Applying Fuzzy Set Approach into Achieving Quality Improvement for Qualitative Quality Response

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Abstract: - Improving quality is essential for manufacturing organizations competing in the global marketplace. Generally, two forms of quality response are available: a quantitative response and a qualitative response. Most studies primarily focus on quantitative quality improvement. Quantitative quality improvement has rarely been reported. The qualitative response is generally represented in the percentage form, or it is classified into several categories. Employing the ordered categorical descriptions (or subjective estimations) to formulate the performance of the qualitative characteristic is also a meaningful approach. Subjective estimation may provide more information for analyzing the problem. However, subjective estimation cannot be directly defined using the conventional binary set for the uncertainties involved. Experimental design techniques and the Taguchi method are two primary approaches used to improve quality. However, these two approaches are inappropriate when the quality response must be subjectively estimated. Hence, a novel approach based on a fuzzy set is proposed in this study to deal with the quality improvement problem of qualitative quality response. The fuzzy set is a well-known approach for dealing with the uncertainties of ordered categorical response. An illustrative example, based on the uniformity of an ion implantation process in Taiwan’s semiconductor industry, demonstrates the effectiveness of the proposed approach.

Key-Words: - Qualitative quality characteristic, linguistic description, fuzzy set, membership function, parameter optimization

1 Introduction
Fierce market competitiveness has driven manufacturers to enhance their product’s quality. Off-line quality control is a cost-effective means of optimizing the product and process design in support of on-line quality control. Under the off-line quality control approach, design parameters and noise parameters heavily influence the responses of a product or manufacturing process. Designers can control design parameters. However, designers cannot always control noise parameters. A robust design is desired to obtain the optimum design parameter settings for a product or an manufacturing process, in such a manner that the product attains its desired target with minimum variation.

In most products, the qualitative quality response is frequently considered owing to the inherent nature of the quality response. Conventional experimental design techniques [4] can be employed to investigate the relationship between the quantitative quality response and the design dimensions (or noise parameters). Additionally, Taguchi’s method [5], [7] combines experimental design techniques with quality loss considerations, making it an efficient approach for off-line quality control. In some cases, the quality response of interest may be a qualitative (or categorical) quality response. However, optimization of a qualitative quality characteristic has seldom been reported [1], [5], [7]. To optimize the qualitative quality characteristic problem, the qualitative response is generally represented using the percentages, or it is classified into several categories. Discriminant analysis [2] can be performed to recognize the relevant factors when analyzing a qualitative response problem. The accumulation analysis (AA) [7] developed by Taguchi can also be performed to optimize the ordered categorical quality response. In a related work, Nair [6] proposed two scoring schemes (SS) to obtain the optimum factor/level combination by compromising location and dispersion effects of each control factor; Jean and Guo [2] also proposed a weighted probability scoring schemes (WPSS) to obtain an optimal factor/level combination with respect to the minimum mean squared deviation (MSD), where MSD is the combination of the location and dispersion effects.

The qualitative form can also be described linguistically. Using a linguistic description allows
us to obtain more information when analyzing the problem. However, the conventional binary set cannot directly deal with the subjective evaluation for the uncertainties involved. Hence, the linguistic description limits the application of the qualitative form [8, 9]. Fuzzy set is a well-known approach used to manage the uncertainties of the qualitative type or linguistic description of response [9, 10]. This study proposes a novel viewpoint of applying fuzzy sets to optimize qualitative quality response. This study focuses mainly on providing an approach for applying fuzzy sets to improve the quality of the ordered categorical response. The proposed fuzzy set approach consider the differences among the nearby linguistic descriptions and analyze the qualitative quality response more flexibly. In addition, the fuzzy-quality-loss-function (FQLF) proposed in this study may be viewed as a criterion for determining the optimum factor/level settings.

2 Quality improvement techniques for qualitative quality response

Taguchi[7] developed accumulation analysis (AA) to effectively resolve the qualitative (categorical) response problems. Taguchi’s AA consists primarily of four steps: (1) define the corresponding cumulative categories, (2) decide the effects of the factor’s levels, (3) plot the cumulative probabilities, and (4) predict the accumulated probabilities of each class under optimum conditions. Taguchi also recommended using the Omega ($\Omega$) transformation to transfer the accumulated probability of the factor level to a corresponding $\Omega$ value, thereby yielding the predicted accumulated probability of the qualitative response. The optimum factor/level combination can be determined by screening the factor effect diagram. However, Taguchi’s AA might lead to an erroneous result under a subjective assessment while attempting to determine the optimum level combination from the factor effect diagram.

Nair[5] presented two scoring schemes (SS) to recognize the dispersion and location effects. His investigation recommended using the mean square to recognize a prominent effect. The optimal condition of dispersion and location effects can be obtained according to the contribution of both effects of each control factor. The final optimal control factor/level combination is obtained by adjusting between the dispersion effect and location effect.

Jean and Guo[1] proposed a weighted probability scoring scheme (WPSS) to counter the drawbacks of Nair’s SS. Their approach is simpler and more straightforward than Nair’s in that they incorporate the dispersion and location effects into a single mean squared deviation (MSD). In addition, the expected mean square deviation for each class can be obtained according to the definition of the categories. The optimal control factor/level combination is obtained by selecting the minimum mean squared deviation.

3 Fuzzy Set Theory

Uncertainties frequently occur in subjectively estimating the qualitative form of a response. The uncertainties can be well described linguistically. Representing uncertainties in binary form, however, may lead to misunderstandings. Zadeh [8, 9] proposed a method based on fuzzy sets, to formalize linguistic evaluations. Accordingly, employing the fuzzy set concept may prevent these misunderstandings. The linguistic evaluation of characteristic can be quantified by using a membership function (MF) in the fuzzy set. That is, the linguistic description can be defuzzied. The MF can transform the linguistic evaluation into a value in interval [0, 1]. Also, the magnitude of the membership value is the membership degree of a fuzzy term with respect to the set elements in a fuzzy set. The major differences between a fuzzy set and a traditional set are: the traditional set can only take a characteristic function with values of 0 or 1 to describe a set, however, the fuzzy set takes an MF with the interval [0, 1] to describe a set. Therefore, the fuzzy set can be regarded as an expansion of the traditional set.

A universal set $U$ consists of the possible linguistic set element $u$. Initially, the universal set $U$ must be defined. The MFS of $\mu_A, \mu_B, \ldots$ lying in the interval [0, 1] are then determined for various fuzzy terms: $\hat{A}, \hat{B}, \ldots$ with respect to the possible set elements in the universal set $U$. To make the fuzzy set more useful, several operators are developed [8, 9]. The relationship between the linguistic terms and the mathematical form can be represented as follows:

\[
[\text{very}] \quad \hat{A} = \hat{A}^2 = \sum_{u \in U} \frac{\mu_A(u)}{u}
\]

\[
[\text{not}] \quad \hat{A} = \setminus \hat{A} = \sum_{u \in U} \frac{1 - \mu_A(u)}{u}
\]

\[
[\text{or}] \quad \hat{A} \cup \hat{B} = \sum_{u \in U} \max[\mu_A(u), \mu_B(u)]
\]

\[
[\text{and}] \quad \hat{A} \cap \hat{B} = \sum_{u \in U} \min[\mu_A(u), \mu_B(u)]
\]
Where $\tilde{A}$, $\tilde{B}$, ... denote the fuzzy terms; $\mu_\tilde{A}$, $\mu_\tilde{B}$, ... denote the membership function for the fuzzy terms $\tilde{A}$, $\tilde{B}$, ...; $\tilde{A}$ represents the complement set of set $\tilde{A}$; $\tilde{A} \cap \tilde{B}$ and $\tilde{A} \cup \tilde{B}$ stands for the intersection and the union, respectively, of set $\tilde{A}$ and set $\tilde{B}$; and max$[\tilde{A}, \tilde{B}]$ and min$[\tilde{A}, \tilde{B}]$ are meant to take the largest and the smallest set value between set $\tilde{A}$ and set $\tilde{B}$. The linguistic quality characteristic can be described quite flexibly by employing the above rules.

4 Propose Approach

The classifications or categorical types of the qualitative response characteristic must be initially recognized before applying the fuzzy set to optimize the qualitative quality response, that is, the linguistic descriptions of the qualitative response. The definition of the linguistic description or the categories of the qualitative response can be determined using engineering knowledge and experience. To perform off-line quality improvement, a suitable evaluation criterion, such as Taguchi’s signal-to-noise ratio (SN) [6, 7], must be constructed. The quality loss is the most widely used index for evaluating quality performance [3]. The concept of the Taguchi’s loss function is applied to formulate the quality loss of the linguistic data using the MF. Leon et al. [3] demonstrated that maximizing SN is the same as minimizing the quality loss. In this study, we use Leon’s concept to develop a quality loss function based on the fuzzy set. The detailed procedure of the proposed approach is summarized as follows:

Step 1. Define the universal set $U$, the set element $u$, and the target $U_{\text{target}}$ of the qualitative response.

The universal set $U$ is a set consisting of the elements affecting the performance of the qualitative response, that is, $u \in U = \{0, 1, 2, \ldots\}$, where $u = 0, 1, \ldots$ represents the coded values of the possible elements affecting the qualitative response’s performance.

The target value $U_{\text{target}}$ of the qualitative response can be determined with respect to the user’s requirement.

Step 2. Determine the MF of the elementary fuzzy terms.

The elementary fuzzy terms can be defined as the terms constructing other possible fuzzy terms. For example, the uniformity of the ion implanting can be described as Very Good, Good, Not good and not bad, Bad, Very Bad. The Good and Bad can be chosen as the elementary fuzzy terms in this example. According to the set element $u$, the MFs of the elementary fuzzy terms can be determined.

Step 3. Construct the MF for evaluating the performance of each category or each classification of the qualitative response.

Each class or each classification of the qualitative response can be represented by the elementary fuzzy terms. Thereby, the MF of each category or each classification will then be determined according to the MF of the elementary fuzzy terms. Equations (1)–(4) can be employed to build the MF for evaluating the performance of each class or each classification of the qualitative response.

Step 4. Perform the designed experiments and accumulate the experimental data.

The FQLF value is calculated based on the quality loss of each of the class or the classifications of the qualitative response. The FQLF is obtained by the following equation:

$$FQLF = \sum_{u \in U} \left[ (u - u_{\text{target}})^2 \times \mu(u) \right]$$

where $u_{\text{target}}$ denotes the target value or the desired value of the qualitative response.

Step 5. Compute the fuzzy-quality-loss-function (FQLF) for each experimental run.

The FQLF value of each factor’s level can be computed by taking the average FQLF value with respect to the experimental runs involving the corresponding factor level. Then, plotting the FQLF value of each factor’s level on the diagram. The optimum factor/level settings can be determined by selecting the settings with the minimum FQLF value on the response diagram. The following example illustrates the computation of FQLF. Suppose a designed experiment is given as follows:

If the count of the experimental run for level-1 of factor A is 2, the FQLF value of the level-1 for factor A can be computed as $7(=(5+9)/2)$. If the count of the
Step 7. Perform the confirmed experiments and compute the improvement contribution ratio (ICR) of the quality loss for the quality response.

The ICR of the quality loss is designed as:

\[
ICR = \frac{LOSS_{Initial\ setting} - LOSS_{Optimal\ setting}}{LOSS_{Optimal\ setting}} \times 100\%
\]

The larger the ICR value, the better is the quality improvement. An ICR value is pre-determined as the requirement of the lowest acceptance criteria. If the ICR value cannot satisfy the requirement, go back to Step 2 to re-define the MF and repeat the procedure until the ICR value achieves the required value.

## 5 Illustrative Example

A uniformity optimization example is illustrated here; the example is taken from the ion implantation process of a Taiwanese manufacturer of integrated circuits (IC). The quality response of interest is the uniformity of the ion implantation on the sensitivity area. From engineering knowledge, the uniformity is frequently quantified from the quality characteristic, e.g. the uniformity can be referred as [very]good, [good], [not][good][and][not][bad], [bad], [very][bad]. From brainstorming discussion with process engineers, the elementary fuzzy terms are chosen as the elementary fuzzy terms. Through a brainstorming discussion with process engineers, the MFs of the elementary fuzzy terms are determined. Accordingly, the MFs of both elementary fuzzy terms are defined as

\[
\tilde{A} = [\text{Good}] = \left\{(0.1, 0.3, 0.5), (0.7, 0.9, 1.0)\right\}
\]

and

\[
\tilde{B} = [\text{Bad}] = \left\{(0.0, 0.7, 0.6), (0.4, 0.2, 0.1)\right\}.
\]

In Step 3, the MFs of the linguistic description of the five categories are then determined according to the elementary fuzzy terms \(\tilde{A}\) and \(\tilde{B}\). Table 2 lists the membership value of each category corresponding to each element in the universal set \(U\). The detailed computation procedures for each category are given as follows.

<table>
<thead>
<tr>
<th>Category</th>
<th>Linguistic description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>[very]good</td>
</tr>
<tr>
<td>II</td>
<td>[good]</td>
</tr>
<tr>
<td>III</td>
<td>[not][good][and][not][bad]</td>
</tr>
<tr>
<td>IV</td>
<td>[bad]</td>
</tr>
<tr>
<td>V</td>
<td>[very][bad]</td>
</tr>
</tbody>
</table>

Thirty-six sensitivity areas are assigned on each wafer in the ion implantation process. The engineer expects that the counts lying in the class I–V are from the largest to the smallest, that is, the ideal result of the counts lying in each category is (36, 0, 0, 0, 0). Six control factors (A–F) are considered in this process (they can not be clearly described here for reasons of business secrecy). Among these control factors, only one factor (factor A) has two levels; the other factors have three levels. The initial parameter settings are A1B1C3D3E1F2. The engineer expects to achieve an improvement in quality of 50% or more, the larger the better. The example is analyzed using the proposed approach. In Step 1, a universal set \(U\) is initially constructed from engineering experience: \(u \in U = \{0, 1, 2, 3, 4, 5\}\); where \(u\) represents the different defect grade, and \(u = 0\) denotes the worst defect grade and \(u = 5\) denotes the best defect grade. The best grade of defect represents the best uniformity of ion implanting. Hence, the target value, or the desired value, of the linguistic description is equal to 5 from the definition. In Step 2, we decide the elementary fuzzy terms. There are five categories of the qualitative characteristic. The terms of Good and Bad can be used to build the other fuzzy terms from the quality characteristic, therefore, they are chosen as the elementary fuzzy terms. Through a brainstorming discussion with process engineers, the MFs of the elementary fuzzy terms are determined.
To cut the experimental time and cost, an L18 orthogonal array (OA) is used. The FQLF of each run is then computed. Table 3 lists the results of FQLF values. The FQLF value of the factor/level effect can be obtained, and the results are summarized in Table 4. The response diagram of factor/level effect is given in Figure 1.

Examining the response diagram of factor/level effect, the optimum parameter setting is obtained by choosing the factor level with minimum FQLF value. The optimum setting is A2B1C2D3E1F1. Confirmed experiments are performed to verify the effectiveness of the found optimum parameter condition. Results of the confirmed experiments for both of the initial settings and the optimum settings are listed in Table 5. The average FQLF value of the initial settings and the optimum settings are found to be 15.19 and 7.25, respectively. The improvement contribution (ICR value) is about 52.3% \(=\frac{(15.19 - 7.25)}{15.19} \times 100\%). This is quite a large improvement. In addition, the engineering requirement (the improvement contribution ratio \(\geq 50\%\)) is achieved so no other improvement activities are needed.

This case can be also viewed as an ordered categorical problem in Taguchi’s experiment and the Taguchi’s AA can be employed to perform the analysis. According to Taguchi’s AA, the accumulated probability of each category must be computed first. Next, the accumulated probability of various factor levels will be obtained. Table 6 lists the accumulated count and the accumulated probability (in parenthesis) of each category denoted by \(<-\) (e.g. \(<III>\) represents the summation of the probabilities for I, II and III). The accumulated probabilities of each factor’s levels are listed in Table 7. From this table 7, the optimum factor/level combination can be obtained as A2B1C3D2E1F1.

Figure 1. The response diagram of the factor effect.

This illustrative example is re-analyzed by employing Jean and Guo’s WPSS method. According to their method, two scores \([1]\) indicating the location and dispersion effects of each experimental run must be initially computed. Herein, the dispersion effect is regarded as the discrepancy between the location effect and the target of each category. In addition, the weight value of each category must be assigned at the same time, and it is regarded as the location effect in their method. There are five categories in this illustrative example, the location score of each category is given as 5, 4, 3, 2 and 1 for I to V, respectively. In this example, the target of the weight values can be given as \((5, 0, 0, 0, 0)\) for categories (I, II, III, IV, V), respectively. The formulas for calculating the location score,
dispersion score and the performance measure are given as follows:

\[ W_n = \sum_{i=1}^{n} w_i p_i, \quad n = 1, 2, \ldots, 18 \]

The location score:

\[ d_i = \sum_{i=1}^{n} \left( w_i p_i - T_{\text{target}} \right)^2, \quad n = 1, 2, \ldots, 18 \]

The performance measure:

\[ E(\text{MSD}) = \frac{1}{W_i} \left( 1 + 3 \frac{d_i}{W_i} \right) \]

The E(MSD) results of factor/levels are summarized in Table 8. From these results, the optimum factor/level combination can be determined by selecting the minimum E(MSD) value. Therefore, the optimum parameter setting is A2B1C2D3E2F1.

<table>
<thead>
<tr>
<th>Factor/level Combination</th>
<th>F1</th>
<th>F2</th>
<th>F3</th>
<th>F4</th>
<th>F5</th>
<th>F6</th>
<th>FQLF</th>
</tr>
</thead>
<tbody>
<tr>
<td>The optimum settings of Taguchi's AA, A0B0C0D0E0F0</td>
<td>32</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7.75%</td>
</tr>
<tr>
<td>The optimum settings of Jean and Guo's WPSS, A0B0C0D0E0F0</td>
<td>32</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7.675%</td>
</tr>
<tr>
<td>The optimum settings of Jean and Guo's WPSS, A0B0C0D1E1F1</td>
<td>32</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7.3%</td>
</tr>
</tbody>
</table>

* the bold face numbers in the Table are the optimum factor/level combination.

To make the comparisons, we performed the confirmed experiments of the optimum condition found by Taguchi’s AA and Jean and Guo’s WPSS. Table 12 lists the results of the confirmed experiments employing Taguchi’s AA, Jean and Guo’s WPSS, and the proposed approach. According to Table 12, the optimum settings are different for each of the above three methods, and the ICR values of Taguchi’s AA, Jean and Guo’s WPSS, and the proposed approach are approximately 49.5% (=|(15.19-7.675)/15.19| × 100%), 51.7% (=|(15.19-7.33)/15.19| × 100%) and 52.3%, respectively. Although these improvement ratios obtained using all three methods are rather close, the proposed approach has the lowest FQLF. In addition, the proposed approach is more flexible than the other two methods, because it takes the linguistic description or the subjective estimation into account.

### 6 Concluding Remarks

Most studies of off-line quality control have largely focused on the optimization of the quantitative quality response. The qualitative characteristic has rarely been reported. The qualitative form can be described by means of linguistic description. Linguistic description can provide more information to analyze the problem. However, the conventional binary set cannot directly deal with subjective evaluation for the uncertainties involved, and therefore, the conventional experimental design techniques and the Taguchi method cannot be directly applied. Fuzzy set is a well-known approach to managing the uncertainties of the qualitative type or linguistic description [8, 9]. This study present a novel approach based on fuzzy set techniques, to improve quality when the quality response needs to be subjectively estimated. From experience demonstrating an illustrative example, the following two concluding remarks can be made:

1. The proposed approach can effectively describe the difference among the nearby linguistic description when we rate the qualitative quality response.
2. Subjective engineering evaluation for evaluating the qualitative quality response using linguistic description can be included in the proposed approach. Hence, the decision selecting of the optimum parameter setting will be more precise.

### References:


