i(t)\in\Re^r$ is the controlled output. $\mathbf{B}_i\in\Re^{n\times m}$ and $\mathbf{C}_i\in\Re^{r\times n}$ are constant matrices. $\hat{\mathbf{A}}_i\in\Re^{n\times n}$ and $\hat{\mathbf{D}}_i\in\Re^{n\times n}$ are matrices whose elements vary in some prescribed ranges; e.g., $\hat{\mathbf{A}}_i$ and $\hat{\mathbf{D}}_i$ are such that

$$\begin{aligned}\hat{\mathbf{A}}_i&=[a^i_{jk}],\quad \underline{a}^i_{jk}\le a^i_{jk}\le\bar{a}^i_{jk}\\ \hat{\mathbf{D}}_i&=[d^i_{jk}],\quad \underline{d}^i_{jk}\le d^i_{jk}\le\bar{d}^i_{jk}\end{aligned}\qquad i=1,2,\ldots,p \tag{3}$$

where a^i_{jk} is the *jk*-th element of the matrix $\hat{A}_i$, $\underline{a}^i_{jk}$ and $\bar{a}^i_{jk}$ denote its low bound and upper bound, respectively, d^i_{jk} is the *jk*-th element of the matrix $\hat{D}_i$, and $\underline{d}^i_{jk}$ and $\bar{d}^i_{jk}$ denote its low bound and upper bound, respectively. Those bounds $\underline{a}^i_{jk}$, $\bar{a}^i_{jk}$, $\underline{d}^i_{jk}$, and $\bar{d}^i_{jk}$ are known real values.

The design goal is to find a matrix **F** such that the static output feedback controller

$$u_i(t)=\mathbf{F}y_i(t)\quad,\quad i=1,2,\ldots,p \tag{4}$$

ensures all the closed-loop interval time-delay systems being asymptotically stable.

We now introduce several properties about matrix measure as follows. The matrix measure of a constant matrix **M** is defined as

$$\mu_v(\mathbf{M})\equiv\lim_{\theta\to0^+}\frac{\left(\|\mathbf{I}+\theta\mathbf{M}\|_v-1\right)}{\theta} \tag{5}$$

where $\|\cdot\|_V$ is a suitable matrix norm (see [5]).

Lemma 2.1 [5]**:** The matrix measure has following properties.

(a). $\mu_V(\cdot)$ is convex; i. e.,

$$\mu_V\left(\sum_{j=1}^{k}\alpha_j\mathbf{M}_j\right)\le\sum_{j=1}^{k}\alpha_j\mu_V\left(\mathbf{M}_j\right)\quad\text{for all }\alpha_j\ge0. \tag{6}$$

(b). For any norm and any constant matrix **M**

$$-\|\mathbf{M}\|_V\le-\mu_V(-\mathbf{M})\le\operatorname{Re}\lambda(\mathbf{M})\le\mu_V(\mathbf{M})\le\|\mathbf{M}\|_V \tag{7}$$

(c). Suppose m_{ij} is the *ij*-th element of **M**, then

$$\mu_1(\mathbf{M})=\max_j\left[\operatorname{Re}(m_{jj})+\sum_{i\ne j}|m_{ij}|\right], \tag{8}$$

$$\mu_2(\mathbf{M})=\max_i\left[\lambda_i(\mathbf{M}+\mathbf{M}^*)/2\right], \tag{9}$$

$$\mu_\infty(\mathbf{M}) = \max_i \left[\mathrm{Re}(m_{ii}) + \sum_{i \neq j} | m_{ij} | \right]. \quad (10)$$

3. Main Results

In this section, we first show that the considered problem is solvable if a corresponding matrix measure assignment problem is solvable. From (1), (2) and (3), the collection of closed-loop systems can be described as:

$\dot{x}(t) = (\hat{\mathbf{A}}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i)x_i(t) + \hat{\mathbf{D}}_i x_i(t-h_i)$, $i = 1,2,\ldots,p$

$y_i(t) = \mathbf{C}_i x_i(t)$, $i = 1,2,\ldots,p$

Denote

$$\underline{\mathbf{A}}_i = [\underline{a}^i_{jk}], \ \bar{\mathbf{A}}_i = [\bar{a}^i_{jk}], \ i = 1,2,\ldots,p \quad (11)$$

$$\underline{\mathbf{D}}_i = [\underline{d}^i_{jk}], \ \bar{\mathbf{D}}_i = [\bar{d}^i_{jk}], \ i = 1,2,\ldots,p \quad (12)$$

and let

$$\mathbf{A}_i = \frac{1}{2}(\underline{\mathbf{A}}_i + \bar{\mathbf{A}}_i), \ \mathbf{D}_i = \frac{1}{2}(\underline{\mathbf{D}}_i + \bar{\mathbf{D}}_i), i = 1,2,\ldots,p \quad (13)$$

$$\mathbf{M}_i = \bar{\mathbf{A}}_i - \mathbf{A}_i, \ \mathbf{N}_i = \bar{\mathbf{D}}_i - \mathbf{D}_i, i = 1,2,\ldots,p \quad (14)$$

where $\mathbf{A}_i$ and $\mathbf{D}_i$ are the average matrices of $\underline{\mathbf{A}}_i$ and $\bar{\mathbf{A}}_i$, and of $\underline{\mathbf{D}}_i$ and $\bar{\mathbf{D}}_i$, respectively. Furthermore, $\mathbf{M}_i$ and $\mathbf{N}_i$ are the maximal bias matrices between $\hat{\mathbf{A}}_i$ and $\mathbf{A}_i$, and between $\hat{\mathbf{D}}_i$ and $\mathbf{D}_i$, respectively. Then, we have the following results.

Theorem 3.1: Suppose that the matrix F satisfies the following conditions

$$\mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i) < -\alpha - \| \mathbf{M}_i \| - e^{\alpha h_i}(\| \mathbf{D}_i \| + \| \mathbf{N}_i \|),$$
$$i = 1,2,\ldots,p \quad (15)$$

then the collection of systems

$\dot{x}(t) = (\hat{\mathbf{A}}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i)x_i(t) + \hat{\mathbf{D}}_i x_i(t-h_i)$, $i = 1,2,\ldots,p$

are all robustly stable with a decay rate α ($\alpha > 0$).

Proof: This theorem can be easily been proved from the results of [14] or from the results of Lemma 2.1.

For simplicity of notation, let

$$\gamma_i = -\alpha - \| \mathbf{M}_i \| - e^{\alpha h_i}(\| \mathbf{D}_i \| + \| \mathbf{N}_i \|),$$
$$i = 1,2,\ldots,p, \quad (16)$$

then (15) becomes

$$\mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i) < \gamma_i, \ i = 1,2,\ldots,p. \quad (17)$$

Therefore, this problem described by Section 2 is solved if the matrix measure assignment problem (17) is solved. Define $\Im_i(\gamma_i) \equiv \left\{ \mathbf{F} \in \Re^{m \times r} \mid \mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i) < \gamma_i \right\}$, for $i = 1,2,\ldots,p$. The admissible solution set is $\Im \equiv \Im_1(\gamma_1) \bigcap \Im_2(\gamma_2) \bigcap \cdots \bigcap \Im_p(\gamma_p)$. Then, we can have the following theorem.

Theorem 3.2: The admissible solution set $\Im$ is convex.

Proof: Sine the intersection of convex sets is convex, we only need to prove that $\Im_i(\gamma_i)$ is convex for each i.

Assume $\mathbf{F}_1 \in \Im_i(\gamma_i)$ and $\mathbf{F}_2 \in \Im_i(\gamma_i)$, which means $\mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}_1\mathbf{C}_i) < \gamma_i$ and $\mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}_2\mathbf{C}_i) < \gamma_i$. Then, to prove that $\Im_i(\gamma_i)$ is convex is same as to prove $\alpha\mathbf{F}_1 + (1-\alpha)\mathbf{F}_2 \in \Im_i(\gamma_i)$, or equivalently to prove $\mu_2(\mathbf{A}_i + \mathbf{B}_i(\alpha\mathbf{F}_1 + (1-\alpha)\mathbf{F}_2)\mathbf{C}_i) < \gamma_i$, for all $0 \le \alpha \le 1$. Note that

$$\begin{aligned}
&\mu_2(\mathbf{A}_i + \mathbf{B}_i(\alpha\mathbf{F}_1 + (1-\alpha)\mathbf{F}_2)\mathbf{C}_i) \\
&= \mu_2(\alpha\mathbf{A}_i + (1-\alpha)\mathbf{A}_i + \alpha\mathbf{B}_i\mathbf{F}_1\mathbf{C}_i + (1-\alpha)\mathbf{B}_i\mathbf{F}_2\mathbf{C}_i) \\
&= \mu_2(\alpha(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}_1\mathbf{C}_i) + (1-\alpha)(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}_2\mathbf{C}_i)) \\
&\le \alpha\mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}_1\mathbf{C}_i) + (1-\alpha)\mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}_2\mathbf{C}_i) \\
&< \gamma
\end{aligned}$$

This completes the proof.

From the above discussions, it is concluded that the matrix measure assignment problem can be considered as a convex feasibility problem. Thus, we now turn our attention to reduce the matrix measure assignment problem to an LMI feasibility problem.

For a matrix U, define $\mathbf{U}_\perp$ as a matrix whose columns form bases of the null bases of U. Then, we can have the following theorem, which is the main result of this paper.

Theorem 3.3:

(1). The matrix F satisfies

$$\mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i) < \gamma_i, \ i = 1,2,\ldots,p. \quad (18)$$

if and only if F satisfies LMIs

$$(\mathbf{A}_i + \mathbf{A}_i^T - 2\gamma_i\mathbf{I}) + \mathbf{B}_i\mathbf{F}\mathbf{C}_i + \mathbf{C}_i^T\mathbf{F}^T\mathbf{B}_i^T < \mathbf{0},$$
$$i = 1,2,\ldots,p. \quad (19)$$

(2). There exists F satisfies (19) if and only if

$$(\mathbf{B}_i)_\perp(\mathbf{A}_i + \mathbf{A}_i^T - 2\gamma_i\mathbf{I})(\mathbf{B}_i^T)_\perp < \mathbf{0}, \ i = 1,2,\ldots,p \quad (20)$$

and

$$(\mathbf{C}_i^T)_\perp(\mathbf{A}_i + \mathbf{A}_i^T - 2\gamma_i\mathbf{I})(\mathbf{C}_i)_\perp < \mathbf{0}, \ i = 1,2,\ldots,p \quad (21)$$

Proof: We first prove part (1). From (9), it can be shown that $\mu_2(\mathbf{A}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i) < \gamma_i$, $i = 1,2,\ldots,p$, are equivalent to

$$(\mathbf{A}_i+\mathbf{B}_i\mathbf{FC}_i)+(\mathbf{A}_i+\mathbf{B}_i\mathbf{FC}_i)^*-2\gamma_i\mathbf{I}<\mathbf{0},\quad i=1,2,\ldots,p. \tag{22}$$

which are equivalent to

$$(\mathbf{A}_i+\mathbf{A}_i^T-2\gamma_i\mathbf{I})+\mathbf{B}_i\mathbf{FC}_i+\mathbf{C}_i^T\mathbf{F}^T\mathbf{B}_i^T<\mathbf{0},\ i=1,2,\ldots,p.$$

This completes the proof of part (1). For the part (2), recall the result in [19]. Given a symmetric matrix $\mathbf{\Psi}\in\Re^{n\times n}$ and two matrices U and V both with a column dimension n, there exists a matrix $\mathbf{\Theta}$ of a compatible dimension such that

$$\mathbf{\Psi}+\mathbf{U}^T\mathbf{\Theta}^T\mathbf{V}+\mathbf{V}^T\mathbf{\Theta}\mathbf{U}<\mathbf{0}$$

if and only if $\mathbf{U}_\perp^T\mathbf{\Psi}\mathbf{U}_\perp<\mathbf{0}$ and $\mathbf{V}_\perp^T\mathbf{\Psi}\mathbf{V}_\perp<\mathbf{0}$.

Letting $\mathbf{\Psi}_i=(\mathbf{A}_i+\mathbf{A}_i^T-2\gamma_i\mathbf{I})$, $\mathbf{V}_i=\mathbf{B}_i^T$, $\mathbf{U}_i=\mathbf{C}_i$, and $\mathbf{\Theta}=\mathbf{F}$, the part (2) is obvious.

Theorem 3.3 tells us that if (20) and (21) hold, then there exists a matrix F that satisfies LMIs (19). In fact, such an F also solves (18). This means that if (20) and (21) hold, then the admissible solution set $\Im$ is not empty. Note that a matrix F satisfying LMIs (19) can easily be obtained by using Matlab's LMI Control Toolbox if $\Im$ is not empty. The obtained F then can also solve the considered problem.

Remark 3.1: The approach described above can be applied to solve the simultaneous output feedback stabilization problem for a collection of uncertain systems:

$\dot{x}_i(t)=(\mathbf{A}_i+\Delta\mathbf{A}_i)x_i(t)+\mathbf{B}_iu_i(t)$, i=1, 2, …, p

$y_i(t)=\mathbf{C}_ix_i(t)$, i=1, 2, …, p

$\|\Delta\mathbf{A}_i\|\le\rho_i$, i=1, 2, …, p

where $x_i\in\Re^n$ is the state, $u_i\in\Re^m$ is the control input, and $y_i\in\Re^r$ is the output; and $\mathbf{A}_i$, $\mathbf{B}_i$, and $\mathbf{C}_i$ are constant matrices of appropriate dimensions. The design goal is to find a matrix F such that the static output feedback controller

$u_i(t)=\mathbf{F}y_i(t)$, i=1, 2, …, p

can stabilize all the closed loop systems in the presence of uncertainty $\Delta\mathbf{A}_i$.

Since $\mu_2(\Delta\mathbf{A}_i)\le\|\Delta\mathbf{A}_i\|$, it is known that if we can find a feedback matrix F such that

$$\mu_2(\mathbf{A}_i+\mathbf{B}_i\mathbf{FC}_i)<-\rho_i,\quad \text{i=1, 2, …, p} \tag{23}$$

then all the closed-loop systems are asymptotically stable. This problem can be easily solved via our approach.

4. Illustrative Examples

Consider two interval time-delay systems described by (2.1)-(2.3) with the following data:

System 1:

$$\underline{\mathbf{A}}_1=\begin{bmatrix}3.8&-5.6&3.5\\-4.3&19.1&-19.2\\29.4&3.6&-36.5\end{bmatrix},\quad \bar{\mathbf{A}}_1=\begin{bmatrix}4.6&-4.2&4.5\\-3.7&22.9&-16.8\\34.6&4.4&-30.5\end{bmatrix},$$

$$\underline{\mathbf{D}}_1=\begin{bmatrix}-0.4&0.7&-0.8\\0.3&-3.2&-2.1\\-3.3&-0.5&-2.3\end{bmatrix},\quad \bar{\mathbf{D}}_1=\begin{bmatrix}-0.2&1.0&-0.3\\0.8&-1.8&0.1\\-0.7&0.5&-1.2\end{bmatrix},$$

$$\mathbf{B}_1=\begin{bmatrix}1&-7\\5&-3\\3&-4\end{bmatrix},\quad \mathbf{C}_1=\begin{bmatrix}-7&8&1\\2&-5&-3\end{bmatrix}.$$

System 2:

$$\underline{\mathbf{A}}_2=\begin{bmatrix}5.2&-6.7&7.1\\-9.3&11.9&-15.9\\21.9&8.6&-27.3\end{bmatrix},\quad \bar{\mathbf{A}}_2=\begin{bmatrix}5.6&-6.2&9.3\\-8.2&15.1&-13.7\\25.3&9.9&-24.6\end{bmatrix},$$

$$\underline{\mathbf{D}}_2=\begin{bmatrix}-0.7&0.2&-1.1\\0.9&-5.0&-4.0\\-2.3&-0.9&-4.2\end{bmatrix},\quad \bar{\mathbf{D}}_2=\begin{bmatrix}-0.3&0.8&-0.7\\1.3&-3.0&-1.0\\-1.1&0.2&-3.4\end{bmatrix},$$

$$\mathbf{B}_2=\begin{bmatrix}2&-7\\4&-1\\2&-3\end{bmatrix},\quad \mathbf{C}_2=\begin{bmatrix}-5&9&1\\3&-7&-2\end{bmatrix}.$$

The delay times $h_1=1$ and $h_2=1$, and the decay rate $\alpha=0.26$. The problem is to find F such that $\mu_2(\mathbf{A}_i+\mathbf{B}_i\mathbf{FC}_i)<\gamma_i$ for $i=1,\ 2$, where

$$\mathbf{A}_i=\frac{1}{2}(\underline{\mathbf{A}}_i+\bar{\mathbf{A}}_i),\ \mathbf{D}_i=\frac{1}{2}(\underline{\mathbf{D}}_i+\bar{\mathbf{D}}_i),\ i=1,\ 2,$$

$$\mathbf{M}_i=\bar{\mathbf{A}}_i-\mathbf{A}_i,\ \mathbf{N}_i=\bar{\mathbf{D}}_i-\mathbf{D}_i,\ i=1,\ 2,$$

$$\gamma_i=-\alpha-\|\mathbf{M}_i\|-e^{\alpha h_i}(\|\mathbf{D}_i\|+\|\mathbf{N}_i\|),\ i=1,\ 2.$$

For $\alpha=0.26$, we can obtain $\gamma_1=-10.6508$ and $\gamma_2=-12.2831$. Then we can easily compute a solution F from the following LMIs using Matlab's LMI Control Toolbox.

$$(\mathbf{A}_1+\mathbf{A}_1^T-2\gamma_1\mathbf{I})+\mathbf{B}_1\mathbf{FC}_1+\mathbf{C}_1^T\mathbf{F}^T\mathbf{B}_1^T<\mathbf{0}$$

$$(\mathbf{A}_2+\mathbf{A}_2^T-2\gamma_2\mathbf{I})+\mathbf{B}_2\mathbf{FC}_2+\mathbf{C}_2^T\mathbf{F}^T\mathbf{B}_2^T<\mathbf{0}$$

A solution is obtained as follows:

$$\mathbf{F}=\begin{bmatrix}-16.9437&-3.7400\\-11.3197&-6.1597\end{bmatrix}.$$

It is easy to check that $\mu_2(\mathbf{A}_1+\mathbf{B}_1\mathbf{FC}_1)=-11.4358$, which is less that $\gamma_1=-10.6508$. Similarly, $\mu_2(\mathbf{A}_2+\mathbf{B}_2\mathbf{FC}_2)=-15.1686<\gamma_2=-12.2831$. It then can be inferred from Theorem 3.1 that the collection of systems

$$\dot{x}(t) = (\hat{\mathbf{A}}_i + \mathbf{B}_i\mathbf{F}\mathbf{C}_i)x_i(t) + \hat{\mathbf{D}}_i x_i(t-h_i), \quad i = 1,2,\ldots,p,$$

are all robustly stable.

5. Conclusions

The problem of simultaneously stabilizing controller design via static output feedback for a collection of interval time-delay systems was solved by finding an admissible solution to the matrix measure assignment problem. We presented an LMI approach to solve the matrix measure assignment problem. It was shown that the admissible solution set of the matrix measure assignment problem is convex. It is also shown that the matrix measure assignment problem is equivalent to an LMI feasibility problem. A necessary and sufficient condition for the existence of output feedback controllers to the matrix measure assignment problem is obtained. Finally, an illustrative example is given to show the correctness of the proposed approach.

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