A Modified Hydro-Thermo-Diffusive Theory of Shock Waves

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Abstract: - Scale-invariant forms of conservation equations are applied to present a modified hydro-thermo-diffusive theory of normal shock waves. The internal structure of a normal shock is described at the scale of laminar molecular-dynamics. The predicted shock thickness and temperature profile are found to be in good agreement with the experimental observations.

Key-Words: - Theory of shock waves; Normal shock structure; Supersonic flows; Gas dynamics.

1 Introduction

The universality of turbulent phenomena from stochastic quantum fields to classical hydrodynamic fields resulted in recent introduction of a scale-invariant model of statistical mechanics and its application to the field of thermodynamics [6-7]. The invariant forms of conservation equations were subsequently employed to present a modified theory of laminar flames [8]. The scale-invariant model of statistical mechanics for the intermediate scales of eddy-, cluster-, and molecular-dynamic is schematically shown in Fig.1.

In the present study, the invariant forms of the conservation equations are applied to investigate the hydro-thermo-diffusive structure of normal shock waves. Because the thickness of shock wave is known to be of the order of a few molecular mean-free-paths, the continuum assumption of classical fluid dynamics is clearly not applicable to the study of the internal structure of shock waves. However, according to the scale invariant model of statistical mechanics [6, 7], the phenomena of Brownian motions suggest the existence of a new equilibrium statistical field at an intermediate scale called equilibrium cluster-dynamics ECD that separates the field of equilibrium eddy-dynamics EED (isotropic homogeneous turbulence) from equilibrium molecular-dynamics EMD (Fig.1). Therefore, the model allows one to move to the adjacent statistical field within the hierarchy at the next smaller scale called equilibrium molecular-dynamics EMD (Fig.1). The invariant forms of conservation equations are then applied at this new scale in order to investigate the internal structure of the shock wave.

2 Invariant Forms of the Conservation Equations for Reactive Fields

Following the classical methods [1-5], the invariant definitions of the density ρ_β, and the velocity of *atom* $\mathbf{u}_\beta$, *element* $\mathbf{v}_\beta$, and *system* $\mathbf{w}_\beta$ at the scale β are given as [7]

$$\rho_\beta = n_\beta m_\beta = m_\beta \int f_\beta du_\beta \quad , \qquad \mathbf{u}_\beta = \mathbf{v}_{\beta-1} \tag{1}$$

$$\mathbf{v}_\beta = \rho_\beta^{-1} m_\beta \int \mathbf{u}_\beta f_\beta d\mathbf{u}_\beta \quad , \qquad \mathbf{w}_\beta = \mathbf{v}_{\beta+1} \tag{2}$$

Similarly, the invariant definition of the peculiar and diffusion velocities are introduced as

$$\mathbf{V}'_\beta = \mathbf{u}_\beta - \mathbf{v}_\beta \quad , \qquad \mathbf{V}_\beta = \mathbf{v}_\beta - \mathbf{w}_\beta = \mathbf{V}'_{\beta+1} \tag{3}$$

Following the classical methods [1-5], the scale-invariant forms of mass, thermal energy and momentum conservation equations at scale β are given as [8]

$$\frac{\partial \rho_\beta}{\partial t} + \nabla.(\rho_\beta \mathbf{v}_\beta) = \Omega_\beta \tag{4}$$

$$\frac{\partial \varepsilon_\beta}{\partial t} + \nabla.(\varepsilon_\beta \mathbf{v}_\beta) = 0 \tag{5}$$

$$\frac{\partial \mathbf{p}_\beta}{\partial t} + \nabla.(\mathbf{p}_\beta \mathbf{v}_\beta) = -\nabla.\mathbf{P}_\beta \tag{6}$$

involving the *volumetric density* of thermal energy $\varepsilon_\beta = \rho_\beta h_\beta$ and linear momentum $\mathbf{p}_\beta = \rho_\beta \mathbf{v}_\beta$. Also,

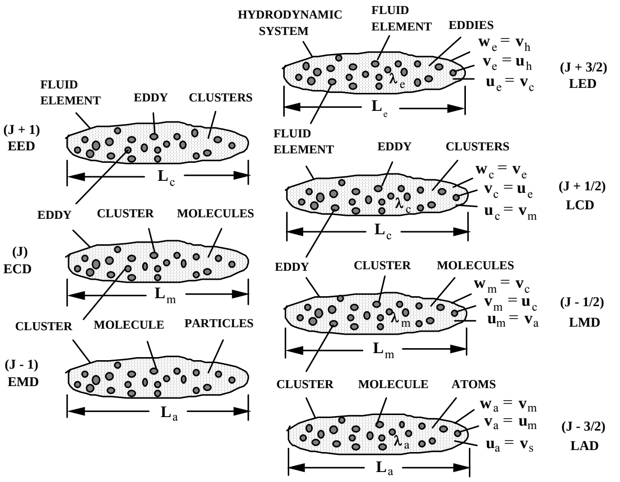

Fig.1 Hierarchy of statistical fields for equilibrium eddy-, cluster-, and molecular-dynamic scales and the associated laminar flow fields.

Ω_β is the chemical reaction rate, h_β is the absolute enthalpy [6], and $\mathbf{P}_\beta$ is the partial stress tensor [1]

$$\mathbf{P}_\beta = m_\beta \int (\mathbf{u}_\beta - \mathbf{v}_\beta)(\mathbf{u}_\beta - \mathbf{v}_\beta) f_\beta du_\beta \qquad (7)$$

In the derivation of (6) we have used the definition of the peculiar velocity (3) along with the identity

$$\overline{\mathbf{V}'_{\beta i}\mathbf{V}'_{\beta j}} = \overline{(\mathbf{u}_{\beta i} - \mathbf{v}_{\beta i})(\mathbf{u}_{\beta j} - \mathbf{v}_{\beta j})} = \overline{\mathbf{u}_{\beta i}\mathbf{u}_{\beta j}} - \mathbf{v}_{\beta i}\mathbf{v}_{\beta j} \quad (8)$$

The transport of mass, linear momentum, and thermal energy are considered to occur by both convection and diffusion. Hence, the local velocity $\mathbf{v}_\beta$ in (4)-(6) is expressed in terms of the convective $\mathbf{w}_\beta$ and the diffusive $\mathbf{V}_\beta$ velocities [8]

$$\mathbf{v}_\beta = \mathbf{w}_\beta + \mathbf{V}_{\beta g} \quad , \quad \mathbf{V}_{\beta g} = -D_\beta \nabla \ln(\rho_\beta) \qquad (9a)$$

$$\mathbf{v}_\beta = \mathbf{w}_\beta + \mathbf{V}_{\beta tg} \quad , \quad \mathbf{V}_{\beta tg} = -\alpha_\beta \nabla \ln(\varepsilon_\beta) \qquad (9b)$$

$$\mathbf{v}_\beta = \mathbf{w}_\beta + \mathbf{V}_{\beta hg} \quad , \quad \mathbf{V}_{\beta hg} = -\nu_\beta \nabla \ln(\mathbf{p}_\beta) \qquad (9c)$$

where ($\mathbf{V}_{\beta g}$, $\mathbf{V}_{\beta tg}$, $\mathbf{V}_{\beta hg}$) are respectively the diffusive, the thermo-diffusive, the linear hydro-diffusive velocities.

By substitutions from (9) in (4)-(6) one obtains, for constant transport coefficients, the scale-invariant forms of conservation equations [8].

$$\frac{\partial \rho_\beta}{\partial t} + \mathbf{w}_\beta . \nabla \rho_\beta - D_\beta \nabla^2 \rho_\beta = \Omega_\beta \qquad (10)$$

$$\frac{\partial T_\beta}{\partial t} + \mathbf{w}_\beta . \nabla T_\beta - \alpha_\beta \nabla^2 T_\beta = -\frac{h_\beta \Omega_\beta}{\rho_\beta c_{p\beta}} \qquad (11)$$

$$\frac{\partial \mathbf{v}_\beta}{\partial t} + \mathbf{w}_\beta . \nabla \mathbf{v}_\beta - \nu_\beta \nabla^2 \mathbf{v}_\beta = -\frac{\nabla p_\beta}{\rho_\beta} - \frac{\mathbf{v}_\beta \Omega_\beta}{\rho_\beta} \qquad (12)$$

An important feature of the modified equation of motion (12) is that it is linear since it involves a convective velocity $\mathbf{w}_\beta$ that is different from the local fluid velocity $\mathbf{v}_\beta$.

The classical form of the continuity equation (4), while being equivalent to (10), does not contain a diffusion term and hence cannot clearly reveal the separate roles of convection versus diffusion within the shock structure. As a result, in the classical theory of shocks in a pure gas, density discontinuity could only be presented as jump condition across the shock. However, because the modified form of the continuity equation (10) does contain a diffusion term, it allows for the analysis of the internal structure of shock waves that is the objective of the present study.

It is now shown that by summation of (4)-(6) over (β) one can arrive at the conservation equations at the next higher scale of (β+1) of the hierarchy (Fig.1). The summation of (4) gives

$$\sum_\beta \rho_\beta = \rho_{\beta+1} \qquad (13)$$

and

$$\sum_\beta \rho_\beta \mathbf{v}_\beta = \sum_\beta \mathbf{p}_\beta = \sum_\beta \rho_{\beta+1} Y_\beta \mathbf{v}_\beta =$$

$$\rho_{\beta+1} \sum_\beta Y_\beta \mathbf{v}_\beta = \rho_{\beta+1} \mathbf{v}_{\beta+1} = \mathbf{p}_{\beta+1} \qquad (14)$$

For (6), the summation of the first term is the same as (14) above. To treat the summation of the second term of (6), one starts with (3)

$$\mathbf{v}_\beta = \mathbf{w}_\beta + \mathbf{V}_\beta = \mathbf{v}_{\beta+1} + \mathbf{V}'_{\beta+1} \qquad (15)$$

Multiplying (15) by ($Y_{\beta+1}\, \rho_\beta \mathbf{v}_\beta$) and summing over ($\beta$+1) and ($\beta$) leads to

$$\sum_\beta \sum_{\beta+1} Y_{\beta+1} \rho_\beta \mathbf{v}_\beta \mathbf{v}_\beta = \sum_\beta \sum_{\beta+1} \rho_\beta \mathbf{v}_\beta Y_{\beta+1} (\mathbf{w}_\beta + \mathbf{V}'_{\beta+1}) =$$

$$\sum_\beta \rho_\beta \mathbf{v}_\beta \mathbf{v}_\beta =$$

$$\sum_\beta \sum_{\beta+1} \rho_\beta \mathbf{v}_\beta Y_{\beta+1} (\mathbf{v}_{\beta+1} + \mathbf{V}'_{\beta+1}) =$$

$$\sum_{\beta} \rho_\beta \mathbf{v}_\beta (\mathbf{w}_{\beta+1} + \mathbf{V}_{\beta+1}) = \rho_{\beta+1} \mathbf{v}_{\beta+1} \mathbf{v}_{\beta+1} = \mathbf{p}_{\beta+1} \mathbf{v}_{\beta+1} \quad (16)$$

Where Y is mass fraction and use was made of the relation $\mathbf{v}_{\beta+1} = \mathbf{w}_{\beta+1} + \mathbf{V}_{\beta+1}$ from (3) in the last step.

For the summation of the energy equation (5) one first notes that

$$\sum_{\beta} \varepsilon_\beta = \varepsilon_{\beta+1} \quad (17)$$

and next multiplying (15) by $Y_{\beta+1}\varepsilon_\beta$ one obtains

$$\sum_{\beta} \sum_{\beta+1} Y_{\beta+1} \varepsilon_\beta \mathbf{v}_\beta = \sum_{\beta} \varepsilon_\beta \mathbf{v}_\beta =$$

$$\sum_{\beta} \sum_{\beta+1} Y_{\beta+1} \varepsilon_\beta (\mathbf{w}_\beta + \mathbf{V}_\beta) = \sum_{\beta} \sum_{\beta+1} \varepsilon_\beta Y_{\beta+1} (\mathbf{v}_{\beta+1} + \mathbf{V}'_{\beta+1}) =$$

$$\sum_{\beta} \varepsilon_\beta (\mathbf{w}_{\beta+1} + \mathbf{V}_{\beta+1}) = \sum_{\beta} \varepsilon_\beta \mathbf{v}_{\beta+1} = \varepsilon_{\beta+1} \mathbf{v}_{\beta+1} \quad (18)$$

3 Connection to the Navier-Stokes Equation of Motion

The original form of the *Navier-Stokes* equation with constant coefficients is given as [1, 9]

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v}.\nabla \mathbf{v} = -\nabla P + \mu \nabla^2 \mathbf{v} + \frac{1}{3} \mu \nabla (\nabla . \mathbf{v}) \quad (19)$$

The pressure P in (19) is related to the thermodynamic pressure p through the total stress tensor $T_{ii} = -p\delta_{ij} + \tau_{ij}$ and is called *mechanical pressure* defined as [10]

$$P = P_m = -(1/3)T_{ii} = p - (1/3)\tau_{ii} \quad (20)$$

The normal viscous stress is given by the flux of momentum $(1/3)\tau_{ii} = -(1/3)\mu\nabla.\mathbf{v}$ such that the gradient of (20) reduces to

$$\nabla P = \nabla P_m = \nabla p + \frac{1}{3} \mu \nabla (\nabla . \mathbf{v}) \quad (21)$$

Substituting from (21) in (19), the original *Navier-Stokes* equation assumes the form

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}.\nabla \mathbf{v} - \nu \nabla^2 \mathbf{v} = -\frac{\nabla p}{\rho} \quad (22)$$

that is almost identical to the modified equation of motion (12) with $\Omega_\beta = 0$ except for the important fact that the latter is linear since the convective velocity **w** is different from the local velocity **v**. However, because (22) includes a diffusion term and **w** and **v** are related by $\mathbf{v} = \mathbf{w} + \mathbf{V}$, it is clear that (22) should in fact be written as (12).

In order to facilitate the future application to the study of detonation waves, the one-dimensional form of invariant conservation equations for reactive fields are described and later simplified by neglecting the reaction terms. For propagation of a planar laminar flame, one introduces the dimensionless parameters

$$\theta = (T - T_u)/(T_b - T_u) \quad , \quad y = Y_F / Y_{Fu}$$

$$\Lambda = [\nu_F W_F B\alpha / (\rho v_o'^2)] e^{-\beta/\chi} \quad (23)$$

The adiabatic flame temperature T_b, the *Zeldovich* number β, and the coefficient of thermal expansion χ are

$$T_b = T_u + QY_{Fu} / (\nu_F W_F c_p)$$

$$\beta = E(T_b - T_u) / RT_b^2 \quad (24)$$

$$\chi = (T_b - T_u) / T_b$$

and one assumes that $\beta >> 1$. Equations (10)-(12) for laminar cluster-dynamic scale β = c that corresponds to conventional gas dynamics become [8]

$$\frac{\partial y}{\partial t'} + w'_x \frac{\partial y}{\partial x'} = D \frac{\partial^2 y}{\partial x'^2} - \Lambda y e^{\beta(\theta-1)} \delta(x'_f) \quad (25)$$

$$\frac{\partial \theta}{\partial t'} + w'_x \frac{\partial \theta}{\partial x'} = \alpha \frac{\partial^2 \theta}{\partial x'^2} + \Lambda y e^{\beta(\theta-1)} \delta(x'_f) \quad (26)$$

$$\frac{\partial v'}{\partial t'} + w'_x \frac{\partial v'}{\partial x'} = \nu \frac{\partial^2 v'}{\partial x'^2} - \frac{1}{\rho'} \frac{\partial p}{\partial x'} + v' \Lambda y e^{\beta(\theta-1)} \delta(x'_f) \quad (27)$$

In the following, the conservation equations (25)-(27) are employed to study of internal structure of normal shock waves in non-reactive fields.

4 Hydro-Thermo-Diffusive Theory of Normal Shock Waves

For description of the structure of a one-dimensional shock wave propagating in an otherwise quiescent $w'_x = 0$ non-reactive $\Omega_\beta = 0$ ideal gas, the invariant conservation equations (25)-(27) reduce to

$$\frac{\partial f}{\partial t'} = \nu_\beta \frac{\partial^2 f}{\partial x'^2} \qquad f = \rho \ , \theta \ , p \qquad (28)$$

$$\frac{\partial v}{\partial t'} = \nu_\beta \frac{\partial^2 v}{\partial x'^2} - \frac{1}{\rho' w'_s}\frac{\partial p'}{\partial x'} \qquad (29)$$

with the dimensionless variables defined as

$$\rho = \frac{\rho' - \rho'_{-\infty}}{\rho'_\infty - \rho'_{-\infty}} \quad , \qquad \theta = \frac{T - T_{-\infty}}{T_\infty - T_{-\infty}}$$

$$p = \frac{p' - p'_{-\infty}}{p'_\infty - p'_{-\infty}} \quad , \qquad v = \frac{v'}{w'_s} \qquad (30)$$

The equation for pressure can be directly deduced from that of density and temperature and the ideal gas law $p' = \rho' RT$ when R is the gas constant.

The conventional gas dynamics corresponds to scale of laminar cluster dynamics LCD (Fig.1) with the characteristic (atomic, element, system) lengths $(l_c = 10^{-7},\ \lambda_c = 10^{-5},\ L_c = 10^{-3})$ m and the associated velocities $(\mathbf{u}_c, \mathbf{v}_c, \mathbf{w}_c)$, where the subscript β = c refers to LCD. Also, the relevant kinematic viscosity for this scale is $\nu_c = l_c u_c/3 = \lambda_m v_m/3$ [6]. At LCD scale the shock wave appears as a mathematical surface of discontinuity separating the supersonic flow $LCD_{-\infty}$ with the temperature $T_{-\infty}$ before the shock from a subsonic flow field LCD_∞ with the temperature T_∞ after the shock as shown in Fig.2a.

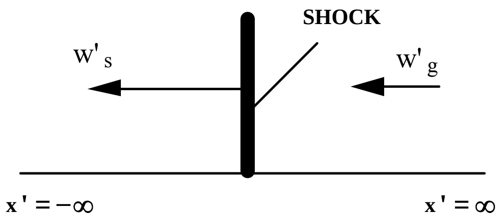

Fig.2a Propagating shock wave at the scale of laminar molecular-dynamics LMD, β = m.

The gas behind the shock moves in the same direction as the shock at the velocity w'_g [11]. The unsteady problem of shock propagating in quiescent gas can be converted to a steady problem of a stationary shock by the introduction of the moving coordinate $z' = x' + w't'$ as shown in Fig.2b

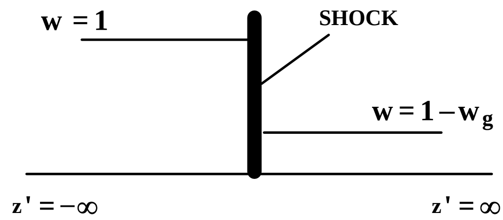

Fig.2b Stationary shock wave at scale of laminar molecular-dynamics LMD, β = m.

For the study of structure of shocks, as opposed to that of laminar flames, one must move to the lower scale of laminar molecular dynamics LMD (Fig.1) with the characteristic (atomic, element, system) lengths $(l_m = 10^{-9},\ \lambda_m = 10^{-7},\ L_m = 10^{-5})$ m and the associated velocities $(\mathbf{u}_m, \mathbf{v}_m, \mathbf{w}_m)$. The kinematic viscosity for this scale is $\nu_m = l_m u_m/3 = \lambda_a v_a/3$ [6]. The conservation equations (28)-(29) in terms of the steady coordinate $z' = x' + w't'$ become

$$w' \frac{df}{dz'} = \nu_m \frac{d^2 f}{dz'^2} \qquad f = \rho \ , \theta \ , p \qquad (31)$$

$$w' \frac{dv}{dtz'} = \nu_m \frac{d^2 v}{dz'^2} - \frac{1}{\rho' w'_s}\frac{dp'}{dz'} \qquad (32)$$

The velocity w' in (31)-(32) is considered to be the appropriate constant mean velocity that makes the shock wave stationary. In the following, it will be shown that this constant velocity is the average of the coordinate-dependent velocity within the internal aerodynamic shock structure. Because the shock thickness is of the order of a few mean free paths of molecules, its internal structure is not yet revealed in terms of the dimensional physical coordinate z' as shown in Fig.2b.

To finally reveal the hydro-thermo-diffusive structure of the shock, one introduces the stretched coordinate and time

$$z = z'/l_H \quad , \qquad t = t'/(l_H / w'_s) \qquad (33)$$

where the hydro-diffusive thickness is defined as

$$l_H = \nu_m / w'_s \ll 1 \qquad (34)$$

such that (31)-(32) become

$$w \frac{df}{dz} = \frac{d^2 f}{dz^2} \qquad f = \rho \ , \theta \ , p \qquad (35)$$

$$w\frac{dv}{dz} = \frac{d^2v}{dz^2} - \frac{R[T(T'_{\infty} - T'_{-\infty}) + T'_{-\infty}]}{w_s'^2\ p}\frac{dp}{dz} \qquad (36)$$

that are subject to the boundary conditions

$$z \to \infty \quad \rho = \theta = p = 1 \quad , \quad v = 1 - w_g \qquad (37a)$$

$$z \to -\infty \quad \rho = \theta = p = 0 \quad , \quad v = 1 \qquad (37b)$$

The calculated structure of the shock wave is finally revealed under the spatial resolution of the stretched coordinate z as shown in Fig.3.

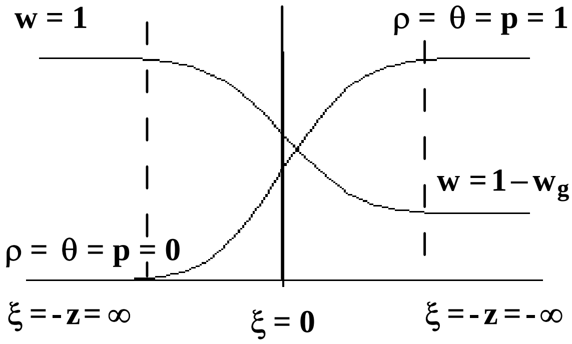

Fig.3 Hydro-thermo-diffusive structure of a stationary shock wave at LMD scale β = m.

At the LMD scale, dissipation occurs through transformation of mean molecular motions to molecular motions $\mathbf{v}_m = \mathbf{u}_c \Rightarrow \mathbf{u}_m = \mathbf{v}_a$ thus accounting for the irreversible nature of flow across the shock. *Hirschfelder* and *Curtiss* [12] emphasized the significance of diffusion process to the classical theory of detonation

> **"Some time ago, *George B. Kistiakowsky* bet one of the authors (J. O. H.) a case of American champagne against a bottle of French champagne that indeed the transport properties do not appreciably affect the behavior of a detonation. This bet has served as an incentive for the present work"**

The important role of transport phenomena at "atomic" scales is also evidenced by the observed correlation between the detonation velocities of various combustible mixtures and the velocity of certain "atomic" radical species discussed in an early investigation of chain reaction theory of explosion by *Lewis* [13].

The classical objection against possible role of diffusion in detonation waves is that the wave thickness is of the order of the molecular mean-free-path. However, this objection is no longer relevant because dissipative processes in conventional gas dynamics will involve $\mathbf{v}_c = \mathbf{u}_e \Rightarrow \mathbf{u}_c = \mathbf{v}_m$ that is now associated with the intermediate statistical field of ECD (Fig.1). For shock and detonation waves, on the other hand, dissipative effects are herein suggested to correspond to $\mathbf{v}_m = \mathbf{u}_c \Rightarrow \mathbf{u}_m = \mathbf{v}_a$ that occur at the smaller scale of LMD (Fig.1). This is also harmonious with the observed correlation between detonation velocities and the mean thermal speeds of "atoms" as was noted by *Lewis* [13].

The higher spatial resolution of the stretched coordinate z also reveals the coordinate-dependence of the convective velocity w in (35)-(36) that in view of the boundary conditions (37) becomes

$$w = 1 - w_g(z + 1/2) \qquad (38)$$

The velocity at the center of the shock (Fig.3) will correspond to the mean value of (38) that is $w_{av} = 1 - w_g / 2$. In supersonic flows the velocity of sound usually exceeds its value at the standard conditions such that the inequality $w_s' \geq 350$ m/s holds. Therefore, the last term of (36) involving the logarithmic gradient of the thermodynamic pressure that multiplies $1/w_s'^2 \ll 1$ can be neglected. Substituting (38) and the new coordinate transformation

$$\xi = (2w_g)^{-1/2}[1 - w_g(z + 1/2)] \qquad (39)$$

into the conservation equations (35)-(37) results in

$$\frac{d^2f}{d\xi^2} + 2\xi\frac{df}{d\xi} = 0 \qquad f = \rho\ ,\ \theta\ ,\ p\ ,\ v \qquad (40)$$

$$\xi \to -\infty \quad \rho = \theta = p = 1 \quad , \quad v = 1 - w_g \qquad (40a)$$

$$\xi \to \infty \quad \rho = \theta = p = 0 \quad , \quad v = 1 \qquad (40b)$$

that lead to the solutions

$$\rho = \theta = p = (1/2)\mathrm{erfc}\xi \qquad (41)$$

$$v = 1 - (w_g / 2)\mathrm{erfc}\xi \qquad (42)$$

The solutions (41) represent the steady shock structure and the calculated temperature profile involving error function is in close agreement with the experimental data of *Sherman* [14] as shown in Fig.4

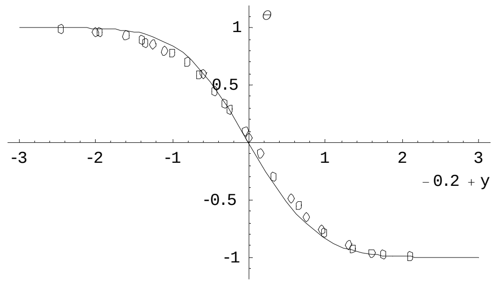

Fig.4 Comparisons between measured temperature θ versus position (y – 0.2) for a normal shock [14] and the prediction in (41).

According to the solutions (41)-(42), upstream and downstream edges of the shock wave will be respectively at $\xi_+ = 2$ and $\xi_- = -2$ to an accuracy of 0.995 such that the predicted shock thickness $(\xi_- - \xi_+ = -4)$ that in view of (33), (34) and (39) is $\delta_s = (z'_+ - z'_-)$ becomes

$$\delta_s = 4\sqrt{2}\frac{\nu_m}{\sqrt{w'_g w'_s}} = 4\sqrt{2}\frac{\nu_m}{\sqrt{w'^2_s - w'_s w'_a}} \quad (43)$$

that is analogous to the laminar flame thickness [8]

$$\delta_f = 4\sqrt{2}\frac{\nu_m}{w'_f} = 4\sqrt{2}\ell_T \quad (44)$$

where w'_f is the laminar flame speed. According to (43), the shock thickness δ_s linearly increases with the viscosity ν_m and decreases with the shock velocity w'_s. The result (43) is also harmonious with the expression suggested by *Granger* [15] for estimating the thickness of shocks on the basis of phenomenological arguments

$$\delta_s = \frac{\nu_m}{w'_s} \quad (45)$$

For the typical value of *Mach* number $Ma_b = 2.5$, the velocities before and after the stationary shock $w'_b = w'_s = 875$ m/s, $w'_a = 262$ m/s are related by [11]

$$\frac{w'_b}{w'_a} = \frac{(\gamma+1)Ma_1^2}{(\gamma-1)Ma_1^2 + 2} \quad (46)$$

and lead to $w'_g = w'_s - w'_a = 613$ m/s. Assuming a molecular diffusivity of $\nu_m = 0.2\ \text{cm}^2/\text{s}$ for air one calculates from (43) the shock thickness

$$\delta_s \simeq 1.5\times10^{-7}\ \text{m} \quad (47)$$

that is consistent with the typical values reported in the literature [11, 14, 15].

5 Concluding Remarks

The invariant forms of conservation equations were solved to present a modified hydro-thermo-diffusive theory of steady shock waves. The predicted temperature profile and thickness of normal shock were found to be in good agreement with the experimental observations.

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