The optimization of kinematical response of gear transmission

VINCENZO NIOLA
Department of Mechanical Engineering for Energetics
University of Naples "Federico II"
Via Claudio 21, 80125, Napoli
ITALY
http://niola.dime.unina.it

GIUSEPPE QUAREMBA
Department of Mechanical Engineering for Energetics
University of Naples "Federico II"
Via Claudio 21, 80125, Napoli
ITALY

Abstract: - A brief theory of wavelet transforms and their effective computation method with an emphasis on the considerations of the choice of pressure angle are presented in this paper. This is followed by the numerical results with related graphs. The procedure used in this paper include the numerical simulation of the dynamics of a gear transmission system. Important advancements in preventive maintenance of gear transmission systems are currently being sought for the development of an accurate machine health diagnostic system. Such a diagnostic system should use vibration or acoustic signals derived directly from the gear transmission. Finally, the improvement of the kinematical quality of gear transmission was tested by means of the Wavelet Transform.

Key-Words: - Spur gear, Discrete Wavelet Transform, numerical simulation, gear transmission, kinematical quality, mechanical diagnostics.

1 Introduction

Gears are one of the most common and important machine components in many advanced machines. The aim of the work is to show the importance of determining the shape of the line of action in order to obtain the best kinematical characteristics in terms of quality performance. Starting from a procedure for designing spur gear sets using the quadratic parametric tooth profiles a numerical simulation was developed [1]. Finally, the improvement of the kinematical quality of gear transmission was tested by means of the Wavelet Transform (WT). In this case the application of Wavelet Transform is also able for the identification and quantification of damaged tooth based on the numerically generated vibration signal. In fact the objective of this work represents only the first step in order to perform the vibration signature analysis procedures for health monitoring and diagnostics of a gear transmission system. The procedure used in this paper include the numerical simulation of the dynamics of a gear transmission system. The advantage of using both a mathematical model of gear profile derived from the line of action and the Wavelet Transform for processing the regularity of such a line is that the mathematical model comprises only one single variable of the line of action and the multiresolution analysis, performed by means of wavelet transform, provides more information regarding the kinematical quality of the designed gear pair.

2 Gear Transmission

Gears are machine elements used to transmit rotary motion between two shafts, normally with a constant ratio. The pinion is the smallest gear and the larger gear is called the gear wheel. A rack is a rectangular prism with gear teeth machined along one side; it is in effect a gear wheel with an infinite pitch circle diameter. In practice the action of gears in transmitting motion is a cam action each pair of mating teeth acting as cams. Gear design has evolved to such a level that throughout the motion of each contacting pair of teeth the velocity ratio of the gears is maintained fixed and the velocity ratio is still fixed as each subsequent pair of teeth come into contact. When the teeth action is such that the driving tooth, moving at constant angular velocity, produces a proportional constant velocity of the driven tooth the action is said to constitute a *conjugate action*. The teeth shape universally selected for the gear teeth is the involute profile.

The vast majority of gear applications use the standard 20° involute system because of its good combination of bending and surface pressure strength, involute insensitivity to errors in center distance and relative ease of manufacturing. Most gears are produced by hobbing or other generation-type processes, where a straight-tooth rack or equivalent tool produces the involute working gear tooth surface as well as a trochoidal root fillet.

Despite the benefits of this system, it is generally felt that a higher bending strength and hence load carrying capacity should be obtained. This is especially true with small numbers of teeth (less than 14 or 17 depending on the tip radius of the hob), where the standard involute teeth are susceptible to undercutting. This is a situation where the tip of the cutter removes material from the involute profile in a secondary cutting action. The resulting teeth have smaller thicknesses near their roots, where the critical section is usually located, and this severely hampers load carrying capacity.

From an other point of view, the characteristics of motion are studied by the techniques of tooth contact analysis in the fixed coordinate system. Many authors proposed model and solution in order to approach the problem of designing an optimum tooth profile. For example [2,3] proposed a mathematical model of parametric tooth profiles for spur gears using pressure angle as a parametric variable. In [4] was proposed a method of designing high-contact-ratio spur gears using quadratic parametric tooth profiles for the shorter addendum without undercut and [5-7] studied the effects of the linear profile modification on the dynamic tooth load and stress for high-contact ratio gearing. In [8] was proposed an optimum tooth profile of spur gear rotary pumps method to reduce the delivery fluctuation.

The advantage of using both a mathematical model of gear profile derived from the line of action and the Wavelet Transform for processing the regularity of such a line is that the mathematical model comprises only one single variable of the line of action and the multiresolution analysis, performed by means of wavelet transform, provides more information regarding the kinematical quality of the designed gear pair [9].

3 Mathematical background

The word wavelet is used in mathematics to denote a kind of orthonormal bases in L^2 with remarkable approximation properties. Wavelets allow to simplify the description of a complicated function in terms of small number of coefficients. Often there are less coefficients necessary than in the classical Fourier analysis. Wavelets are adapted to local properties of functions to a larger extent than the Fourier basis. The adaptation is done automatically in view of the existence of a second degree of freedom: the localization in time (or space, if multivariate functions are considered). The vertical axis in the next graphs denotes always the level, *i.e.*, the partition of the time axis into finer and finer resolutions. The advantage of this "multiresolution analysis" is that we can see immediately local properties of data and thereby influence our further analysis. There were attempts in the past to modify the Fourier analysis by partitioning the time domain into pieces and applying different Fourier expansions on different pieces (*e.g.*, Fourier Fast Transform). But the partitioning is always subjective. Wavelets provide an elegant and mathematically consistent realization of this intuitive idea [10].

In summary, wavelets offer a frequency/time representation of data that allows us time (respectively, space) adaptive filtering, reconstruction and smoothing.

Recall that a mother wavelet ψ is a function of zero h-th moment (*e.g.*, see [10], [11], [12])

$$\int_{-\infty}^{+\infty} x^h \psi(x)dx = 0\,, \quad h \in \mathbf{N}. \quad (1)$$

From this definition, it follows that, if ψ is a wavelet whose all moments are zero, also the function $\psi_{ik}(x)$: $= 2^{-j/2}\, \psi(2^{-j}x - k)$ is a wavelet.

Now consider a wavelet ψ and a function φ such that $\{\{\varphi_{j_0k}\}, \{\psi_{jk}\}, k \in \mathbf{Z}, j = 0, 1, 2, \ldots\}$ is a complete orthonormal system. In this case, a given signal $s(t)$, decomposed by wavelet (*i.e.*, Continuous Wavelet Transform) is represented in the following detail function coefficients

$$d_{jk} = \int_{-\infty}^{+\infty} s(\tau) \cdot \frac{1}{\sqrt{2^j}} \psi\left(\frac{\tau - k}{2^j}\right) d\tau \quad (2)$$

and in the approximating scaling coefficients as follows

$$a_{j_0 k} = \int_{-\infty}^{+\infty} s(\tau) \cdot \varphi(\tau - k) d\tau \,. \qquad (3)$$

Note that, for any j, d_{jk} can be regarded, as a function of k. Consequently, if the signal $s(t)$ is a smooth function, then the relative details are zero, since, as said before, a wavelet has zero moments (for a detailed argumentation see [10]).

The sequence of spaces $\{V_j, j \in Z\}$, generated by φ is called a multiresolution analysis (*MRA*) of $L^2(R)$ if it satisfies the following properties

$$V_j \subset V_{j+1}, j \in Z \text{ and}$$

$$\bigcup_{j \geq 0} V_j \text{ is dense in } L^2(R).$$

It follows that if $\{V_j, j \in Z\}$, is a *MRA* of $L^2(R)$, we say that the function φ generates a *MRA* of $L^2(R)$, and we call φ the father wavelet.

Besides, based on Parseval theorem, for any $s \in L^2(R)$, it follows that

$$s(t) = \sum_k a_{j_0 k} \varphi_{j_0 k}(t) + \sum_{j=j_0}^{j_1} \sum_k d_{jk} \psi_{jk}(t) \,. \quad (4)$$

The relation (4) is called a multiresolution expansion of s. This means that any $s \in L^2(R)$ can be represented as a series (convergent in $L^2(R)$), where $a_{j_0 k}$ and d_{jk} are some coefficients, and $\{\psi_{jk}\}$, $k \in Z$, is a basis for W_j, where we define

$$W_j = V_{j+1} - V_j, j \in Z \,.$$

In (1) $\{\psi_{jk}(t)\}$ is a general basis for W_j. The space W_j is called resolution level of multiresolution analysis. In the following, by abuse of notation, we frequently write "resolution level *j*" or simply "level *j*". We employ these words mostly to designate not the space W_j itself, but rather the coefficients d_{jk} and the function ψ_{jk} "on the level *j*". As the Fourier Fast Transform (FFT), the Discrete Wavelet Transform (DWT) is a fast and linear operation operating on a data array of length equal to a power of 2 and that transforms it in an array of equal length but numerically different. Both FFT and DWT could be considered as a transformation from the original dominion (*i.e.*, time) to a different dominion. In both the cases the functions used to operate the transformation form a Complete Orthonormal System (CONS). Unlike trigonometrical basis, which defines one only Fourier transform, infinite wavelet bases exist that differ for their localization in the dominion of the time and for their regularity.

A particular wavelet basis is characterized by numerical filters. In the present work it has been applied the filter proposed by Daubechies, which includes both wavelets strongly localized and wavelets strongly regular. A filter is characterized by L coefficients denoted as: $h_0, \ldots, h_{L-1}$.

We considered the Daubechies family of length $L = 4, h_0, \ldots, h_3$. The first step of wavelet transform was represented by the calculation of the following product $\boldsymbol{w}^{J-1} = W^J \boldsymbol{x}$ where $\boldsymbol{x} \equiv \{x_0, x_1, \ldots, x_{N-1}\}$ is the vector of $N = 2^J$ data of which the wavelet transform have to be calculated. While $\boldsymbol{w}^{J-1}$ is the wavelet vector transform (of length L) after the first step of calculation; W^J is the N order wavelet transformation matrix

$$W^J = \begin{pmatrix} h_0 & h_1 & h_2 & h_3 & & & & & & \\ g_0 & g_1 & g_2 & g_3 & & & & & & \\ & & h_0 & h_1 & h_2 & h_3 & & & & \\ & & g_0 & g_1 & g_2 & g_3 & & & & \\ & & & & \ddots & & & & & \\ & & & & & & h_0 & h_1 & h_2 & h_3 \\ & & & & & & g_0 & g_1 & g_2 & g_3 \\ h_2 & h_3 & & & & & & & h_0 & h_1 \\ g_2 & g_3 & & & & & & & g_0 & g_1 \end{pmatrix} \quad (5)$$

where the white elements are zero. It is important to observe the matrix structure. The first raw generates the first element of convolution between $\boldsymbol{x}$ and the $\boldsymbol{h}$ filter.

Likewise the third, fifth..., and generally the odd raws of matrix generate the third, fifth..., element of

convolution respectively.
The even raws generate the same type of convolution but with the filter $\boldsymbol{g}$ rather than $\boldsymbol{h}$. The filter $\boldsymbol{g}$ is also called the conjugated one of $\boldsymbol{h}$ and it represents a pass-high filter.
It is uniquely determined by means of $\boldsymbol{h}$ as the following relation

$$g_k = (-1)^k h_{L-k-1},\ k = 0,\ldots,L-1. \quad (6)$$

The **h** and **g** filters are also named as *quadrature mirror filters* (QMF). Note that $\boldsymbol{g}$ is such to return null values if the vector of which we want to calculate the transform is sufficiently regular: in practical the coefficients g_k have $p = L/2$ null moments (in the following it will be esplicitate such a condition named as "p-order approximation").
Therefore the output of the filter $\boldsymbol{h}$ is the vector $\boldsymbol{x}$ represented in a *coarse* shape, while the output of the filter $\boldsymbol{g}$ represents the *detail* that added to the *coarse* information allows to reconstruct the original vector.
We still notice that in the last two raws the coefficient h_2 and the correspondent high-pass filter g_2 are present due to the regularity conditions stated for the vector $\boldsymbol{x}$.
By means of the inverse transform it is possible to reconstruct the original vector $\boldsymbol{x}$ of N length by means of vectors of $N/2$ length composed of output of the convolution with the low-pass filter $\boldsymbol{h}$ and the high-pass filter $\boldsymbol{g}$.
The value of the elements of the vector filter $\boldsymbol{h}$ can be obtained by imposing the orthonormality condition for the matrix W^J as follows

$$\begin{aligned} & h_0^2 + h_1^2 + h_2^2 + h_3^2 = 1 \\ & h_0 h_2 + h_1 h_3 = 0 \end{aligned} \quad (7)$$

and the "approximation condition of $p = L/2 = 2$ order"

$$\begin{aligned} & g_0 + g_1 + g_2 + g_3 = 0 \\ & 0g_0 + 1g_1 + 2g_2 + 3g_3 = 0. \end{aligned} \quad (8)$$

In the present work (*i.e.*, $L = 4$) the solution of condition is [10]

$$\begin{aligned} h_0 &= \frac{1+\sqrt{3}}{4\sqrt{2}} \\ h_1 &= \frac{3+\sqrt{3}}{4\sqrt{2}} \\ h_2 &= \frac{3-\sqrt{3}}{4\sqrt{2}} \\ h_3 &= \frac{1-\sqrt{3}}{4\sqrt{2}}. \end{aligned} \quad (9)$$

The DWT consists in applying the W^j matrix in a hierarchical way to the vector $\boldsymbol{x}\left(W^J\right)$ of length $N = 2^J$, then to the *coarse* vector obtained by the convolution of $\boldsymbol{x}$ with the low-pass filter $\boldsymbol{h}$ (of $N/2=2^{J-1}$ length, with the W^{J-1} matrix), therefore still to the vector of $N/4$ length obtained from the next convolution with the filter $\boldsymbol{h}$, and so on until to a prefixed level J_0 or when the convolution with the low-pass filter supplies a single element.
The last procedure takes the name of pyramidal algorithm. In order to explain the procedure let us consider the case $N=16=2^4$.
Therefore the procedure is synthesized as follows

$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{pmatrix} W^4 \rightarrow \begin{pmatrix} c_0^{(3)} \\ d_0^{(3)} \\ c_1^{(3)} \\ d_1^{(3)} \\ c_2^{(3)} \\ d_2^{(3)} \\ c_3^{(3)} \\ d_3^{(3)} \\ c_4^{(3)} \\ d_4^{(3)} \\ c_5^{(3)} \\ d_5^{(3)} \\ c_6^{(3)} \\ d_6^{(3)} \\ c_7^{(3)} \\ d_7^{(3)} \end{pmatrix} P \rightarrow \begin{pmatrix} c_0^{(3)} \\ c_1^{(3)} \\ c_2^{(3)} \\ c_3^{(3)} \\ c_4^{(3)} \\ c_5^{(3)} \\ c_6^{(3)} \\ c_7^{(3)} \\ d_0^{(3)} \\ d_1^{(3)} \\ d_2^{(3)} \\ d_3^{(3)} \\ d_4^{(3)} \\ d_5^{(3)} \\ d_6^{(3)} \\ d_7^{(3)} \end{pmatrix} W^3 \rightarrow \begin{pmatrix} c_0^{(2)} \\ d_0^{(2)} \\ c_1^{(2)} \\ d_1^{(2)} \\ c_2^{(2)} \\ d_2^{(2)} \\ c_3^{(2)} \\ d_3^{(2)} \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{pmatrix} P \rightarrow$$

$$\begin{pmatrix} c_0^{(2)} \\ c_1^{(2)} \\ c_2^{(3)} \\ c_3^{(2)} \\ d_0^{(2)} \\ d_1^{(2)} \\ d_2^{(2)} \\ d_3^{(2)} \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{pmatrix} W^2 \rightarrow \begin{pmatrix} c_0^{(1)} \\ d_0^{(1)} \\ c_1^{(1)} \\ d_1^{(1)} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{pmatrix} P \rightarrow \begin{pmatrix} c_0^{(1)} \\ c_1^{(1)} \\ d_0^{()1} \\ d_1^{(1)} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{pmatrix} W^1 \rightarrow \begin{pmatrix} c_0^{(0)} \\ d_0^{(0)} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{pmatrix}. \quad (10)$$

where P is a permutation matrix of elements of vector **x** which orders all the coefficients of type "c"

(*i.e.*, *coarse* coefficients) and type "*d*" (*i.e.*, *detail* coefficients). Note that the W^j matrix of order j acts on *coarse* coefficients of j level, while the *detail* coefficients of the same level are unchanged.
Therefore at the end the wavelet transform vector will be formed as following

$$\left(c_0^{(0)}d_0^{(0)}d_0^{(1)}d_1^{(1)}d_0^{(2)}d_1^{(2)}d_2^{(2)}d_3^{(2)}d_0^{(3)}d_1^{(3)}d_3^{(3)}d_4^{(3)}d_5^{(3)}d_6^{(3)}d_7^{(3)}\right)^T \quad (11)$$

where $c_0^{(0)}$ means the *coarse* coefficient obtained at the fourth step of wavelet transform, $d_0^{(0)}$ indicates the *detail* coefficient obtained on the same step, $d_0^{(1)}, d_1^{(1)}$ are the *detail* coefficients obtained at the third step of transform, the $d_k^{(2)}, k = 0,\ldots,3$ represent the *detail* coefficients obtained at the second step and finally $d_k^{(1)}, k = 0,\ldots,7$ the *detail* coefficients obtained at the first step of transform. Since the procedure is based on orthogonal linear operations equally the WT will show the same feature.
For the calculation of the inverse transform, it will be sufficient to repeat the steps of the transform in the inverse order.
In [1][9] it was studied the problem of minimizing the specific sliding ratio of meshing profiles functions for both the pinion and gear as well as the sliding work. The function $\alpha(\lambda)$ which minimize the above functions for all λ values, will provide the optimum shape of the line of action, where λ is the parametric variable of the line of action and α the angle between λ and x-axis. By deriving the quoted functions we obtained

$$\begin{aligned}&\lambda^2 \frac{d^2\alpha(\lambda)}{d\lambda^2} - \lambda^2 \tan\alpha(\lambda)\left(\frac{d\alpha(\lambda)}{d\lambda}\right)^2 + \\ &+\lambda\frac{d\alpha(\lambda)}{d\lambda} - \tan\alpha(\lambda) = 0\end{aligned} \quad (12)$$

where $\cos\alpha(\lambda) \neq 0 \rightarrow \alpha(\lambda) \neq \mp \pi/2$.

Finally, for evaluating the features of the signal, a parameter (entropy) was defined. Given a set S: = $\{x_i,\ I \in \{1,2,\ldots,n\}\}$ and a function $c: x_i \in S \rightarrow c(x_i) \in \mathbf{R}$, the entropy $H(c)$ of c is defined as follows

$$H(c) := -\sum_{c(x_i)\neq m} \frac{1}{s}\cdot\frac{c(x_i)-m}{M-m}\cdot\ln\left(\frac{1}{s}\cdot\frac{c(x_i)-m}{M-m}\right) \quad (13)$$

where

$$s = \sum_{i\in I}\frac{c(x_i)-m}{M-m},$$

M: = max $\{c(x_i), i \in \{1,2,\ldots,n\}\}$

and

m: = min $\{c(x_i), i \in \{1,2,\ldots,n\}\}$.

The entropy measures the best ratio between the maximum dynamic showed by signal and the smallest uniformity of signal. Given $|S| = n$, the entropy, as before defined, riches its maximum value at $ln(n)$ iff, for any $i \in S$, $c(x_i) = const$. Finally $H(c) = 0$ iff, for any $i \in \{1, 2,\ldots,n\}$, $c(x_i) = S$ and, for any $j \in \{1, 2,\ldots,n\}-\{i\}$, $c(x_j) = 0$.

In the Fig.1 is shown a schematic Block Diagram employed for the simulation gear box. It was used for estimating the response of DWT applied to the model of Gear Box.

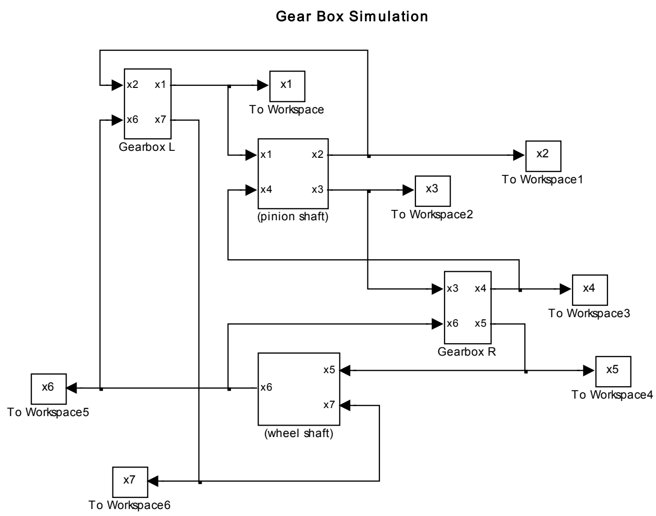

Fig. 1 Schematic Block Diagram

4 Results

In the Fig. 2 and 3, reported below, it is shown the ability of wavelet transform in order to detect and to localize, starting from the (12) the instants where the entropy, defined in (13), riches consistent values indicating the potential sliding and stress surface of profile.

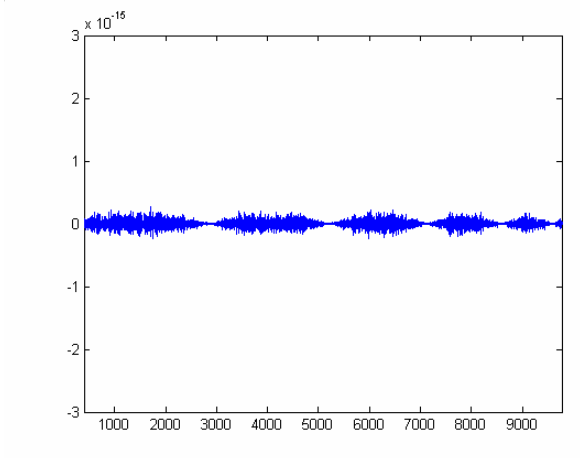

Fig. 2 Entropy distribution

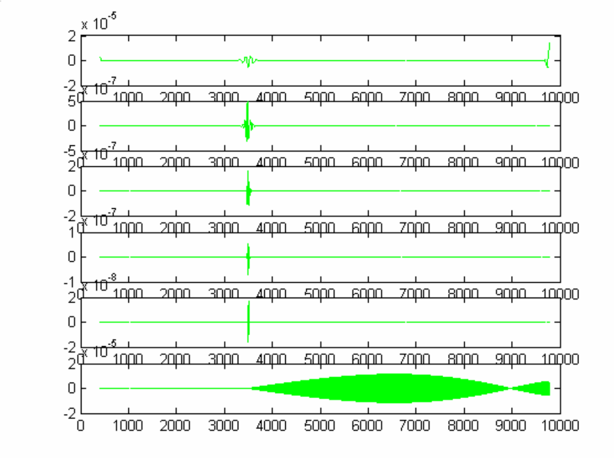

Fig. 3 MRA analysis of entropy distribution

In particular in the Fig. 3 it is well represented the instants where the contact stress riches its maximum value (*i.e.*, from 3000 to 4000 units of time) and consequently the surface pressure of tooth profiles. The results are very interesting and show that if we design a line of action that fulfils the kinematical behavior requirements of the gear pair it will be much easier, by applying the wavelets, to ensure the kinematical quality of the designed gear pair.

5 Conclusions

Gears are one of the most common and important machine components in many advanced machines.

Modern gear design is generally based on standard tools. This makes gear design quite simple (almost like selecting fasteners), economical, and available for everyone, reducing tooling expenses and inventory. At the same time, it is well known that universal standard tools provide gears with less than optimum performance and— in some cases—do not allow for finding acceptable gear solutions. Application specifics, including low noise and vibration, high density of power transmission (lighter weight, smaller size) and others, require gears with non-standard parameters [13,14,15].

On the other side, an improved understanding of sliding work and surface contact stress is required both for the early detection of incipient gear failure and to achieve high reliability.

A brief theory of wavelet transforms and their effective computation method with an emphasis on the considerations of the choice of pressure angle are presented in this paper.

This is followed by the numerical results with related graphs.

It is shown the ability of wavelet transform in order to detect and to localize, starting from the line of action, the area where the energy and the entropy assume consistent value indicating the potential sliding and stress surface of profile.

In order to avoid such a problem, probably, we have to modify the pressure angle and corresponding arches of circumference, constituting the profile, until the first derivative. The results are interesting and show that if we design a line of action that fulfils the kinematical behavior requirements of the gear pair it will be much easier, by applying the wavelets, to ensure the kinematical quality of the designed gear pair.

Then the proposed methodology will be investigated in order to investigate deeper on the regularity of the line of action.

The objective is to calculate the shape of the required modified hobbing tools and to show that the generation process will be no more complicated than that used currently for the production of standard gears.

The work suggests some directions for future investigations:

- reduction of undercutting and interference problem
- reduction of slipping speeds
- increase loading capacity
- increase rigidity of toothing
- reduction of noise and radial forces.

The application of Wavelet Transform is also able for the identification and quantification of damaged tooth based on the numerically generated vibration signal.

The procedure used in this paper include the numerical simulation of the dynamics of a gear transmission system.

Important advancements in preventive maintenance of gear transmission systems are currently being sought for the development of an accurate machine health diagnostic system. Such a diagnostic system would use vibration [16]or acoustic signals from the gear transmission system for

- rapid on-line evaluation of gear wear or damage status
- prediction of remaining gear life.

Such health diagnostic capabilities would be essential for effective machine event/life management and advance warning before critical component failures.

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