The Growth Forecast in the Number of Broadband Services Subscribers Using Grey Model

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Abstract: - Considering the need to prepare short to mid-term plan and budget for building and maintaining costly infrastructure for the fast growing broadband services in Taiwan, this paper attempted to forecast the growth in the number of broadband subscribers in Taiwan with the help of Grey Model, GM(1,1), developed from Grey system theory. A forecast system was designed and built based on GM(1,1) followed by an experiment using the data published by DGT Taiwan. The experimental results show that a rolling GM(1,1) can satisfactorily forecast the growth in the number of broadband subscribers based on only a small sample of data. A set of four sequential records can accurately forecast the fifth record with less than 2.54% error in average and can adequately forecast the sixth, seventh, eighth and ninth records with a maximum of 12.98% error.

Key-Words: Broadband services, Broadband subscriber, Budget, Forecast, Grey Model.

1 Introduction

Recent development in the Internet has initiated a high demand for broadband services [1]. To fulfill such demand, telecommunication services providers have to deal with the capital investment on more and more infrastructure such as servers and cable links. In addition, to enhance customer services, many telecommunication services providers has adopted advanced information and communication technology to improve their infrastructure.

The investment on building and maintaining costly infrastructure has become a critical issue for telecommunication services providers. However, the preparation of short-term to mid-term budget has turned out to be a new challenge for management and technical team [2]. An accurate or satisfactory forecast of demand, i.e., the future growth in the number of telecommunication services subscribers, plays a key role in planning for services providers [3]. A satisfactory forecast is therefore crucial but such forecast is hard to describe by mathematic model.

Although the number of broadband subscriber is influenced by some certain factors such as price and the number of competitors, there are some uncertain factors that cannot be clearly identified or calculated. For a forecast involving certain and uncertain factors, a grey system is required. Grey models developed from the Grey theory [4, 5], have been widely used in many fields for forecasting. Most of the forecasts were proven with high prediction accuracy.

The most commonly used Grey model is the GM(1,1) which denotes a single variable and the first-order linear dynamic model. However, a GM(1,1) model is theorized to deal with non-negative sequences. As the number of broadband subscriber is always positive, the GM(1,1) can be applied to forecast the number.

This paper attempted to forecast the growth in the number of broadband subscribers in Taiwan with the help of Grey Model, GM(1,1), developed from Grey system theory. The GM(1, 1) is reviewed in Section 2. The forecast and predication error measurement methodology are described in Section 3. Experimental data and results are shown in Section 4. Conclusions of this paper are given in Section 5.

2 GM(1, 1) Model

The Grey system theory was proposed by Deng in 1982 to solve grey system problems [4, 5]. The Grey system theory treats all variables as a Grey quantity within a certain range. Grey prediction model (GM) then collects available data to obtain the internal regularity.

The GM examines the nature of internal regularity in managing the disorganized primitive data. The most commonly used model is the GM(1, 1). It is widely used in various fields: agriculture, meteorology, economy, manufacturing, and others [6, 7, 8, 9, 10]. The GM can be obtained in 5 steps [11].

Step 1: Generating

Accumulated Generation Operation (AGO): Accumulating obtained systematic regularity discrete time-series data. Using the AGO technique efficiently reduces noise by converting ambiguous primitive time-series data to a monotonically increased series.

The AGO technique is capable of identifying the systematic regularity quickly and easily. The Grey system must apply one-order accumulated generating operation (AGO) to the primitive sequence in order to provide the middle message of building a model.

$$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)) \quad (1)$$

$x^{(1)}$ is $x^{(0)}$ one-order accumulated generating operation (AGO) sequence, that is,

$$x^{(1)} = (\sum_{k=1}^{1} x^{(0)}(k), \sum_{k=1}^{2} x^{(0)}(k), \ldots, \sum_{k=1}^{n} x^{(0)}(k)) \quad (2)$$

Class Ratio: $\sigma(k) = \frac{x(k-1)}{x(k)}$ $k \geq 2$, when $\sigma(k) \in (0,1]$, the Grey model can be built.

Inverse-accumulated generating operation (IAGO): The acquired sequence one-order inverse-accumulated generating operation (IAGO) is acquired and the sequence that must be reduced as following equation in order to obtain statistics confirm the efficiency of the proposed prediction model.

$$\hat{x}^{(0)}(k) = x^{(1)}(k) - x^{(1)}(k-1)$$

Step 2: Grey Derivatives

When $x^{(1)} = AGOx^{(0)}$, the Grey derivatives is $D(k)$, while $x^{(1)}$ is at k^{th} $D(k)$. $D(k) = x^{(0)}(k) = x^{(1)}(k) - x^{(1)}(k-1)$, $x^{(1)}$ of Grey derivatives $D(k)$, respect for $D(k)$ one half of existence within components, corresponding relation, their corresponding relation as following equation: $z^{(1)} = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1)$ (3)

Step 3: Grey Difference Equation Derivatives

$D(k) = x^{(1)}(k) - x^{(1)}(k-1)$, while $x^{(1)}(k)$ and $x^{(1)}(k-1)$ are white indicate systems, however, an incomplete information of $D(k)$ herein creating a sequence of one-order linear moving GM(1,1), the first order differential equation of GM(1,1) model is $\frac{dx}{dt} + ax = b$,where t denotes the independent variables in the system, a represents the developed coefficient, b is the Grey controlled variable, moreover a and b denoted the parameters requiring determination in the model.

When a model is constructed, the differential equation is $x^{0}(k) + az^{(1)}(k) = b$, including $k = 2, 3, \ldots, n$, where a, b denoted standby substantial number, this differential equation $x^{(0)}(k) + az^{(1)}(k) = b$ is called as GM(1,1) model.

$$x^{0}(2) + az^{(1)}(2) = b$$

$$x^{0}(3) + az^{(1)}(3) = b$$

$$\vdots$$

$$x^{0}(n) + az^{(1)}(n) = b$$

where $z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1)$,

let

$$Y_N = \left[x^{(0)}(2) \ldots\ldots x^{(0)}(n)\right]^T \quad B = \begin{bmatrix} -z^{(1)}(2) \ldots -z^{(1)}(n) \\ 1 \quad \ldots \quad 1 \end{bmatrix}^T \quad A = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$Y_N = BA \quad , \quad B^T Y_N = B^T BA \quad , \quad A = (B^T B)^{-1} B^T Y_N$$

Furthermore, accumulated matrix a and b are as below expand equations:

$$a = \frac{\sum_{k=2}^{n} z^{(1)}(k) \sum_{k=2}^{n} x^{(0)}(k) - (n-1) \sum_{k=2}^{n} z^{(1)}(k) x^{(0)}(k)}{(n-1) \sum_{k=2}^{n} \left[z^{(1)}(k)\right]^2 - \left[\sum_{k=2}^{n} z^{(1)}(k)\right]^2} \quad (4)$$

$$b = \frac{\sum_{k=2}^{n} [z^{(1)}(k)]^2 \sum_{k=2}^{n} x^{(0)}(k) - \sum_{k=2}^{n} z^{(1)}(k) \sum_{k=2}^{n} z^{(1)}(k) x^{(0)}(k)}{(n-1) \sum_{k=2}^{n} [z^{(1)}(k)]^2 - [\sum_{k=2}^{n} z^{(1)}(k)]^2} \quad (5)$$

Step 4: Whitening Equation

Differential equation $\frac{dx^{(1)}}{dt} + ax^{(1)} = b$ model becomes $x^{(1)}(t) = ce^{-at} + \frac{b}{a}$

Step 5: Utilize IAGO to return to primitive equation

$$\hat{x}^{(0)}(k+1) = x^{(1)}(k+1) - x^{(1)}(k) \quad (6)$$

$$\hat{x}^{(0)}(k+1) = (1 - e^{a}) \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak}$$

3 Methodology

The rolling GM(1, 1) was used. A set of four sequential records was used to forecast the fifth record. In experiment 1, only actual data was used. No forecasted data was used in forecasting. For example, the actual 1st, 2nd, 3rd and 4th records were used to forecast the forecasted 5th record; the actual 2nd, 3rd, 4th and 5th records were used to forecast the forecasted 6th record; and so on.

In experiment 2, if the actual data was less than 4 records, the forecasted data was used instead. For example, the actual 1st, 2nd, 3rd and 4th records were used to forecast the forecasted 5th record; the actual 2nd, 3rd, 4th and the forecasted 5th records were used to forecast the forecasted 6th record (as there was no actual 5th record); the actual 3rd, 4th , the forecasted 5th , and the forecasted 6th records were used to forecast the forecasted 7th record (as there was not actual 5th and 6th records); and so on.

For residual error checking, the following formula was used.

$$e(k) = \left| \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \right| \times 100\% \quad (7)$$

4 Results

4.1 Experiment 1

The results of experiment 1 were shown in Table 1. The average accuracy of the forecasted number of broadband subscribers was satisfactorily.

Table1 Results of Experiment 1

Quarter/Year	Actual Number of Broadband Subscribers	Forecasted Number of Broadband Subscribers	Residual Error
4th Q./2001	11,640	N/A	N/A
1st Q./2002	14,320	N/A	N/A
2nd Q./2002	16,260	N/A	N/A
3rd Q./2002	18,710	N/A	N/A
4th Q./2002	21,160	21,315	0.0073
1st Q./2003	22,780	24,124	0.0590
2nd Q./2003	24,690	25,241	0.0223
3rd Q./2003	27,150	26,632	0.0191
4th Q./2003	30,410	29,568	0.0277
1st Q./2004	32,428	33,645	0.0375
2nd Q./2004	33,964	35,605	0.0483
3rd Q./2004	35,867	35,968	0.0028
4th Q./2004	37,512	37,670	0.0042
Average			0.0254

The average residual error rate for the 9 forecasted records was 0.0254, i.e., 2.54%. The accuracy rate, 97.46%, indicated the effectiveness of GM(1, 1) in forecasting the growth number in broadband subscribers in Taiwan [12].

The best forecasted result was the 3rd quarter of 2003. The residual error was 0.0028, i.e., 0.28%. The accuracy rate, 99.72%, was nearly perfect. It was noted that even the worst forecast had an accuracy rate of 95.17% (the 2nd quarter of 2004).

4.2 Experiment 2

The results of experiment 2 were shown in Table 2. In experiment 2, it was assumed that the data from the 4th quarter of 2003 and onward was missing. One or more forecasted data were used in the subsequent forecasts. However, the results showed that the average accuracy of the forecasted number of broadband subscribers was still satisfactorily even though forecasted data were used instead of the actual data.

Table 2 Results of Experiment 2

QuarterYear	Actual Number of Broadband Subscribers	Forecasted Number of Broadband Subscribers	Residual Error
4th Q./2001	11,640	N/A	N/A
1st Q./2002	14,320	N/A	N/A
2nd Q./2002	16,260	N/A	N/A
3rd Q./2002	18,710	N/A	N/A
4th Q./2002	21,160	N/A	N/A
1st Q./2003	22,780	N/A	N/A
2nd Q./2003	24,690	N/A	N/A
3rd Q./2003	27,150	N/A	N/A
4th Q./2003	30,410	*29598*	0.0267
1st Q./2004	32,428	*32480*	0.0016
2nd Q./2004	33,964	*35468*	0.0443
3rd Q./2004	35,867	*38812*	0.0821
Average 1			0.0387
4th Q./2004	37,512	*42380*	0.1298

The average residual error rate for the 4 forecasted records was 0.0387, i.e., 3.87%. The accuracy rate, 96.13%, indicated the effectiveness of GM(1, 1) in forecasting the growth number in broadband subscribers even though some forecasted data were reused in the subsequent forecasts.

Interestingly, the forecast of the 4th quarter of 2004 was still acceptable as the residual error was 0.1298, i.e., 12.98% and the accuracy rate was 87.02%. The former forecast used only forecasted data from previous forecast.

4 Conclusions

This paper shows that the GM(1, 1) model can satisfactorily forecast the growth in the number of broadband subscribers despite the limited number of sampling. When actual data and forecasted data were combined to forecast the subsequent record, GM(1, 1) is still proved useful and satisfactory. This paper also shows that the first set of four forecasted data can adequately forecast the subsequent record. Instead of merely based on gut feeling or luck, telecommunication services providers may use GM(1, 1) in helping them to plan short-term to mid-term infrastructure investment and prepare their budget..

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