Wheeled mobile robot actuations of a Multiple Degrees-of-Freedom Parallel manipulator

MARIE BERNARD SIDIBE, YI LI FU and YULIN MA
Advanced manufacturing and technology Center, Department of mechatronics
Harbin Institute of Technology
92 West Dazhi Street Nangang district, Harbin150001 P.R. China

Abstract: -This paper introduces a new design combining wheeled mobile robots and parallel mechanism for the purpose of manipulation. The movable platform consists of a hexagonal platform, which is rigidly fixed to three inclined planes regularly fixed at three of its sides. Contrarily to the previous similar works, the platform is not linked to mobile robots but lies down on three free different ball wheels located at the tip of a vertical and rigid link fixed at the top of each mobile robot. The mobile robot motions in horizontal plane are transformed through the frictional contact between ball wheels and inclined planes to transport or orient the platform.

Key-words: -Parallel manipulator, platform, wheeled mobile robot, kinematics-

1. Introduction

Parallel manipulators have attracted robotics designers since some decades ago and most of the presented manipulators have been based on the principle of connecting a movable platform to the base through actuated and passive joints [1], [2]. Generally this strategy leads to manipulator with restricted mobility i.e. with a number of degree-of-freedom less than six [3]. They also suffer from smaller workspace of the platform. Recently researches have been focused on parallel manipulator supported by wheeled mobile robots (WMR) to solve this mobility limitation problem of parallel mechanisms. Regarding this topic only a small number of works can be found in robotic literature and one of them is ref. [4] of this paper. But in ref. [4] design, the links connecting each rolling machine (cart) to the platform have been accomplished through a spherical joint and a revolute joint so that the directions of the static forces vary and depend on the platform position and orientations. This phenomenon has brought a complex dynamic to the design. The parallel mechanism we introduce in this paper does not have any joint and its performance lays down on the mobility of the three supporting WMR.

Some WMR equipped with a combination of some specified wheels have full mobility. WMR mobility has been discussed in several papers [5], [6]. Their collaborations also have been object of several researches [7]. These topics are not the aim of this paper but we consider that the three supporting WMRs have full mobility, which allows them to reach any point without complex maneuvers. At the top of each of these robots is fixed a vertical link with constant height. A free ball wheel connected to each link tip is able to rotate about the ball center in any direction. The pressure of the platform weight maintains the contacts ball-inclined planes. The platform adopts a new position and orientation anytime when at least one of the robots changes its posture. The stability of the system depends on platform weight and the WMRs position with respect to the inclined planes. The workspace depends on the platform dimensions and the inclination angle of inclined planes. The inverse and direct kinematics of the system are presented in section.3.

2. Model descriptions and its mobility

The model of parallel manipulator presented here consists of two main parts: the movable platform and three rolling machines (WMR) as shown in Fig.1. The movable platform is composed of a hexagonal platform P and three inclined planes P_i ($i=1,2,3$) having same inclination angle θ with respect to platform P. They are fixed to three of the six sides of the hexagon (Fig.2 and 3). The rolling machines numbered as $Robot_i$ ($i=1,2,3$) are able to move into any direction in a horizontal plane (O,X,Y) of the world coordinate system $\mathrm{R}:(O,X,Y,Z)$ attached to the ground.

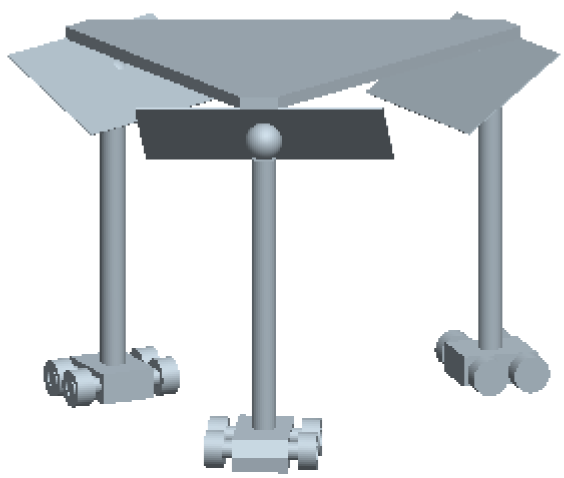

Fig.1: model of the manipulator

A rigid link is fixed to the top of each WMR. A free ball wheel is located at the link top, which is in contact with the corresponding inclined plane. Simply the movable platform can be just considered as relied down on the robots through the contact ball wheels-inclined planes. The purpose of the ball wheels is to allow relative rolling motion without slipping between the inclined planes and WMRs. The entire system can be assimilated with three fingers grasping and manipulating an object. A coordinate system $\mathrm{R}_i:(O_i,X_i,Y_i,Z_i)$ is attached to each Robot_i $(i{=}1,2,3)$ and the last coordinate system to be defined is $\mathrm{R}_p:(O_p,X_p,Y_p,Z_p)$ witch is connected to the platform.

When the three WMRs move in translation having a same direction and velocity in plane (O,X,Y) of frame R, the platform will be transported at the same direction with the same velocity. The translation of the movable platform with respect to Z-axis is obtained when each robot relates to or moves away from the hexagonal platform. The rotation of the movable platform is possible about any axis passing by point A_i and A_j when Robot_i and Robot_j have fixed postures and Robot_k moves $(i \neq j \neq k)$. The rotation about Z-axis is obtained when each WMR adopts a circular motion about Z-axis with a synchronized velocity.

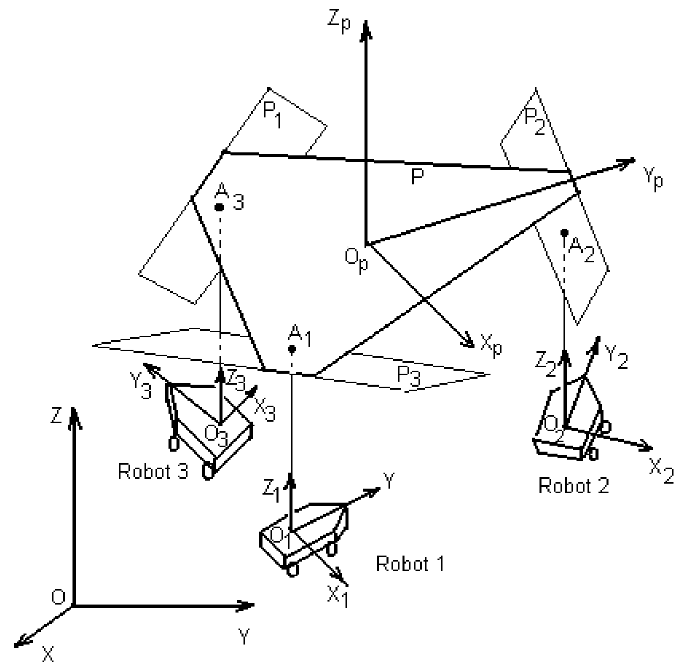

Fig.2: Kinematics model

3. Kinematics

This section deals with the kinematics of parallel manipulator. It contains two parts, inverse and forward kinematics. The inverse kinematics involves mapping a known posture (position and orientation) of the platform to a set of postures of the three supporting WMR. The forward or direct kinematics can be stated as follow: given a posture to each WMR, compute the position and orientation of the platform. The kinematics is built upon the following assumptions:

-The robot wheels move on a horizontal plane.

-The wheels are not deformable and their contacts with the ground are a point. - Robot wheels and ball wheels motions are pure rolling leading to a null velocity at the contact point.

-No slipping, skidding, sliding or friction for rotation around the contact point.

The hexagon parameters are defined by points B_i and $C_i\,(i{=}1,2,3)$ located at its six vertices. Each couple (B_i, C_i) defines the two extremities of platform P and plan P_i intersection line (Fig.3). In the same figure

Point D represents the common intersection point of each plane P_i (i=1,2,3) and Z_p axis of R_p. In Fig.2 point A_i (i=1,2,3) defines the contact point between plane P_i and the corresponding Robot_i top. We define h as point A_i height with respect to the ground. Height h is constant.

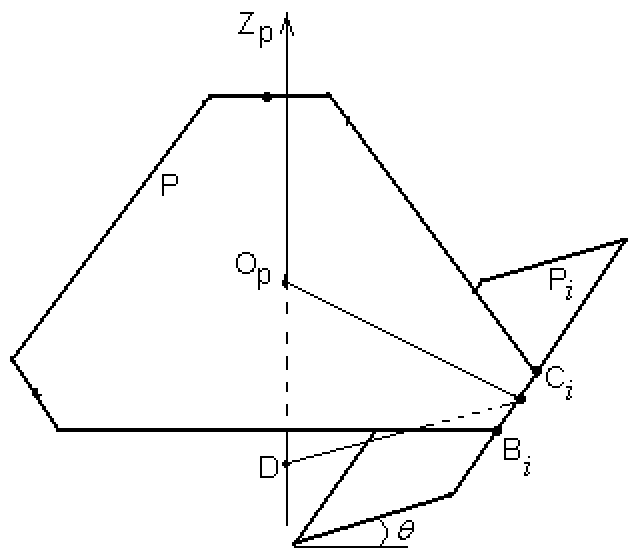

Fig.3: movable platform vertices

3.1 Inverse kinematics

The different orientations and the position of origin O_p, $[O_p]_R = (x \quad y \quad z)^T$ of the platform frame are known. With them we have to find we postures x_i and y_i (i=1,2,3) of the three mobile robots. We define angles α, β and γ as respectively the platform yaw, pitch and roll. Then the rotation matrices $R(\alpha), R(\beta), R(\gamma)$ expressing the orientation of R_p respectively with respect to Z-axis, Y-axis and X-axis of frame R are given by:

$$R(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (1)$$

$$R(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \qquad (2)$$

$$R(\gamma) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix} \qquad (3)$$

The complete rotation matrix of R_p with respect to R is given by multiplying equations (1), (2) and (3) and expressed as follow

$$R\,(\alpha, \beta, \gamma) = R(\alpha)R(\beta)R(\gamma) \qquad (4)$$

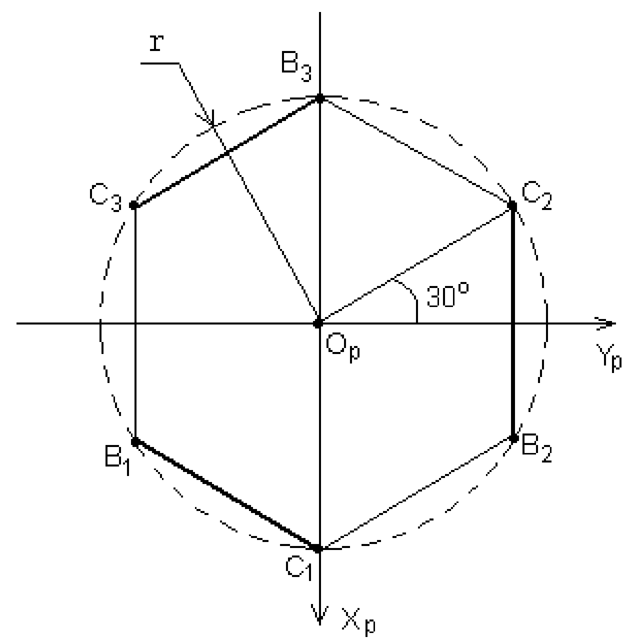

Fig.4: hexagonal platform vertices

We assume that the hexagonal form of P is regular and can be inscribed in a circle of radius r (Fig.4). The coordinates of points B_i, C_i and D in frame $R_p:(O_p,X_p,Y_p,Z_p)$ can be described by vectors $[B_i]_{R_p}$, $[C_i]_{R_p}$ (i=1,2,3) and $[D]_{R_p}$. We obtain:

$$[B_1]_{R_p} = \begin{bmatrix} r\sin 30^o \\ -r\cos 30^o \\ 0 \end{bmatrix}; [C_1]_{R_p} = \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix};$$

$$[B_2]_{R_p} = \begin{bmatrix} r\sin 30^o \\ r\cos 30^o \\ 0 \end{bmatrix}; \quad [C_2]_{R_p} = \begin{bmatrix} -r\sin 30^o \\ r\cos 30^o \\ 0 \end{bmatrix};$$

$$[B_3]_{R_p} = \begin{bmatrix} -r \\ 0 \\ 0 \end{bmatrix}; [C_3]_{R_p} = \begin{bmatrix} -r\sin 30^o \\ -r\cos 30^o \\ 0 \end{bmatrix} \text{ and}$$

$$[D]_{R_p} = \begin{bmatrix} 0 \\ 0 \\ -r\sin\theta\cos 30^o \end{bmatrix} \qquad (6)$$

The coordinates of the above points in the fixed frame R are expressed as follow:

$$[B_i]_R = R(\alpha, \beta, \gamma)[B_i]_{R_p} + [O_p]_R \qquad (7)$$

$$[C_i]_R = R(\alpha, \beta, \gamma)[C_i]_{R_p} + [O_p]_R \qquad (8)$$

$$[D]_R = R(\alpha, \beta, \gamma)[D]_{R_p} + [O_p]_R \qquad (9)$$

Where vector $[O_p]_R$ represents the coordinates of O_p in the fixed frame. Three non-aligned points are enough to define a plane in a space. With equation (7), (8) and (9) we have completely defined for each plane P_i three non-aligned points B_i, C_i and D. Robot$_i$ posture x_i and y_i represent the coordinates of the origin O_i (i=1,2,3) of the frame $R_i:(O_i,X_i,Y_i,Z_i)$ in the fixed frame R. The coordinates of the contact points A_i (i=1,2,3) are given by the following vector:

$$[A_i]_R = \begin{bmatrix} x_i \\ y_i \\ h \end{bmatrix} \qquad (10)$$

Hence, for a given posture and orientations of the platform the inputs, which are the three robots postures, are computed by considering that vectors $[B_i, C_i]_R$, $[B_i, D]_R$ and $[B_i, A_i]_R$ (i=1,2,3) are linearly independent. They constitute a basis of plane P_i. Finally the inputs are given by the following equation

$$\det\left|\overrightarrow{B_iC_i}, \overrightarrow{B_iD}, \overrightarrow{B_iA_i}\right|_R = 0, \quad i=1,2,3 \qquad (11)$$

Equation (11) is a system of three independents equations. Each of them represents the equation of a line in horizontal plane (O,X,Y) of the world coordinate system $R:(O,X,Y,Z)$ attached to the ground. Infinity postures (x_i, y_i) verifying equation (11) can be found for each mobile robot. The conclusion is that each mobile robot can move in a corresponding line represented by equation (11) without changing the platform position and orientations.

3.2 Direct kinematics

Robots postures x_i and y_i are known; the different orientations α, β, γ and the position of origin O_p, $[O_p]_R = (x \quad y \quad z)^T$ of the platform frame are unknown. To solve the direct kinematics problem we need to build at least six equations because the total unknowns are six. For that we consider the system of three equations forming by equation (11) to which we have to add three more. To proceed in this way we define the platform stability constraint as follow: the platform has a maximum stability when the perpendicular projection of each robot posture on the projection of $[B_i, C_i]$ segment in plane (O,X,Y) coincides with the same segment middle. If point M_i is the middle of $[B_i, C_i]_R$ then the dot product of vectors $\overrightarrow{B_iC_i}_R$ and $\overrightarrow{O_iM_i}_R$ is equal to zero which is:

$$\overrightarrow{B_iC_i}_R . \overrightarrow{O_iM_i}_R = 0 \qquad i=1,2,3 \qquad (12)$$

Equation (12) is computed only with the X and Y components of the two vectors in plane (O,X,Y). The system of equations (11) and (12) consists of the direct kinematics of the mechanism.

4. Example

In this section we illustrate with an example the inverse kinematics of the system. Table.1 represents the given values of the platform parameters.

r	θ	h
1m	30^0	2m

Table.1: parameter values

We also give the following values to the platform orientations and the position of origin O_p

$\alpha = 0^0$; $\beta = 0^0$; $\gamma = 30^0$ and $[O_P]_R = \begin{pmatrix} 0 & 0 & \frac{7}{4} \end{pmatrix}$.With these values $R(\alpha, \beta, \gamma)$ becomes

$$R(\alpha, \beta, \gamma) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30^0 & -\sin 30^0 \\ 0 & \sin 30^0 & \cos 30^0 \end{bmatrix} \quad (13)$$

The coordinates of the platform vertices are computed using equations (7),(8) and (9) then the inverse kinematics is partially solved by the following three equations

$$\det\left|\overrightarrow{B_1C_1}, \overrightarrow{B_1D}, \overrightarrow{B_1A_1}\right|_R = \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} & x_1 - \frac{1}{2} \\ \frac{3}{4} & \frac{6+\sqrt{3}}{8} & y_1 + \frac{3}{4} \\ \frac{\sqrt{3}}{4} & \frac{2\sqrt{3}-3}{8} & \frac{\sqrt{3}+1}{4} \end{vmatrix} = 0 \quad (14)$$

$$\det\left|\overrightarrow{B_2C_2}, \overrightarrow{B_2D}, \overrightarrow{B_2A_2}\right|_R = \begin{vmatrix} 1 & -\frac{1}{2} & x_2 - \frac{1}{2} \\ 0 & \frac{\sqrt{3}-6}{8} & y_2 - \frac{3}{2} \\ \frac{\sqrt{3}-1}{4} & \frac{-3}{8} & 0 \end{vmatrix} = 0$$

$$\det\left|\overrightarrow{B_3C_3}, \overrightarrow{B_3D}, \overrightarrow{B_3A_3}\right|_R = \begin{vmatrix} \frac{3}{2} & 1 & x_3 + 1 \\ -\frac{3}{4} & \frac{\sqrt{3}}{8} & y_3 \\ -\frac{\sqrt{3}}{4} & \frac{-3}{8} & \frac{1}{4} \end{vmatrix} = 0 \quad (16)$$

Finally equations (14), (15) and (16) become respectively

$$\left(\frac{3}{16} - \frac{2\sqrt{3}}{8}\right) y_1 - \frac{6}{16} x_1 + \frac{36-\sqrt{3}}{64} = 0 \quad (17)$$

$$-\left(\frac{2+\sqrt{3}}{8}\right) y_2 + \frac{7\sqrt{3}-9}{32} x_2 + \frac{5\sqrt{3}-3}{64} = 0 \quad (18)$$

$$-\left(\frac{4\sqrt{3}-9}{16}\right) y_3 + \frac{3}{8} x_3 + \frac{12+3\sqrt{3}}{64} = 0 \quad (19)$$

Equations (17), (18) and (19) represent the three robot postures. They are line equations in plane (in Fig.5). Point N_1, N_2 and N_3 consist of their intersections. The robot postures are totally defined by choosing the middle of segment $[N_i N_j]$.

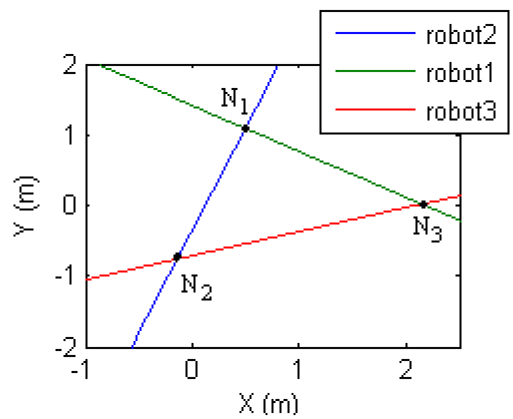

Fig.5: robot postures

5. Conclusion

The quality of a parallel manipulator performance depends mainly on its number of degree-of-freedom and its workspace. The parallel manipulator presented in this paper has these two qualities. It can perform any motion in space. Even if the workspace have not been highlighted in this paper it is easy to find out that it depends mainly on the inclination angle of the inclined planes and their size. It also has the particularity to not have connecting joints. The direct and inverse kinematics of the system have been analyzed.

This present study can be a framework for future works such as the system dynamics, which leads to the coordinated control of the three WMRs.

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