A BMI Approach to Robust Controller Design for Systems with Real Parametric Uncertainties

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Abstract: - In this work, we propose two robust controller design methods based on new iterative schemes for the systems with real parametric uncertainties. The methods are derived on the basis of a recently developed method for computing optimal stability multipliers. The salient property that there is no need to choose the poles of the multipliers a priori is retained. Beside the benefit of that the controller order would not increase without bounds, a hybrid combination of the proposed methods is possible. Comprehensive comparisons of the proposed methods and those designs which mimic the well known D-K iteration are made. A numerical example is given to illustrate the results.

Key-Words: - robust stability, stability multiplier, linear matrix inequality (LMI), real parametric uncertainty

1 Introduction

µ synthesis is [1-6] is a powerful tool for designing robust controllers for systems subject to multiple sources of parametric and/or dynamic uncertainties. In 1985 the well known D-K iteration [1] was proposed to produce robust controllers for the systems with structured dynamic uncertainties, which is essentially based on iterating between the analysis phase of computing the complex µ upper bound (or the optimal multipliers) with the controller fixed, and the synthesis phase of $H_\infty$ optimization with the multiplier fixed.

In early research curve fitting is widely used in the analysis phase, however, error might occur due to approximation. Later, two basis function methods were proposed in Ly et al [7] and [8]. While curve fitting of the scalings is no longer required and good estimates of the µ upper bound could be rendered by both of the methods, both of them rely heavily on a proper choice of the bases, equivalently, the choice of the poles of the multipliers. Later, in [9] a skillful LMI method was proposed for computing the optimal multipliers. Particularly, there is no need to choose the poles of the multipliers a priori, yet a lower bound constraint on the multiplier order (at least as large as the order of closed-loop system) is assumed for the approach. This generally improves the computation of the real µ upper bound. New robust controller design based on the method [9] has been presented in [6], which inherits the advantage of [9] at the expense that the order of the resulting controllers dramatically increases as the iteration number of the synthesis procedure goes up. Therefore, it is our purpose to propose new robust controller design to alleviate the problem. The paper is organized as follows. Section 2 gives the formal problem statement and some preliminaries for future developments. Section 3 presents the two proposed approaches for the robust controller synthesis problem. Comprehensive comparisons of the controller designs are made. In Section 4, a numerical example is provided to demonstrate the results. Section 5 is the conclusions.

2 Problem Formulation

Notation

Most notation used in this paper is fairly standard, see e.g., [6].

Problem Description

The $\Delta - P - K$ paradigm is considered, where it is assumed throughout that $\Delta$ belongs to the parametric structured uncertainty set defined as follows: $\Delta = \{\text{block - diag}(\delta_1 I_m, \ldots, \delta_L I_m): \delta_i \in R, i = 1, \ldots, L\}$

The symbol $P$ denotes the generalized plant, including the nominal plant, weighting functions, etc, described by

$$
\begin{align*}
\dot{x} &= Ax + B_1 w + B_2 u \\
z &= C_1 x + D_{11} w + D_{12} u \\
y &= C_2 x + D_{21} w + D_{22} u
\end{align*}
$$

where $x \in R^{n}, \quad w \in R^{n}, \quad u \in R^{m}, \quad z \in R^{n},$ and $y \in R^{n}$ with $m := m_1 + \cdots + m_u$. The symbol $K$ denotes a dynamic controller of the form...

\[ K \left\{ \begin{array}{l}
x_k = A_k x_k + B_k y \\
u = C_k x_k + D_k y
\end{array} \right. \] (2)

where \( x_k \in \mathbb{R}^{n_k} \), to be designed. Our goal is to find a dynamic controller \( K(s) \) of form (2) to enlarge the robust stability margin of the perturbed system in Fig. 1. Specifically, for all \( \Delta \in \Delta_r \) with \( A : \mathbb{R}^{n} \rightarrow \mathbb{R}^{n} \), (i.e., \( \Delta \leq \gamma \) for all \( i \) ) we want to design a dynamic controller of form (2) to maximize the value \( \gamma \) such that the closed-loop system (1)-(2) is robustly stable.

To reduce the conservatism, an improved technique which employs passivity theorem with multipliers has been addressed in [2,4,8,10], as shown in Fig. 2, where \( \hat{\gamma} = (I + \gamma^2 \Delta)(I - \gamma^2 \Delta)^{-1} \), and \( \tilde{M}_r := (I - \gamma M)(I + \gamma M)^{-1} = F_1(\tilde{P}_r, K) \) [4].

![Passivity framework with multipliers](image)

It is easy to check that for any \( \Delta \in \Delta_r \) with \( A : \mathbb{R}^{n} \rightarrow \mathbb{R}^{n} \), \( \hat{\gamma} \) is a constant diagonal matrix with entries in \([0, \infty)\). A well known sufficient condition for the robust synthesis problem is thus to maximize the value \( \gamma \) via finding a dynamic controller \( K(s) \) of form (2) and suitable stability multipliers \( W_1(s) \) and \( W_2(s) \) in the set \( S_\gamma(\mathbb{R}^n) \), satisfying (i) \( W_1, W_2, W_1^{-1} \) and \( W_2^{-1} \) are in \( RH_\infty \), (ii) \( W_1 W_2 \) is strictly positive real (SPR), and (iii) \( W_1^{-1} \tilde{M}_r W_2^{-1} \) is SPR [4]. By letting \( W = W_1^{-1} W_2^{-1} \) (possibly non-causal), it has been shown in [2,4,8,10] that the conditions described above are equivalent to the following frequency domain conditions with \( \tilde{M}_r := F_1(\tilde{P}_r, K) \) which hold for some \( \epsilon > 0 \) and for all \( \forall \omega \in \mathbb{R} \cup \{ \infty \} \),

\[ \tilde{M}_r(j\omega) W(j\omega) + W(j\omega)^* \tilde{M}_r(j\omega)^{**} > 2\epsilon I \] (3)

\[ W(j\omega) + W(j\omega)^* > 0 \] (4)

In summary, the robust synthesis problem we consider is to maximize the value \( \gamma \) via finding a dynamic controller \( K(s) \) of form (2) and a generalized stability multiplier \( W \in S_\gamma(\mathbb{R}^n) \) such that conditions (3) and (4) hold. For the latter development, it is required to know the state-space realization of the sector transformed plant \( \tilde{P}_r = S \otimes \Gamma P \), where

\[ S = \begin{pmatrix} I_n & \sqrt{\gamma} \mathbf{I} \\ \sqrt{\gamma} \mathbf{I} & -I_n \end{pmatrix}, \quad \Gamma = \begin{pmatrix} \gamma I_n & 0 \\ 0 & I_s \end{pmatrix} \]

and the symbol \( \otimes \) means star product [11]. A tedious algebraic manipulation shows that

\[ \tilde{P}_r(s) \ll \begin{bmatrix} \hat{A} & \hat{B}_1 & \hat{B}_2 \\ \hat{C}_1 & \hat{D}_{11} & \hat{D}_{12} \\ \hat{C}_2 & \hat{D}_{21} & \hat{D}_{22} \end{bmatrix} \]

where

\[ \hat{A} = A - \gamma B_1 (I + \gamma D_{11})^{-1} C_1; \]

\[ \hat{B}_1 = \sqrt{\gamma} B_1 (I + \gamma D_{11})^{-1} D_{12}; \]

\[ \hat{C}_1 = -\sqrt{\gamma} (I + \gamma D_{11})^{-1} C_1; \]

\[ \hat{C}_2 = C_2 - \gamma D_{11} (I + \gamma D_{11})^{-1} C_1; \]

\[ \hat{D}_{11} = (I + \gamma D_{11})^{-1} (I - \gamma D_{11}); \]

\[ \hat{D}_{12} = -\sqrt{\gamma} (I + \gamma D_{11})^{-1} D_{12}; \]

\[ \hat{D}_{21} = \sqrt{\gamma} (I + \gamma D_{11})^{-1} D_{21}; \]

\[ \hat{D}_{22} = D_{22} - \gamma D_{21} (I + \gamma D_{11})^{-1} D_{12}. \]

Method 1:
The design procedure of method 1 involves iterations between computing the real \( \mu \) upper bound (analysis phase) and solving for a SPR controller (synthesis phase) [12,13]. For details, see [6].

Method 2:
The design is similar to method 1. The only difference is the replacement of the analysis phase with the method proposed in [9]. A brief review of the approach [9] is introduced as follows: Consider the stable system \( \tilde{M}_r := F_1(\tilde{P}_r, K) \). As before, let \( n_{w_1} \) denote the order of the stable system \( \tilde{M}_r \) and \( n_w \) denote the order of generalized stability multiplier \( W \) to be computed. Under the constraint \( n_w \geq n_{w_1} - n_x + n_x \), where \( n_x \) denote the order of the generalized plant \( P \) and the controller \( K \) respectively, suppose that there exist a positive number \( \epsilon \), matrices \( P = P_T \in \mathbb{R}^{(n_y, n_{w_1})} \), and block-diagonal matrices \( X \in \mathbb{R}^{(n_y, n_{w_1})} \), \( Y \in \mathbb{R}^{(n_w, n_{w_1})} \), \( P_w = P_T \in \mathbb{R}^{(n_w, n_{w_1})} \), \( B_w \in \mathbb{R}^{(n_w, n_{w_1})} \), \( D_w \in \mathbb{R}^{(n_w, n_{w_1})} \), with \( P_w \) non-singular, such that the following LMIs hold

\[
\begin{bmatrix}
X^T + X & -B_w & -Y^T \\
-B_w^T & 2\epsilon I - (D_w + D_w^T) & -Y \\
L_{11} & L_{12} & L_{22}
\end{bmatrix} < 0
\] (5)

where
Then \( W(s) = Y(sP_w - X)^{1/2}B_w + D_w \) is a generalized stability multiplier satisfying conditions (3) and (4).

In comparison with [8] in which the poles are chosen by ad hoc work, the method in [9] has the advantage of not necessary to choose the multiplier poles a priori, which in turn could reduce the conservatism of computing optimal multipliers and thus gets a tighter estimate of the robust stability margin. However, the price is that the order of the resulting controllers by method 2 dramatically increases as the iteration number of the synthesis procedure goes up. To alleviate this problem a new procedure employing a fixed-order controller synthesis scheme is proposed on the basis of the analysis conditions (5)-(6).

### 3 Main Results

In this section we present two design methods originating from the analysis condition presented in [9] which leads to a nonlinear optimization problem involving bilinear matrix inequalities (BMIs) when controller synthesis is considered. On the basis of the new BMI formulation, different ways of partitioning the BMI variables into two groups are discussed and iterative LMI (ILMI) method is introduced to solve the problem. Algorithms based on state-space manipulation will be given and comprehensive comparison will be made between the proposed methods and the two D-K iteration like synthesis methods presented in [6] will be provided.

To proceed, let \( Q_{eq} \) be the system matrix of the transformed controller \( K_{eq} \) where

\[
K_{eq} = K(I - \hat{D}_{22}K)^{-1};
\]

it follows that the closed-loop system \( \hat{F}(\hat{P},K_{eq}) \) has the following system matrix depending affinely on \( Q_{eq} \).

\[
\begin{bmatrix}
A_{\hat{M}_T}(Q_{eq}) & B_{\hat{M}_T}(Q_{eq}) \\
C_{\hat{M}_T}(Q_{eq}) & D_{\hat{M}_T}(Q_{eq})
\end{bmatrix}
\]

with

\[
A_{\hat{M}_T}(Q_{eq}) = \begin{bmatrix}
\hat{A} & 0 & 0 \\
0 & \hat{B}_T & 0 \\
0 & 0 & \chi
\end{bmatrix} \quad B_{\hat{M}_T}(Q_{eq}) = \begin{bmatrix}
\hat{B}_T & 0 \\
0 & \hat{B}_T & 0 \\
0 & 0 & 0
\end{bmatrix} \quad Q_{eq} = \begin{bmatrix}
I & 0 \\
0 & I \\
0 & 0
\end{bmatrix}
\]

\[
B_{\hat{M}_T}(Q_{eq}) = \begin{bmatrix}
\hat{B}_T & 0 \\
0 & \hat{B}_T & 0 \\
0 & 0 & 0
\end{bmatrix} \quad Q_{eq} = \begin{bmatrix}
I & 0 \\
0 & I \\
0 & 0
\end{bmatrix}
\]

\[
C_{\hat{M}_T}(Q_{eq}) = \begin{bmatrix}
\hat{C}_1 & 0 & 0 & \hat{C}_2 \\
0 & \hat{C}_1 & 0 & 0 \\
0 & 0 & \hat{C}_2 & 0
\end{bmatrix} \quad D_{\hat{M}_T}(Q_{eq}) = \begin{bmatrix}
\hat{D}_{11} & 0 & 0 \\
0 & \hat{D}_{12} & 0 \\
0 & 0 & \hat{D}_{12}
\end{bmatrix}
\]

Substituting the above information into (5) and (6), the robust stability margin optimization problem becomes a nonlinear optimization problem subject to BMI constraints as described as follows:

Maximize \( \gamma \) in the variables \( \varepsilon \) (a positive number), the block-diagonal matrix variables \( X \in \mathbb{R}^{n_x \times n_x} \), \( Y \in \mathbb{R}^{n_y \times n_y} \),

\[
P_w = P_w^T \in \mathbb{R}^{n_w \times n_w} \quad P_{M_T} = P_{M_T}^T \in \mathbb{R}^{(n_p + n_u)(n_p + n_u)} \]

\[
B_w \in \mathbb{R}^{n_w \times n_w} \quad D_w \in \mathbb{R}^{n_y \times n_w} \]

matrix variables \( Q_{eq} \) and \( P = P^T \in \mathbb{R}^{(n_p + n_u)(n_p + n_u)} \) subject to the following BMI constraints (7)-(10).

\[
P_{M_T} > 0 \quad \text{(7)}
\]

\[
P_{M_T} A_{\hat{M}_T}(Q_{eq}) + P_{M_T} A_{\hat{M}_T}(Q_{eq})^T < 0 \quad \text{(8)}
\]

\[
\begin{bmatrix}
X^T & X & B_w & -Y^T \\
B_w^T & Y & 2\varepsilon I & -(D_w + D_{eq})^T \\
G_{11} & G_{12} & G_{13} \\
G_{12}^T & G_{22} & G_{23} \\
G_{13}^T & G_{23} & G_{33}
\end{bmatrix} < 0 \quad \text{(9)}
\]

where

\[
G_{11} := X^T + X; \quad G_{12} := Y^T B_{\hat{M}_T}(Q_{eq}) + P_w \Gamma_0 A_{\hat{M}_T}(Q_{eq}) + XT\gamma^2; \quad G_{13} := B_w - Y^T D_{\hat{M}_T}(Q_{eq}) - P_w \Gamma_0 C_{\hat{M}_T}(Q_{eq}); \quad G_{22} := P A_{\hat{M}_T}(Q_{eq}) + A_{\hat{M}_T}(Q_{eq}) P
\]

\[
+ \Gamma_0 Y^T B_{\hat{M}_T}(Q_{eq}) + B_{\hat{M}_T}(Q_{eq}) Y \Gamma_0 \\
G_{23} := B_{\hat{M}_T}(Q_{eq}) D_w - Y^T D_{\hat{M}_T}(Q_{eq}) - P C_{\hat{M}_T}(Q_{eq}); \quad G_{33} := 2\varepsilon I - D_{\hat{M}_T}(Q_{eq}) D_w - D_{eq}^T D_{\hat{M}_T}(Q_{eq})
\]

Since there is no efficient algorithm for solving general BMIs, instead we use ILMI method by which the relevant LMI problem of each phase can be efficiently solved by the existing softwares, e.g., [14]. Following this thought, how to partition the BMI variables \( X, Y, P_w, P, P_{M_T}, D_w, B_w, Q_{eq} \) into two groups of variables matters much. By inspection, it is easy to find that one of the following two groups...
of variables, \((Y, P_\text{w}, P, P_{M_\text{r}}, D_\text{w})\) and \(Q_{eq}\), should be kept fixed when the other one is to be solved. Furthermore, the rest of the variables, \(X\) and \(B_\text{w}\), can be either set free to solve or kept constant in each phase. With these observations, two fixed-order robust controller synthesis methods are proposed as follows.

**Method 1:**
The method involves iteratively solving the two groups of variables, \((X, Y, P_\text{w}, P, P_{M_\text{r}}, D_\text{w}, B_\text{w})\) and \(Q_{eq}\), i.e., alternatively computing the optimal \(X\) and \(B_\text{w}\), can be either set free to solve or kept constant in each phase. The design procedure is quite flexible mechanical system example presented in [3], the generalized plants have the special property that \(D_{21} = 0\). As can be checked, this additional information reduces several nonlinear coupled terms in (16) to be affinely dependent only upon a single variable. Specifically, with \(D_{11} = 0\) the term \(\tilde{D}_{21}\) is null too (check the formula in Section 2), which in turn implies that the terms \(B_{M_\text{r}}\) and \(D_{M_\text{r}}\) are no longer dependent upon \(Q_{eq}\). In order to solve the BMIs (14) and (16) by iterative LMI method, one has to solve the group of variables \((P_\text{w}, P, P_{M_\text{r}}, B_\text{w}, D_\text{w})\) in different phase. While this is the only constraint for applying ILMI method to solve the BMI problem, each of the rest of the variables \(X, Y, D_\text{w}\), and \(B_\text{w}\) can be either treated as a variable or kept fixed as a constant in each phase. With these observations, several iterative (synthesis) schemes like method 2 are possible for this class of special cases.

**Remark 2.** For some practical problems, e.g., the flexible mechanical system example presented in [3], the generalized plants have the special property that \(D_{21} = 0\). As can be checked, this additional information reduces several nonlinear coupled terms in (16) to be affinely dependent only upon a single variable. Specifically, with \(D_{11} = 0\) the term \(\tilde{D}_{21}\) is null too (check the formula in Section 2), which in turn implies that the terms \(B_{M_\text{r}}\) and \(D_{M_\text{r}}\) are no longer dependent upon \(Q_{eq}\). In order to solve the BMIs (14) and (16) by iterative LMI method, one has to solve the group of variables \((P_\text{w}, P, P_{M_\text{r}})\) and \(Q_{eq}\) in different phase. While this is the only constraint for applying ILMI method to solve the BMI problem, each of the rest of the variables \(X, Y, D_\text{w}\), and \(B_\text{w}\) can be either treated as a variable or kept fixed as a constant in each phase. With these observations, several iterative (synthesis) schemes like method 2 are possible for this class of special cases.

**Remark 3.** While D-K iteration as well as the two methods proposed in [6] and method 1 of this paper involve iteratively solving the multiplier and the
controller in different phases, there is no direct trade off between the two phases. From this viewpoint, it is interesting to note that method 2 provides a new scheme for controller synthesis. Specifically, a phase of method 2 involves solving the variables $Q_{eq}$, $X$, and $B_w$ in which the variable $Q_{eq}$ is directly linked to the controller parameters and the variables $X$, and $B_w$ represent part of the multiplier parameters since the generalized stability multiplier is given by the formula $W(s) = Y(s)P_w(s) - X)^{-1}B_w + D_w$ [9]. This indicates that method 2, in sharp contrast with the conventional schemes, introduces a new mechanism which allows direct trade off between the controller and part of the multiplier parameters in a single phase.

Remark 4. Method 1 of [6] successfully combines the method of [8] searching for suitable multipliers with the method [12] for computing a SPR controller to yield a robustly stabilizing controller. Since the order of the multipliers can be kept fixed, the order of the resulting controllers would keep fixed as well. However, one drawback of the method is that the poles of the multipliers have to be selected a priori which is usually by ad hoc work. Method 2 of [6] alleviates this drawback by employing the method presented in [9] for searching suitable multipliers. But the order of the resulting controllers would dramatically increase as the iteration number goes up. In comparison with method 1 and method 2 of [6], method 1 and method 2 of this paper were proposed to preserve the advantages of the method [9] while keeping the order of the controllers fixed. In addition, spectral factorization is not required in the design procedures of the two methods which in turn reduces the computational load.

Remark 5. Robust stability margin could be enlarged by increasing the order of the controllers. A lot of combinations of the proposed methods aforementioned can be considered. For example, one may take controllers obtained at certain iteration of method 1 and method 2 of [6], then computes the robust stability margin by [9], and then applies method 2 of this paper. Similarly, stability margin could be enlarged by increasing the order of the generalized multipliers.

4 Numerical Example
Consider the example taken from [6]. For ease of exposition, method 1 and method 2 is referred to method 1 and method 2 of [6], and method 3 and method 4 is referred to method 1 and method 2 of this paper. Application of the $H_\infty$ control theory yields a lower bound of the stability margin as $\gamma = 0.6781$ (the inverse of the resulting $H_\infty$ norm). The numerical data obtained by applying the four proposed robust controller designs in this paper is listed in Table 1. The number in the parentheses indicates the number of iterations of each synthesis procedure at which it terminates (i.e., there is no significant increase in $\gamma$).

In method 1 generalized stability multipliers of order four with all of the poles assigned at -1 are used to estimate the robust stability margin. Method 2 outperforms method 1 a little bit. This might be attributed to that method 2 utilizes a more flexible method [9] to estimate the real $\mu$ upper bound (i.e., the inverse of the bound is a lower bound of the stability margin). However, as expected the order of the resulting controller at the 3rd iteration of method 2 is as high as 10. The corresponding generalized stability multiplier also has order as high as 12. In comparison with method 1 and method 2, while the order of the controllers and the generalized stability multipliers (see the third column of Table 1) by applying method 3 and method 4 is much smaller and kept fixed during the design process, the estimates of stability margin are approximately the same. This implies that method 3 and method 4 provide good lower order controller design for this example.

<table>
<thead>
<tr>
<th>Method</th>
<th>Stability margin $\gamma$</th>
<th>Order $(n_K, n_W)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_\infty$ control</td>
<td>0.6781</td>
<td>(2, none)</td>
</tr>
<tr>
<td>Method 1</td>
<td>0.7399 (4$^{th}$)</td>
<td>(6, 4)</td>
</tr>
<tr>
<td>Method 2</td>
<td>0.7424 (3$^{rd}$)</td>
<td>(10, 12)</td>
</tr>
<tr>
<td>Method 3</td>
<td>0.7372 (6$^{th}$)</td>
<td>(2, 4)</td>
</tr>
<tr>
<td>Method 4</td>
<td>0.7465 (6$^{th}$)</td>
<td>(2, 4)</td>
</tr>
</tbody>
</table>

Robust stability margin could be enlarged by increasing the order of the controllers. This can be done by considering a hybrid combination of the proposed methods. For illustration, we take controllers obtained from method 1 and method 2, then compute the stability margin by [9], and then follow the application of method 4. Note that in the first row of Table 2, we take the controller obtained at the 2$^{nd}$ iteration method 1. Then applying method 4 yields a much better value $\gamma = 0.7533$ at the 6$^{th}$ iteration. The order of the resulting controller and the generalized stability multiplier are 6 and 8, respectively. The detailed numerical results are presented in Table 2, which show that the results by
applying the hybrid methods are even better than those of the original ones as listed in Table 1.

<table>
<thead>
<tr>
<th>Method 1(2\textsuperscript{nd}) + Method 4</th>
<th>Stability margin</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method 1(2\textsuperscript{nd}) + Method 4</td>
<td>0.7533 (6\textsuperscript{th})</td>
<td>(6,8)</td>
</tr>
<tr>
<td>Method 1(3\textsuperscript{rd}) + Method 4</td>
<td>0.7665(6\textsuperscript{th})</td>
<td>(6,8)</td>
</tr>
<tr>
<td>Method 2(2\textsuperscript{nd}) + Method 4</td>
<td>0.7606(6\textsuperscript{th})</td>
<td>(6,8)</td>
</tr>
<tr>
<td>Method 2(3\textsuperscript{rd}) + Method 4</td>
<td>0.7635(6\textsuperscript{th})</td>
<td>(10,12)</td>
</tr>
</tbody>
</table>

5 Conclusions

Two robust controller designs are proposed for systems subject to real parametric uncertainties. New BMI formulation was established and different iterative LMI schemes were provided to solve the BMI problem via efficient convex programming. No curve fitting and spectral factorization are required. Furthermore, the difficulty of selecting the poles of the generalized stability multipliers (usually by ad hoc work) is prevented. Comprehensive comparisons concerning the order of the resulting controllers and the multipliers as well as the poles of the multipliers for the proposed methods and those mimic the well known D\textsubscript{K} iteration were made. Hybrid combination of the proposed methods is possible. The presented numerical example demonstrated the effectiveness of the proposed methods.

Acknowledgement

This work was supported in part by the National Science Council, Taiwan, under Grant NSC 91-2213-E-032-009.

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