A BMI Approach to Robust Controller Design for Systems with Real Parametric Uncertainties

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Abstract: - In this work, we propose two robust controller design methods based on new iterative schemes for the systems with real parametric uncertainties. The methods are derived on the basis of a recently developed method for computing optimal stability multipliers. The salient property that there is no need to choose the poles of the multipliers a priori is retained. Beside the benefit of that the controller order would not increase without bounds, a hybrid combination of the proposed methods is possible. Comprehensive comparisons of the proposed methods and those designs which mimic the well known D-K iteration are made. A numerical example is given to illustrate the results.

Key-Words: - robust stability, stability multiplier, linear marix inequality (LMI), real parametric uncertainty

1 Introduction

μ synthesis is [1-6] is a powerful tool for designing robust controllers for systems subject to multiple sources of parametric and/or dynamic uncertainties. In 1985 the well known D-K iteration [1] was proposed to produce robust controllers for the systems with structured dynamic uncertainties, which is essentially based on iterating between the analysis phase of computing the complex μ upper bound (or the optimal multipliers) with the controller fixed, and the synthesis phase of H_∞ optimization with the multiplier fixed.

In early research curve fitting is widely used in the analysis phase, however, error might occur due to approximation. Later, two basis function methods were proposed in Ly et al [7] and [8]. While curve fitting of the scalings is no longer required and good estimates of the μ upper bound could be rendered by both of the methods, both of them rely heavily on a proper choice of the bases, equivalently, the choice of the poles of the multipliers. Later, in [9] a skillful LMI method was proposed for computing the optimal multipliers. Particularly, there is no need to choose the poles of the multipliers a priori, yet a lower bound constraint on the multiplier order (at least as large as the order of closed-loop system) is assumed for the approach. This generally improves the computation of the real μ upper bound. New robust controller design based on the method [9] has been presented in [6], which inherits the advantage of [9] at the expense that the order of the resulting controllers dramatically increases as the iteration number of the synthesis procedure goes up. Therefore, it is our purpose to propose new robust controller design to alleviate the problem. The paper is organized as follows. Section 2 gives the formal problem statement and some preliminaries for future developments. Section 3 presents the two proposed approaches for the robust controller synthesis problem. Comprehensive comparisons of the controller designs are made. In Section 4, a numerical example is provided to demonstrate the results. Section 5 is the conclusions.

2 Problem Formulation

Notation

Most notation used in this paper is fairly standard, see e.g., [6] .

Problem Description

The $\Delta - P - K$ paradigm is considered, where it is assumed throughout that Δ belongs to the parametric structured uncertainty set defined as follows:

$$\Delta_r := \left\{ block-diag(\delta_1 I_{m_1}, \ldots, \delta_L I_{m_L}) : \delta_i \in R, i = 1, \ldots, L \right\}$$

The symbol P denotes the generalized plant, including the nominal plant, weighting functions, etc, described by

$$P \quad \begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z = C_1 x + D_{11} w + D_{12} u \\ y = C_2 x + D_{21} w + D_{22} u \end{cases} \tag{1}$$

where $x \in \mathbb{R}^{n_P}$, $w \in \mathbb{R}^m$, $u \in \mathbb{R}^{n_u}$, $z \in \mathbb{R}^m$, and $y \in \mathbb{R}^{n_y}$ with $m := m_1 + \cdots + m_L$. The symbol K denotes a dynamic controller of the form

$$K \quad \begin{cases} \dot{x}_K = A_K x_K + B_K y \\ u = C_K x_K + D_K y \end{cases} \tag{2}$$

where $x_K \in \mathbb{R}^{n_K}$, to be designed. Our goal is to find a dynamic controller $K(s)$ of form (2) to enlarge the robust stability margin of the perturbed system in Fig. 1. Specifically, for all $\Delta \in \Delta_r$ with $\|\Delta\| \le \gamma$ (i.e., $|\delta_i| \le \gamma$ for all i) we want to design a dynamic controller of form (2) to maximize the value γ such that the closed-loop system (1)-(2) is robustly stable.

To reduce the conservatism, an improved technique which employs passivity theorem with multipliers has been addressed in [2,4,8,10], as shown in Fig. 2, where $\tilde{\Delta} = (I + \gamma^{-1}\Delta)(I - \gamma^{-1}\Delta)^{-1}$, and $\tilde{M}_\gamma := (I - \gamma M)(I + \gamma M)^{-1} = F_l(\tilde{P}_\gamma, K)$ [4].

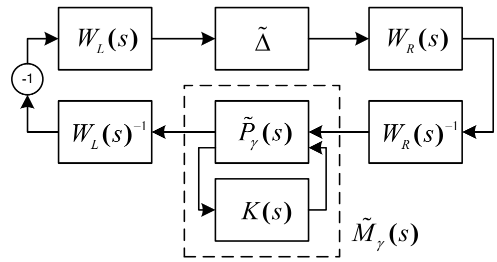

Fig. 2 Passivity framework with multipliers

It is easy to check that for any $\Delta \in \Delta_r$ with $\|\Delta\| \le \gamma$, $\tilde{\Delta}$ is a constant diagonal matrix with entries in $[0, \infty)$. A well known sufficient condition for the robust synthesis problem is thus to maximize the value γ via finding a dynamic controller $K(s)$ of form (2) and suitable stability multipliers $W_L(s)$ and $W_R(s)$ in the set $S_R(RF)$, satisfying (i) W_L, W_R, W_L^{-1} and W_R^{-1} are in RH_∞, (ii) $W_L W_R$ is strictly positive real (SPR), and (iii) $W_L^{-1}\tilde{M}_\gamma W_R^{-1}$ is SPR [4]. By letting $W = W_R^{-1} W_L^{\sim}$ (possibly non-causal), it has been shown in [2,4,8,10] that the conditions described above are equivalent to the following frequency domain conditions with $\tilde{M}_\gamma = F_l(\tilde{P}_\gamma, K)$ which must hold for some $\varepsilon > 0$ and for all $\forall \omega \in \mathbb{R} \cup \{\infty\}$,

$$\tilde{M}_\gamma(j\omega)W(j\omega) + W(j\omega)^* \tilde{M}_\gamma(j\omega)^* > 2\varepsilon I, \tag{3}$$

$$W(j\omega) + W(j\omega)^* > 0. \tag{4}$$

In summary, the robust synthesis problem we consider is to maximize the value γ via finding a dynamic controller $K(s)$ of form (2) and a generalized stability multiplier $W \in S_R(RF)$ such that conditions (3) and (4) hold. For the latter development, it is required to know the state-space realization of the sector transformed plant $\tilde{P}_\gamma = S \star \Gamma P$, where

$$S = \begin{pmatrix} I_m & -\sqrt{2} I_m \\ \sqrt{2} I_m & -I_m \end{pmatrix}, \quad \Gamma = \begin{pmatrix} \gamma I_m & 0 \\ 0 & I_{n_y} \end{pmatrix}$$

and the symbol $\star$ means star product [11]. A tedious algebraic manipulation shows that

$$\tilde{P}_\gamma(s) \underset{=}{ss} \left[\begin{array}{c|cc} \tilde{A} & \tilde{B}_1 & \tilde{B}_2 \\ \hline \tilde{C}_1 & \tilde{D}_{11} & \tilde{D}_{12} \\ \tilde{C}_2 & \tilde{D}_{21} & \tilde{D}_{22} \end{array}\right]$$

where

$\tilde{A} = A - \gamma B_1 (I + \gamma D_{11})^{-1} C_1$;

$\tilde{B}_1 = \sqrt{2} B_1 (I + \gamma D_{11})^{-1}$; $\tilde{B}_2 = B_2 - \gamma B_1 (I + \gamma D_{11})^{-1} D_{12}$;

$\tilde{C}_1 = -\sqrt{2}\gamma (I + \gamma D_{11})^{-1} C_1$; $\tilde{C}_2 = C_2 - \gamma D_{21} (I + \gamma D_{11})^{-1} C_1$;

$\tilde{D}_{11} = (I + \gamma D_{11})^{-1}(I - \gamma D_{11})$; $\tilde{D}_{12} = -\sqrt{2}\gamma (I + \gamma D_{11})^{-1} D_{12}$;

$\tilde{D}_{21} = \sqrt{2} D_{21} (I + \gamma D_{11})^{-1}$; $\tilde{D}_{22} = D_{22} - \gamma D_{21} (I + \gamma D_{11})^{-1} D_{12}$

.

Method 1:

The design procedure of method 1 involves iterations between computing the real μ upper bound (analysis phase) and solving for a SPR controller (synthesis phase) [12,13]. For details, see [6].

Method 2:

The design is similar to method 1. The only difference is the replacement of the analysis phase with the method proposed in [9]. A brief review of the approach [9] is introduced as follows: Consider the stable system $\tilde{M}_\gamma := F_l(\tilde{P}_\gamma, K)$. As before, let $n_{\tilde{M}_\gamma}$ denote the order of the stable system $\tilde{M}_\gamma$ and n_W denote the order of generalized stability multiplier W to be computed. Under the constraint $n_W \ge n_{\tilde{M}_\gamma} = n_P + n_K$ where n_p and n_K denote the order of the generalized plant P and the controller K respectively, suppose that there exist a positive number ε, matrices $P = P^T \in \mathbb{R}^{(n_P + n_K)\times(n_P + n_K)}$, and block-diagonal matrices $X \in \mathbb{R}^{n_W \times n_W}$, $Y \in \mathbb{R}^{m \times n_W}$, $P_W = P_W^T \in \mathbb{R}^{n_W \times n_W}$, $B_W \in \mathbb{R}^{n_W \times m}$, $D_W \in \mathbb{R}^{m \times m}$, with P_W non-singular, such that the following LMIs hold

$$\begin{bmatrix} X^T + X & B_W - Y^T \\ B_W^T - Y & 2\varepsilon I - (D_W + D_W^T) \end{bmatrix} < 0 \tag{5}$$

$$\begin{bmatrix} L_{11} & L_{12} \\ L_{12}^T & L_{22} \end{bmatrix} < 0 \tag{6}$$

where

$$L_{11} = \begin{bmatrix} X^T + X & Y^T B_{\tilde{M}_r}^T + P_W \Gamma_0 A_{\tilde{M}_r}^T + X\Gamma_0 \\ * & PA_{\tilde{M}_r}^T + A_{\tilde{M}_r} P + \Gamma_0^T Y^T B_{\tilde{M}_r}^T + B_{\tilde{M}_r} Y \Gamma_0 \end{bmatrix};$$

$$L_{12} = \begin{bmatrix} B_W - Y^T D_{\tilde{M}_r}^T - P_W \Gamma_0 C_{\tilde{M}_r}^T \\ B_{\tilde{M}_r} D_W - \Gamma_0^T Y^T D_{\tilde{M}_r}^T - PC_{\tilde{M}_r}^T \end{bmatrix};$$

$$L_{22} = 2\varepsilon I - D_{\tilde{M}_r} D_W - D_W^T D_{\tilde{M}_r}^T;$$

$$\Gamma_0 := \begin{bmatrix} I_{n_P+n_K} \\ 0 \end{bmatrix} \in R^{n_W \times (n_P+n_K)}.$$

Then $W(s) := Y(sP_W - X)^{-1} B_W + D_W$ is a generalized stability multiplier satisfying conditions (3) and (4). In comparison with [8] in which the poles are chosen by ad hoc work, the method in [9] has the advantage of not necessary to choose the multiplier poles a priori, which in turn could reduce the conservatism of computing optimal multipliers and thus gets a tighter estimate of the robust stability margin. However, the price is that the order of the resulting controllers by method 2 dramatically increases as the iteration number of the synthesis procedure goes up. To alleviate this problem a new procedure employing a fixed-order controller synthesis scheme is proposed on the basis of the analysis conditions (5)-(6).

3 Main Results

In this section we present two design methods originating from the analysis condition presented in [9] which leads to a nonlinear optimization problem involving bilinear matrix inequalities (BMIs) when controller synthesis is considered. On the basis of the new BMI formulation, different ways of partitioning the BMI variables into two groups are discussed and iterative LMI (ILMI) method is introduced to solve the problem. Algorithms based on state-space manipulation will be given and comprehensive comparison between the proposed methods and the two D-K iteration like synthesis methods presented in [6] will be provided.

To proceed, let Q_{eq} be the system matrix of the transformed controller K_{eq} where $K_{eq} = K\left(I - \tilde{D}_{22}K\right)^{-1}$; it follows that the closed-loop system $\tilde{M}_\gamma = F_l(\breve{P}_\gamma, K_{eq})$ has the following system matrix depending affinely on Q_{eq}.

$$\left[\begin{array}{c|c} A_{\tilde{M}_\gamma}(Q_{eq}) & B_{\tilde{M}_\gamma}(Q_{eq}) \\ \hline C_{\tilde{M}_\gamma}(Q_{eq}) & D_{\tilde{M}_\gamma}(Q_{eq}) \end{array}\right]$$

with

$$A_{\tilde{M}_\gamma}(Q_{eq}) = \begin{bmatrix} \tilde{A} & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & \tilde{B}_2 \\ I & 0 \end{bmatrix} Q_{eq} \begin{bmatrix} 0 & I \\ \tilde{C}_2 & 0 \end{bmatrix}$$

$$B_{\tilde{M}_\gamma}(Q_{eq}) = \begin{bmatrix} \tilde{B}_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & \tilde{B}_2 \\ I & 0 \end{bmatrix} Q_{eq} \begin{bmatrix} 0 \\ \tilde{D}_{21} \end{bmatrix}$$

$$C_{\tilde{M}_\gamma}(Q_{eq}) = \begin{bmatrix} \tilde{C}_1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & \tilde{D}_{12} \end{bmatrix} Q_{eq} \begin{bmatrix} 0 & I \\ \tilde{C}_2 & 0 \end{bmatrix}$$

$$D_{\tilde{M}_\gamma}(Q_{eq}) = \begin{bmatrix} \tilde{D}_{11} \end{bmatrix} + \begin{bmatrix} 0 & \tilde{D}_{12} \end{bmatrix} Q_{eq} \begin{bmatrix} 0 \\ \tilde{D}_{21} \end{bmatrix}$$

Substituting the above information into (5) and (6), the robust stability margin optimization problem becomes a nonlinear optimization problem subject to BMI constraints as described as follows:

Maximize γ in the variables ε (a positive number), the block-diagonal matrix variables $X \in \mathbb{R}^{n_W \times n_W}$, $Y \in \mathbb{R}^{n_W \times n_W}$, $P_W = P_W^T \in \mathbb{R}^{n_W \times n_W}$, $P_{\tilde{M}_\gamma} = P_{\tilde{M}_\gamma}^T \in \mathbb{R}^{(n_P+n_K)\times(n_P+n_K)}$, $B_W \in \mathbb{R}^{n_W \times m_W}$, $D_W \in \mathbb{R}^{p_W \times n_W}$, matrix variables Q_{eq} and $P = P^T \in \mathbb{R}^{(n_P+n_K)\times(n_P+n_K)}$ subject to the following BMI constraints (7)-(10).

$$P_{\tilde{M}_\gamma} > 0 \qquad (7)$$

$$P_{\tilde{M}_\gamma} A_{\tilde{M}_\gamma}(Q_{eq}) + \left(P_{\tilde{M}_\gamma} A_{\tilde{M}_\gamma}(Q_{eq})\right)^T < 0 \qquad (8)$$

$$\begin{bmatrix} X^T + X & B_W - Y^T \\ B_W^T - Y & 2\varepsilon I - (D_W + D_W^T) \end{bmatrix} < 0 \qquad (9)$$

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{12}^T & G_{22} & G_{23} \\ G_{13}^T & G_{23}^T & G_{33} \end{bmatrix} < 0 \qquad (10)$$

where

$G_{11} := X^T + X$;

$G_{12} := Y^T B_{\tilde{M}_r}^T(Q_{eq}) + P_W \Gamma_0 A_{\tilde{M}_r}^T(Q_{eq}) + X\Gamma_0 a$;

$G_{13} := B_W - Y^T D_{\tilde{M}_r}^T(Q_{eq}) - P_W \Gamma_0 C_{\tilde{M}_r}^T(Q_{eq})$;

$$G_{22} := PA_{\tilde{M}_r}^T(Q_{eq}) + A_{\tilde{M}_r}(Q_{eq})P + \Gamma_0^T Y^T B_{\tilde{M}_r}^T(Q_{eq}) + B_{\tilde{M}_r}(Q_{eq}) Y\Gamma_0;$$

$G_{23} := B_{\tilde{M}_r}(Q_{eq}) D_W - \Gamma_0^T Y^T D_{\tilde{M}_r}^T(Q_{eq}) - PC_{\tilde{M}_r}^T(Q_{eq})$;

$G_{33} := 2\varepsilon I - D_{\tilde{M}_r}(Q_{eq}) D_W - D_W^T D_{\tilde{M}_r}^T(Q_{eq})$.

Since there is no efficient algorithm for solving general BMIs, instead we use ILMI method by which the relevant LMI problem of each phase can be efficiently solved by the existing softwares, e.g., [14]. Following this thought, how to partition the BMI variables X, Y, P_W, P, $P_{\tilde{M}_\gamma}$, D_W, B_W, Q_{eq} into two groups of variables matters much. By inspection, it is easy to find that one of the following two groups of variables, (Y, P_W, P, $P_{\tilde{M}_\gamma}$, D_W) and Q_{eq}, should be kept fixed when the other one is to be solved. Furthermore, the rest of the variables, X and B_W, can be either set free to solve or kept constant in each phase. With these observations, two fixed-order robust controller synthesis methods are proposed as follows.

Method 1:
The method involves iteratively solving the two groups of variables, (X, Y, P_W, P, $P_{\tilde{M}_\gamma}$, D_W, B_W) and Q_{eq}, i.e., alternatively computing the optimal multipliers in one phase and the controller in another phase. A detailed synthesis procedure is outlined in the following algorithm.

Algorithm 1:

1. Compute a suboptimal H_∞ controller K such that the value γ which satisfies $\|\gamma F(P,K)\|_\infty < 1$ is maximized. Then compute $K_{eq} = K\left(I - \tilde{D}_{22}K\right)^{-1}$.
2. Maximize γ by solving (7)-(10) where the transformed controller K_{eq} (i.e., Q_{eq}) is kept fixed. The details are as follows:
   2.1 Increase γ. Compute $A_{\tilde{M}_\gamma}(Q_{eq})$, $B_{\tilde{M}_\gamma}(Q_{eq})$, $C_{\tilde{M}_\gamma}(Q_{eq})$, $D_{\tilde{M}_\gamma}(Q_{eq})$. Set $\Gamma_0 := \begin{bmatrix} I_{n_p+n_K} \\ 0 \end{bmatrix} \in R^{n_W \times (n_P+n_K)}$.
   2.2 Solve (7)-(10) for the variables X, Y, P_W, P, $P_{\tilde{M}_\gamma}$, B_W, D_W and ε. Back to step 2.1 till there is no significant increase in γ.
3. Maximize γ by solving (8)-(10) where the variables X, Y, P_W, P, $P_{\tilde{M}_\gamma}$, B_W, D_W are kept fixed. The details are as follows:
   3.1 Increase γ.
   3.2 Solve (8)-(10) for the variables Q_{eq} and ε. Back to step 2.1 till there is no significant increase in γ.
4. Repeat step 2 and step 3, till there is no significant increase in γ.
5. The resulting controller is given by
$$K = K_{eq}\left(I + \tilde{D}_{22}K_{eq}\right)^{-1}.$$

Method 2:
The method involves iteratively solving the two groups of variables, (X, Y, P_W, P, $P_{\tilde{M}_\gamma}$, B_W, D_W) and (Q_{eq}, X, B_W). The design procedure is quite similar to Method 1 except that in step 3.2 of Algorithm 1 two more variables, X, B_W, are considered and computed.

Remark 1. Method 2 is quite similar to method 1 except that there are two more variables X, B_W for consideration in the phase of computing a new controller. Theoretically, this implies that method 2 would get better γ than method 1 when the multipliers obtained in the previous step were the same. On the other hand, while both of the methods were derived from the same thought with different groups of variables to be computed in the controller searching phase, it is noted that there exist other iterative schemes as long as each of the following two groups of variables, (Y, P_W, P, $P_{\tilde{M}_\gamma}$, D_W) and Q_{eq}, was solved in different phase; for example, iteratively computing the two groups of variables, (X, Y, P_W, P, $P_{\tilde{M}_\gamma}$, D_W) and (Q_{eq}, B_W). Note that the variations of the proposed methods stem from that the variables, X and B_W, can be either set free to solve or kept fixed in each individual phase.

Remark 2. For some practical problems, e.g., the flexible mechanical system example presented in [3], the generalized plants have the special property that $D_{21} = 0$. As can be checked, this additional information reduces several nonlinear coupled terms in (16) to be affinely dependent only upon a single variable. Specifically, with $D_{21} = 0$ the term $\tilde{D}_{21}$ is null too (check the formula in Section 2), which in turn implies that the terms $B_{\tilde{M}_\gamma}$ and $D_{\tilde{M}_\gamma}$ are no longer dependent upon Q_{eq}. In order to solve the BMIs (14) and (16) by iterative LMI method, one has to solve the group of variables (P_W, P, $P_{\tilde{M}_\gamma}$) and Q_{eq} in different phase. While this is the only constraint for applying ILMI method to solve the BMI problem, each of the rest of the variables X, Y, D_W, and B_W can be either treated as a variable or kept fixed as a constant in each phase. With these observations, several iterative (synthesis) schemes like method 2 are possible for this class of special cases.

A serial of remarks comparing the proposed methods in this paper and those in [6] are as follows.

Remark 3. While D-K iteration as well as the two methods proposed in [6] and method 1 of this paper involve iteratively solving the multiplier and the controller in different phases, there is no direct trade off between the two phases. From this viewpoint, it is interesting to note that method 2 provides a new scheme for controller synthesis. Specifically, a phase of method 2 involves solving the variables Q_{eq}, X, and B_W in which the variable Q_{eq} is directly linked to the controller parameters and the variables X, and B_W represent part of the multiplier parameters since the generalized stability multiplier is given by the formula $W(s) := Y(sP_W - X)^{-1} B_W + D_W$ [9]. This indicates that method 2, in sharp contrast with the conventional schemes, introduces a new mechanism which allows direct trade off between the controller and part of the multiplier parameters in a single phase.

Remark 4. Method 1 of [6] successfully combines the method of [8] searching for suitable multipliers with the method [12] for computing a SPR controller to yield a robustly stabilizing controller. Since the order of the multipliers can be kept fixed, the order of the resulting controllers would keep fixed as well. However, one drawback of the method is that the poles of the multipliers have to be selected a priori which is usually by ad hoc work. Method 2 of [6] alleviates this drawback by employing the method presented in [9] for searching suitable multipliers. But the order of the resulting controllers would dramatically increase as the iteration number goes up. In comparison with method 1 and method 2 of [6], method 1 and method 2 of this paper were proposed to preserve the advantages of the method [9] while keeping the order of the controllers fixed. In addition, spectral factorization is not required in the design procedures of the two methods which in turn reduces the computational load.

Remark 5. Robust stability margin could be enlarged by increasing the order of the controllers. A lot of combinations of the proposed methods aforementioned can be considered. For example, one may take controllers obtained at certain iteration of method 1 and method 2 of [6], then computes the robust stability margin by [9], and then applies method 2 of this paper. Similarly, stability margin could be enlarged by increasing the order of the generalized multipliers.

4 Numerical Example

Consider the example taken from [6]. For ease of exposition, method 1 and method 2 is referred to method 1 and method 2 of [6], and method 3 and method 4 is referred to method 1 and method 2 of this paper. Application of the H_∞ control theory yields a lower bound of the stability margin as $\gamma = 0.6781$ (the inverse of the resulting H_∞ norm). The numerical data obtained by applying the four proposed robust controller designs in this paper is listed in Table 1. The number in the parentheses indicates the number of iterations of each synthesis procedure at which it terminates (i.e., there is no significant increase in γ). In method 1 generalized stability multipliers of order four with all of the poles assigned at -1 are used to estimate the robust stability margin. Method 2 outperforms method 1 a little bit. This might be attributed to that method 2 utilizes a more flexible method [9] to estimate the real μ upper bound (i.e., the inverse of the bound is a lower bound of the stability margin). However, as expected the order of the resulting controller at the 3rd iteration of method 2 is as high as 10. The corresponding generalized stability multiplier also has order as high as 12. In comparison with method 1 and method 2, while the order of the controllers and the generalized stability multipliers (see the third column of Table 1) by applying method 3 and method 4 is much smaller and kept fixed during the design process, the estimates of stability margin are approximately the same. This implies that method 3 and method 4 provide good lower order controller design for this example.

Table1.

	Stability margin	Order
	γ	(n_K, n_W)
H_∞ control	0.6781	(2,none)
Method 1	0.7399(4th)	(6,4)
Method 2	0.7424 (3rd)	(10,12)
Method 3	0.7372(6th)	(2,4)
Method 4	0.7465(6th)	(2,4)

Robust stability margin could be enlarged by increasing the order of the controllers. This can be done by considering a hybrid combination of the proposed methods. For illustration, we take controllers obtained from method 1 and method 2, then compute the stability margin by [9], and then follow the application of method 4. Note that in the first row of Table 2, we take the controller obtained at the 2nd iteration method 1. Then applying method 4 yields a much better value $\gamma = 0.7533$ at the 6th iteration. The order of the resulting controller and the generalized stability multiplier are 6 and 8, respectively. The detailed numerical results are presented in Table 2, which show that the results by applying the hybrid methods are even better than those of the original ones as listed in Table 1.

Table 2.

	Stability margin γ	Order (n_K, n_W)
Method 1(2nd) + Method 4	0.7533 (6th)	(6,8)
Method 1(3rd) + Method 4	0.7665(6th)	(6,8)
Method 2(2nd) + Method 4	0.7606 (6th)	(6,8)
Method 2(3rd) + Method 4	0.7635(6th)	(10,12)

5 Conclusions

Two robust controller designs are proposed for systems subject to real parametric uncertainties. New BMI formulation was established and different iterative LMI schemes were provided to solve the BMI problem via efficient convex programming. No curve fitting and spectral factorization are required. Furthermore, the difficulty of selecting the poles of the generalized stability multipliers (usually by ad hoc work) is prevented. Comprehensive comparisons concerning the order of the resulting controllers and the multipliers as well as the poles of the multipliers for the proposed methods and those mimic the well known D_K iteration were made. Hybrid combination of the proposed methods is possible. The presented numerical example demonstrated the effectiveness of the proposed methods.

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References:

[1] J.C. Doyle, Structured uncertainty in control system design, In *Proceedings of the 24th IEEE conference on decision and control*, Ft. Lauderdale, Florida ,1985, pp. 260-265.

[2] M.G. Safonov and R.Y. Chiang, Real/complex Km-synthesis without curve fitting, *Control and Dynamic Systems*, New York, Academic Press., Vol.56, 1993, pp. 303-324.

[3] P.M. Young, Controller design with real parametric uncertainty, In *Proceeding American Control Conference*, Baltimore, Maryland, 1994, pp. 2333-2337. also see, *International Journal of Control*, Vol.65, No.3, 1996, pp. 469-509 .

[4] K.C. Goh, M.G. Safonov, and J.H. Ly, Robust synthesis via bilinear matrix inequalities, *International Journal of Robust and Nonlinear Control*, Vol.6, No.9/10, 1996, pp. 1079-1095.

[5] A.L. Tits and Y.S. Chou, On mixed μ synthesis, *Automatica*, Vol.36, No.7, 2000, pp. 1077-1079.

[6] Y.S. Chou, C.C. Lin, Y.H. Chen, A BMI approach to robust controller design for systems with real parametric uncertainties, submitted.

[7] J.H. Ly, M.G. Safonov, and R.Y. Chiang, Real/complex multivariable stability margin computation via generalized Popov multiplier-LMI approach, In *Proceedings of American control Conference*, Baltimore, Maryland, 1994, pp. 425-429.

[8] V. Balakrishnan, Linear matrix inequalities in robustness analysis with multipliers, *System and Control Letters*, Vol.25, No.4, 1995, pp. 265-272.

[9] P.B. Gapski and J.C. Geromel, LMI approach to the real parametric robust stability problem with multipliers, *International Journal of Robust and Nonlinear Control,* Vol.8, No.7, 1998, pp. 599-610.

[10] Y.S. Chou, A.L. Tits, and V. Balakrishnan, Stability multipliers and μ upper bounds: connections and implications for numerical verification of frequency-domain conditions, *IEEE Transactions on Automatic Control*, Vol.44, No.5, 1999, pp. 906-913.

[11] K. Zhou and J.C. Doyle, *Essentials of Robust Control*, Prentice Hall, 1998.

[12] C. Scherer, P. Gahinet, and M. Chilali, Multiobjective Output-feedback Control via LMI Optimization, *IEEE Transactions on Automatic Control*, Vol.42, 1997, pp. 896-911.

[13] L. Turan, M.G. Safonov, and C.H. Huang, Synthesis of positive real feedback system: A Simple Derivation via Parrott's Theorem, *IEEE Transactions on Automatic Control*, 1997, Vol.42 , pp. 1154-1157.

[14] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, Manual of LMI Control Toolbox, Math Works, Inc, 1995.