# Phase field modeling of the conversion from methane hydrate to carbon dioxide hydrate

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*Abstract:* - A ternary phase field theory has been developed to study kinetics of transformation of methane hydrate into  $CO_2$  hydrate is investigated in the presence of fluid  $CO_2$  under conditions characteristic to underwater reservoirs. We present illustrative simulations.

Key-Words: - methane hydrate; CO<sub>2</sub> hydrate; transformation kinetics; phase field modelling

## **1** Introduction

Natural gas hydrates are crystalline solids built of water cages containing gas molecules (mostly methane). These substances can be found in abundance in the Arctic regions and in marine sediments. The methane hydrate is stable at water depths larger than 300 m, and forms sediment layers of hundreds of meters thick. According to conservative estimates, the worldwide amount of carbon in gas hydrates is more twice of the carbon in fossil fuels [1-5]. Gas hydrates are regarded as a new abundant energy source, whose exploitation may become economic with increasing oil prices and after developing the appropriate technologies [3-5]. Under conditions typical to underwater reservoirs the CO<sub>2</sub> hydrate is more stable than the methane hydrate. This raises the possibility that via pumping industrial CO2 into hydrate fields one can gain methane, a process that can make CO<sub>2</sub> deposition economic. It is, however, worth mentioning that methane is about 20 times more efficient green house gas than CO<sub>2</sub>. Therefore, technologies that handle the released methane safely need to be developed. The methane stored in underwater hydrate reservoirs represents itself a natural climatic hazard: if released even a small fraction could cause serious climatic changes [3-5]. Therefore, it is a basic interest of humanity to understand details of methane balance of Earth including the formation and dissolution of gas hydrates [3-5]. Summarizing, gas hydrate reactions (formation and dissolution) are interesting for the following reasons [4]:

(1) Gas hydrates contain methane in abundance that could serve in the

future as a new energy source provided that its economic exploitation is solved.

(2) Gas hydrate reservoirs, as methane source and sink, represent an essential part of the methane balance of the Earth, and may play an essential role in climatic changes.

In accordance with these, we have investigated the kinetics of gas hydrate reactions since years. Our research team has recently developed a model polycrystalline phase field of solidification [6-9] and adapted it for gas hydrate reactions. The kinetic data evaluated from molecular dynamics calculations are used in phase field simulation describing the time and spatial evolution of gas hydrate reactions [10-14]. These extremely computation intensive simulations have been performed on computer clusters consisting of 60 and 120 processors, respectively. In this work present a ternary field phase theory and address the transformation of methane hydrate into CO<sub>2</sub> hydrate in the presence of fluid CO<sub>2</sub>.

# 2 Phase Field Model for Hydrate Conversion

The local state of matter is characterized by four fields: the phase field  $\phi(\mathbf{r})$  that monitors the solid liquid transition, and three concentrations

fields  $\{c_1(\mathbf{r}), c_2(\mathbf{r}), c_3(\mathbf{r})\}$  that specify the local composition. The free energy of the inhomogeneous system is given by the integral

$$F = \int d^{3}r \begin{cases} \frac{\varepsilon_{\phi}^{2}T}{2} |\nabla\phi|^{2} + wTg(\phi) + \\ [1 - p(\phi)]f_{s}(c_{1}, c_{2}, c_{3}, T) + \\ p(\phi)f_{L}(c_{1}, c_{2}, c_{3}, T) \end{cases}$$

where  $c_3 = 1 - c_1 - c_2$ , and the coefficient of the square gradient term  $\varepsilon_{\phi}^2$  and the free energy scale *w* can be related to the free energy and thickness of the hydrate-fluid interface in equilibrium, while the double-well, interpolation and anisotropy functions have the form shown below

$$g(\phi) = \frac{1}{4}\phi^2(1-\phi)^2$$
,  $p(\phi) = \phi^3(10-15\phi+6\phi^2)$ .

Assuming relaxation dynamics, the respective equations of motions are as follows:

$$\begin{split} \dot{\phi} &= -M_{\phi} \frac{\delta F}{\delta \phi} = M_{\phi} \left\{ \nabla \left( \frac{\partial f}{\partial \nabla \phi} \right) - \frac{\partial f}{\partial \phi} \right\} \\ \dot{c}_{1} &= \nabla M_{c,1} \nabla \frac{\delta F}{\delta c_{1}} = \nabla \left\{ M_{c,1}(c_{1},c_{2}) \nabla \left[ \left( \frac{\partial f}{\partial c_{1}} \right) - \nabla \left( \frac{\partial f}{\partial \nabla c_{1}} \right) \right] \right\} \\ \dot{c}_{2} &= \nabla M_{c,2} \nabla \frac{\delta F}{\delta c_{2}} = \nabla \left\{ M_{c,2}(c_{1},c_{2}) \nabla \left[ \left( \frac{\partial f}{\partial c_{2}} \right) - \nabla \left( \frac{\partial f}{\partial \nabla c_{2}} \right) \right] \right\} \end{split}$$

where  $M_{\phi}$  and  $M_{c,j}$  are the respective mobilities determining the time scale of the evolution of the system. These equations are solved simultaneously in a dimensionless form using an explicit finite difference scheme and periodic boundary conditions. Here the same  $M_{c,j} = D$ , where D is the diffusion coefficient has been used for all the concentration fields.

The thermodynamic data for the CO<sub>2</sub> – H<sub>2</sub>O – CH<sub>4</sub> ternary molecular system have been taken from Svandal et.al.[15, 16, 17]. Typical free energy surfaces for the hydrate and fluid phases are displayed in Figure 1. The model parameters  $\varepsilon_{\phi}^2$  and w have been chosen so that they correspond to an interfacial free energy of  $\gamma = 30$  mJ/m<sup>2</sup> and an interface thickness of d = 0.85 nm. The spatial step was taken as  $\Delta x = 0.1$  nm, and the time step as  $\Delta t = 5 \times 10^{-12}$  s. The diffusion coefficient has been assumed to be  $D = 10^{-9}$  m<sup>2</sup>/s in the liquid phase, and about 1000 times smaller in the solid hydrate. The size of the rectangular simulation grid has been  $500 \times 250$ . Two slabs of CO<sub>2</sub> hydrate have been placed into the

simulation window: One to the left, and another to the right. The gap in between has been filled in by a dominantly  $CO_2$  fluid.

#### **3** Discussion and conclusions

The conversion of methane hydrate into CO<sub>2</sub> hydrate in the presence of  $CO_2$  solution is shown in Figure 2. The left side of the simulation window is displayed. In the first 0.1 ns the initially sharp concentration and phase field profiles relax towards the diffuse interfaces analogous to those seen in modelcular dynamics and phase field simulations in binary systems [14]. This leads to a "softening" of the interface layer and a fractional melting of the methane hydrate at its surface. The CO<sub>2</sub> molecules diffuse into hydrate, and replace the methane molecules, which move into the fluid, accumulating in the blue peak on the right hand side. Note that the methane concentration in the methane hydrate is larger ( $\sim 0.14$ ) than the CO<sub>2</sub> concentration in the  $CO_2$  hydrate (~0.11). As time proceeds, the  $CO_2$ hydrate layer thickens, albeit with a continuously decreasing rate. The average thickening rate, taken for the duration of the simulation, is rather high  $\sim 0.1$  m/s. This and the observation that the conversion rate decelerates are consistent with the diffusion controlled process assumed here. Molecular dynamics simulations might, however, be necessary to verify that indeed this is the dominant transformation mechanism. Work is underway to explore how the present results depend on the materials properties and the model parameters.



Figure 1. Conversion of methane-hydrate to  $CO_2$ -hydrate at 1 C and 6.2 GPa pressure. Time evolution of the phase field (black) is shown together with that of the  $CO_2$  concentration (red), and the  $CH_4$  concentration (blue). On the vertical axis, a common scale applies for the phase field and the two concentrations. The five curves correspond to times t = 0.1, 0.5, 1, 1.5 and 2.7 ns, respectively. Numbers 1 and 5 denote the curves corresponding to the first and last instances.

### 4 Summary

We have developed a phase field model for methane hydrate conversion into  $CO_2$  hydrate in the presence of  $CO_2$  fluid, and presented illustrative simulations under conditions typical to underwater reservoirs. Systematic investigation of the transformation kinetics is underway.

#### References:

[1] Sloan E.D. Jr, *Nature*, 2003, 426, 353.

[2] Milich L., *Global Environmental Change – Human and Policy Dimensions*, 1999, 9, 179.

[3] Kvenvolden K. A., Potential effects of gas hydrate on human welfare, *Proc. Natl. Acad. Sci. USA*, 1999, 96, 3420.
[4] Gas hydrate homepage of the US Geological Surwey: http://marine.usgs.gov/fact-sheets/gas-hydrates/title.html

[5] Brewer P., Charter R., Holder G., Holdtitch S., Johnson A., Kastner M., Mahajan D., Parrish W., Report of the Hydrate Advisory Commettee on Methane Hyrdate Issues and Opportunities Including Assessment of Uncertainty of the Impact of Methane Hydrate on Global Climate Changes. *Report for US Congress*, 2002; http://www.netl.doe.gov/technologies/oil-

gas/publications/Hydrates/pdf/CongressReport.pdf

[6] Gránásy L., Börzsönyi T., Pusztai T., Nucleation and bulk crystallization in binary phase field theory, *Phys. Rev. Lett.*, 2002, 88, Art. no. 206105.

[7] Gránásy L., Pusztai T., Warren J. A., Douglas J. F., Börzsönyi T., Ferreiro V., Growth of "dizzy dendrites" in a random field of foreign particles, Nature Materials **2**, 92 (2003).

[8] L. Gránásy, T. Pusztai, T. Börzsönyi, J. A. Warren, J. F. Douglas, *A general mechanism of polycrystalline growth*. Nature Materials **3**, 645 (2004).

[9] L. Gránásy, T. Pusztai, J. A. Warren, *Modelling polycrystalline solidification using phase field theory*. J. Phys.: Condens. Matter. **16**, R1205 (2004).

[10] B. Kvamme, A. Graue, E. Aspenes, T. Kuznetsova, L. Gránásy, G. Tóth, T. Pusztai, Kinetics of solid hydrate formation by carbon dioxide: Phase field theory of hydrate nucleation and magnetic resonance imaging. Phys. Chem. Chem. Phys. **6**, 2327-2334 (2004).

[11] L. Gránásy, T. Pusztai, G. Tegze, T. Kuznetsova, B. Kvamme, Towards a full dynamic model of  $CO_2$  hydrate formation in aqueous solutions: Phase field theory of nucleation and growth. "Advances in the Study of Gas Hydrates", eds. C.E. Taylor, J.T. Kwan (Springer, Berlin, 2004), Chap. 1.

[12] A. Svandal, B. Kvamme L. Gránásy, T. Pusztai, *The influence of diffusion on hydrate growth*. J. Phase Equilib. Diff. **26**, 534-538 (2005).

[13] A. Svandal, B. Kvamme L. Gránásy, T. Pusztai, T. Buanes, J. Hove, *The phase field theory applied to CO*<sub>2</sub> and  $CH_4$  hydrate. J. Cryst. Growth **287**, 486-490 (2006).

[14] G. Tegze, T. Pusztai, G. Tóth, L. Gránásy, A. Svandal, T. Buanes, T. Kuznetsova, B. Kvamme, Multi-scale approach to CO<sub>2</sub>-hydrate formation in aqueous solution: Phase field theory and molecular dynamics. Nucleation and growth. J. Chem. Phys., 2006, 124, Art. no. 234710.

[15] Svandal A., Kuznetsova T., Kvamme B., Thermodynamic properties and phase transitions in the  $H_2O/CO_2/CH_4$  system, Phys. Chem. Chem. Phys., 2006, 8,1707.

[16] Svandal, A., Kuznetsova, T., Kvamme, B., *Thermodynamic properties, interfacial structures and phase transtions in the*  $H_2O/CO_2/CH_4$  *system*, Fluid Phase Equilibria, 2006, in press

[17] B. Kvamme, H. Tanaka, J. Phys. Chem. **99** (1995) 7114-7119