

# ESSENTIAL PRINCIPLE OF DEBRIS FLOW DYNAMICS

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*Abstract:* As one of dominant geological disaster and geomorphologic processes in mountains, debris flow dynamics is essential in designing of structures against debris flow disaster, risk appraisal and mitigation of the disaster. Essential principles of debris flow dynamics are composed of flow velocity, impact and abrasion of debris flow. Based on two-phase theory, both continuum and momentum equations every phase are established. Further, method to calculate velocity of two-phase debris flow is established by introducing parameters  $M$  and  $G_0$ . Impact forces and duration can be identified by impact formality on banks of debris flow valley or structures. According to abrasion forces of slurry and particles in debris flow, methods to calculate abrasion thickness and velocity are provided primitively. These principles have applied effectively in engineering since 2000.

*Key words:* Debris flow dynamics; Flow velocity; Impact; Abrasion

## 1 INTRODUCTION

As one of representative disasters long highways in China, debris flow along highway have caused economic loss at least 50,000 million RMB and killed tens of people every year in western regions of China since 1990(2004a, 2004b). Hitherto, thousands of researchers are engaged in studies and control of debris flow, and have made outstanding contributions. For example, Bagnold (1954, 1956) made experiments on a gravity-free dispersion of large solid spheres in Newtonian fluid under shearing and obtained moving law for particle under gravity. Hampton (1979) put a method forward to calculate buoyancy in debris flow. Major et al. (1992) approached debris flow rheology through fine grained slurries experiment and established rheology equations. Richard (1997) made deep research on physics of debris flow and established some important theories such as temperature variation of particles in debris flow, unsteady surge or series surges of debris flow and pore pressure fluid in deposit. O'Brien et al.(1993) and Ghilardi et al.(2001) made some studies deeply on simulation of propagation and deposition of debris flow by numeric methods such as FEM and FLO-2D. Takahashi (1980) and Ning Chien (1989) established kinematics equations of particle in water and viscous liquid through testing in open channel.

However, due to special features existed in studies and controlling of debris flow along highways and/or railways, dynamics of debris flow must essential. The dynamics must be focus on flow velocity, impact and abrasion to bank or bed of debris flow or control structures. Based on these ideas, Chen HK et al. (2004c, 2006a, 2006b) have established one general outlining dynamics of debris flow.

The purpose of this work is to show the essential principle of debris flow dynamics.

## 2 VELOCITY OF TWO-PHASES DEBRIS FLOW

Generally, variations exist obviously in minerals and size of solid such as particles and rock block in real debris flow and the size dominant the dynamic features of debris flow. In real debris flow, difficulty in calculating velocity of debris flow result from by solids with various diameters in debris flow. According to volume is the same, solids with various diameters in real debris flow is simplified as equivalent particles with the same diameter. In dynamics of debris flow, two-phase fluid is emphasized from the beginning.

### 2.1 Assumptions

The first, debris flow belongs to one dimensional flow in x-direction.

The second, in mobilization of debris flow, all external materials into debris flow are ignored.

The third, interactions such as that between particles, inner action of slurry and interaction between particle and slurry are considered.

### 2.2 Governing equations

Take control volume of two-phase debris flow in motion as an object to analyze, establish conservation equations of solid phase and liquid phase respectively.

(1) Continuum equation

Continuum equations are as follows respectively.

$$\frac{\partial}{\partial t}[\rho_s \alpha] + \nabla \cdot [\rho_s \alpha v_s] = 0 \tag{1}$$

$$\frac{\partial}{\partial t}[\rho_f (1-\alpha)] + \nabla \cdot [\rho_f (1-\alpha) v_f] = 0 \tag{2}$$

(2) Momentum equation

Vector equations in 3D momentum equation of solid phase and liquid phase are provided as follows respectively.

$$\rho_s \left( \frac{\partial v_s}{\partial t} + v_s \cdot \nabla v_s \right) = b_s + f_s - \nabla P_s \tag{3}$$

$$\rho_f \left( \frac{\partial v_f}{\partial t} + v_f \cdot \nabla v_f \right) = b_f + f_f - \nabla P_f \tag{4}$$

Fluid resistance every phase, apparent mass force derived from difference of velocity and acceleration, and interaction between particles.

In 1D flow status, equation.(3) and equation.(4) are simplified as follows.

$$\rho_s \left( \frac{\partial v_s}{\partial t} + v_s \frac{\partial v_s}{\partial x} \right) = b_s + f_s - \frac{\partial P_s}{\partial x} \tag{5}$$

$$\rho_f \left( \frac{\partial v_f}{\partial t} + v_f \frac{\partial v_f}{\partial x} \right) = b_f + f_f - \frac{\partial P_f}{\partial x} \tag{6}$$

(3) Variables calculation

Based on Chen HK et al.(2006a), four variables foregoing are cited as follows.

$$b_f = \rho_f g \cos \theta \tag{7}$$

$$b_s = \alpha(\rho_s - \rho_f)g \cos \theta \tag{8}$$

$$f_s = \frac{3\alpha(1-2\alpha)}{2d_e} P + 0.041\rho_s(\lambda d_e)^2 \left( \frac{dv_s}{dy} \right)^2 \tag{9}$$

$$f_f = \tau_B + \mu b + d_0 - \frac{3\alpha(1-2\alpha)}{2d_e} P \tag{10}$$

(4) Solution

Substitute  $b_s$  and  $f_s$  into equation.(5), and ignore variation of particles along depth of debris flow, then

$$\rho_s v_s \frac{dv_s}{dx} = \alpha(\rho_s - \rho_f)g \cos \theta + \frac{3\alpha(1-2\alpha)}{2d_e} P - \alpha \frac{dP}{dx} \tag{11}$$

And substitute  $b_f$  and  $f_f$  into equation.(6), then

$$\rho_f v_f \frac{dv_f}{dx} = \rho_f g \cos \theta + (\tau_B + \mu b) d_0 - \frac{3\alpha(1-2\alpha)}{2d_e} P - (1-\alpha) \frac{dP}{dx} \tag{12}$$

Combine equation.(11) with equation.(12), so

$$(1-\alpha) \rho_s v_s^2 - \alpha \rho_f v_f^2 = Ax \tag{13}$$

Where, substitution A is

$$A = 2\{\alpha[(1-\alpha)\rho_s - (2-\alpha)\rho_f]g \cos \theta - \alpha(\tau_B + \mu b)d_0 + \frac{3\alpha(1-2\alpha)}{2d_e} P\} \tag{14}$$

Let

$$v_f = M v_s \tag{15}$$

Where,  $M$  is the function of parameter  $\alpha$ , and is groped through many testing in laboratory and real-time field observation (Chen HK, et al., 2006a).

$$M = \frac{1}{(1-\alpha)^4} \tag{16}$$

So

$$v_s = \sqrt{\frac{Ax}{(1-\alpha)\rho_s - M^2 \rho_f}} \tag{17}$$

Define velocity of solid phase calculated in formula (17) as theoretical velocity of solid phase, and both the velocity and real velocity differ certainly in quantity through observation data of tens of debris flow in western China (Chen HK, et al., 2006a). Define the difference as divergent coefficient  $G_0$  of solid phase as follows.

$$G_0 = \frac{V_s}{v_s} \tag{18}$$

Divergent coefficient  $G_0$  of solid phase is function of ratio of solid phase in debris flow ( $\alpha$ ), length of pass area of debris flow valley (L) and thickness of debris flow mass (h), i.e.

$$G_0 = G_0(\alpha, L, h) \tag{19}$$

Chen HK et al. (2004a) obtain overall divergent coefficient  $G_0$  through real-time field observation and analyses in detail to twenty or so of debris flow in western China. For example, when  $\alpha = 0.05$ ,  $G_0$  is shown in Figure 1. Further, the real velocity of solid phase in debris flow can be calculated in the next formula.

$$\bar{v}_s = \frac{V_s}{G_0} \tag{20}$$

Instead of theoretical velocity in formula (15) in real velocity of solid phase, real velocity  $\bar{v}_f$  of slurry in debris flow obtains.

$$\bar{v}_f = M \bar{v}_s \tag{21}$$

Take Pingchuan debris flow locating at highway from Xichang city to Muli county and Niuniuba debris flow in Meigu river basin in Liangshan YI autonomous region of Sichuan province, and K631 debris flow locating at the Tianshan highway in Xinjiang Urgur autonomous region as examples, whose average velocity through observation in real-time field are 9.70m/s, 11.56m/s and 11.60m/s respectively (Chen HK, et al, 2004a). Calculation velocities are fitting to that in observation whose error are less than 10 percent (Chen HK, et al., 2004a; 2006a). However, there is the limit that velocities using present method are fixed in mouth of pass area of debris flow valley.

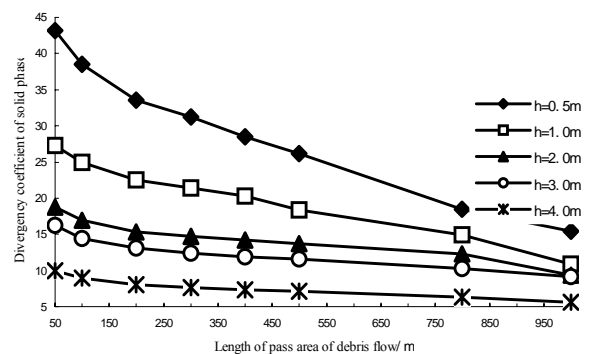


Fig.1 Curves of divergent coefficient of solid phase ( $\alpha = 0.05$ )

### 3 IMPACTION OF DEBRIS FLOW

In accordance of investigations in real time field since 1990s in western regions of China, Chen HK et al (2006b) discover one effective method to identify impact force and impact duration of debris flow through impact formality. Let formality be a semi-elliptical spheroid, length of long axis and short axis and depth of the spheroid are assigned as  $a_0$ ,  $b_0$  and  $c_0$  respectively, then area of mouth plane  $A_k$  and area of interior surface  $S_k$  of an impact formality are calculated in following formulas.

$$A_k = \frac{1}{4}\pi a_0 b_0 \quad (22)$$

$$S_k = \pi(a_0 + b_0)c_0 \quad (23)$$

Impact forces of debris flow are composed of impact both slurry and particle in debris flow mass.

Take unit volume slurry of debris flow as control volume, so maximum impact area on the surface of structure or banks of debris flow valley is about 3.9 square meters. Let  $a_f$  represents kinematical acceleration of debris flow, then mass  $m_f$  of the control volume is  $\rho_f$ . Based on Newtonian's law, impact force  $q_f$  of debris flow slurry on unit impact area obtains in the following formula.

$$q_f = 0.2564\rho_f a_f \quad (24)$$

To an equivalent particle, whose mass  $m_s$  is calculated in the next formula.

$$m_s = \frac{\pi}{6}d_e^3\rho_s \quad (25)$$

Impact  $P_{s0}$  force of the particle is established through Newtonian's law.

$$P_{s0} = m_s a_s \quad (26)$$

Further, numbers  $n_s$  of solid particles on unit impact area can be amounted in the next formula.

$$n_s = \frac{4\alpha}{\pi(1+\alpha)d_e^2} \quad (27)$$

Then impact force  $\bar{P}_s$  of solid phase on unit impact area is identified in the next formula.

$$\bar{P}_s = n_s P_{s0} \quad (28)$$

Combine equation (26) with equation (30), impact force of debris flow on surface of structures or banks of debris flow valley is expressed as follows.

$$P = K_0(q_f + P_s) \quad (29)$$

The bigger of viscosity of debris flow, the greater of reveal coefficient. Methods to calculate  $a_f$  and  $a_s$  are established by Chen HK et al.(2004c).

Further, impact duration of debris flow is expressed as follows.

$$T = \frac{S_k \tau_k}{K_r P A_k \sin \theta} \quad (30)$$

## 4 ABRASION DYNAMICS OF DEBRIS FLOW

According to mechanic equilibrium of control volume of debris flow slurry, and assume the velocity of debris flow in mobilization is in uniform distribution, the

abrasion force of debris flow slurry of unit length along the flow direction is obtained in the next formula.

$$R_f = A \sin \theta + B \cos \theta + C v_f^2 + Da \quad (31)$$

Where

$$A = bh \gamma_c$$

$$B = -bh \gamma_c \tan \varphi_f$$

$$C = 0.02K_0 \gamma_c$$

$$D = -\frac{bh \gamma_c}{g}$$

$$\gamma_c = [\alpha \rho_s + (1 - \alpha) \rho_f] g$$

Shearing force  $R_s$  on unit length surface of structures or bed of debris flow valley by particle is calculated in the following formula.

$$R_s = \frac{b\alpha^2}{d_e^2} [(G + W_0) \sin \theta + P - \frac{G}{g} a] \quad (32)$$

Using dimensionless analysis, abrasion amount can be identified in the following formula.

$$\delta = \zeta t P v \quad (33)$$

Abrasion thickness and velocity can be calculated using the following formulas respectively.

$$e = \frac{\delta}{b} \quad (34)$$

$$v_0 = \frac{e}{t} \quad (35)$$

## 5 CONCLUSIONS AND DISCUSSIONS

The first, dynamics of debris flow is essential in design of structures against debris flow disaster, meanwhile, the evidences in risk appraisal and mitigation of the disaster.

The second, essential principles of debris flow dynamics include three aspects, i.e. flow velocity of debris flow, impact force and abrasion of debris flow. Further, serial of formulas about debris flow dynamics are established in the paper.

The third, velocity of debris flow can be identified by observations and/or investigations in real time field, and theoretical calculation using empirical formulas or semi-empirical formulas. However, hitherto, there isn't a general method applying for any regions or debris flows with various viscosity. The method to calculate velocities of two-phase debris flow presented in the paper is only an initial and valuable try to establish a general method to calculate flow velocity of debris flow.

The forth, impact forces of debris flow provide the loads to design structures such as drainage trough, debris flow tunnel, converge trough, and dams etc. Methods to identify the force are various, for example, observations in-situ and theoretical calculations. However, due to the random for debris flow to break out, the complex constitute of debris flow such as solids in debris flow with various diameters and shapes, and surging of debris flow, which result in difficulty to obtain impact force of debris flow. The authors establish a method to calculate impact force of two-phase debris flow by impact

formality, which must be an valuable contribution in the aspect.

The fifth, as a forgotten topic in realm of debris flow research, abrasion of debris flow must be emphasized. Abrasion of debris flow directly dominates the effective application life of control structures. In the paper, method to calculate abrasive forces of slurry and solid phase in debris flow are established, further, abrasion thickness and abrasion velocity can be calculated too. Of course, the work is primitive at present, more and more works must be operated in future.

The principles providing in the paper have applied in 60 engineering projects against debris flow disasters along highways in western regions of China since 2000.

## APPENDIX: NOTATION

$\rho_s$  is the average density of solid particle  $\text{g.cm}^{-3}$   
 $\rho_f$  is the liquid slurry respectively,  $\text{g.cm}^{-3}$   
 $\rho_c$  is the average density of debris flow,  $\text{g.cm}^{-3}$   
 $\alpha$  is the average volume concentration of solid phase in debris flow  
 $v$  is the average velocity of debris flow,  $\text{m.s}^{-1}$   
 $v_s$  is the average velocities of solid phase along moving direction,  $\text{m.s}^{-1}$   
 $v_f$  is the average velocities of liquid phase along moving direction,  $\text{m.s}^{-1}$   
 $\bar{v}_s$  is the real velocity of solid phase in debris flow,  $\text{m.s}^{-1}$   
 $\bar{v}_f$  is the real velocity of slurry in debris flow,  $\text{m.s}^{-1}$   
 $P$  is the average pressure of debris flow,  $\text{kPa}$   
 $P_s$  is the pressure of solid phase in debris flow, and  $P_s = \alpha P$   
 $P_f$  is the pressure of slurry in debris flow, and  $P_f = (1 - \alpha)P$   
 $b_s$  is the force unit volume on solid phase,  $\text{kN.m}^{-3}$   
 $b_f$  is the volume force unit volume on liquid phase,  $\text{kN.m}^{-3}$   
 $f_s$  is the other forces on solid phase,  $\text{kN.m}^{-3}$   
 $f_f$  is the other forces on liquid phase,  $\text{kN.m}^{-3}$   
 $g$  is the gravity acceleration,  $9.8\text{m.s}^{-2}$   
 $\theta$  is the angle of bed of debris flow valley,  $\text{Deg.}$   
 $d_e$  is the diameter of equivalent particle,  $\text{m}$   
 $\tau_b$  is the yield stress of Bingham media,  $\text{Pa}$   
 $\mu$  is the rigidity of slurry,  $\text{Pa.s}$   
 $c_f$  is the testing coefficient of liquid phase  
 $d_0$  is the equivalent diameter of control volume,  $\text{m}$   
 $M$  is the ratio of solid velocity to liquid velocity in debris flow  
 $G_0$  is the ratio of calculating velocity to real velocity of solid phase.  
 $L$  is the length of pass area of debris flow valley,  $\text{m}$

$h$  is the thickness of debris flow mass,  $\text{m}$   
 $a_0$  is the length of long axis of the spheroid,  $\text{m}$   
 $b_0$  is the length of short axis of the spheroid,  $\text{m}$   
 $c_0$  is the depth of the spheroid,  $\text{m}$   
 $a$  is the average acceleration of debris flow,  $\text{m.s}^{-2}$   
 $a_s$  is the average acceleration of solid phase in debris flow,  $\text{m.s}^{-2}$   
 $a_f$  is the average acceleration of slurry in debris flow,  $\text{m.s}^{-2}$   
 $q_f$  is the impact force of liquid phase,  $\text{Pa}$   
 $m_s$  is the mass of an equivalent particle,  $\text{kg}$   
 $P_{s0}$  is the impact force of a solid particle,  $\text{Pa}$   
 $n_s$  is the number of solid particle on unit impact area  
 $\bar{P}_s$  is the impact force of solid phase on unit impact area,  $\text{Pa}$   
 $T$  is the formation duration of impact formality,  $\text{sec.}$   
 $K_r$  is the anti-impact coefficient of materials, for example, the coefficient of concrete of C30, C25 and C10 are  $1.07 \times 10^{-3}$ ,  $1.32 \times 10^{-3}$  and  $2.54 \times 10^{-3}$  respectively, while that both rock bank and earth bank of debris flow valley are  $1.78 \times 10^{-3}$  and  $3.0 \times 10^{-5}$  respectively.  
 $\tau_k$  is the strength of anti-shear of impact formality,  $\text{kPa}$   
 $K_0$  is the reveal coefficient of impact force, variation between 500 and 550.  
 $R_f$  is the abrasion force of slurry in debris flow,  $\text{kN/m}$   
 $R_s$  is the abrasion force of shearing force of solid phase in debris flow,  $\text{kN/m}$   
 $\varphi_f$  is the inertial friction angle of slurry in debris flow,  $\text{Deg.}$   
 $G$  is the weight of particles of unit length,  $\text{kN/m}$   
 $W_0$  is the weight of slurry over particle,  $\text{kN/m}$   
 $\delta$  is the volume abrasion amount,  $\text{m}^3/\text{s}$   
 $\xi$  is the anti-abrasion coefficient of surface concrete or others material,  $\text{m}^3/\text{kN}$ . For example, the anti-abrasion coefficient of surface concrete or others material of C15 C25 C30 and C40 are  $1.13 \times 10^{-11}$   $\text{m}^3/\text{kN}$ ,  $4.39 \times 10^{-12}$   $\text{m}^3/\text{kN}$ ,  $2.74 \times 10^{-12}$   $\text{m}^3/\text{kN}$  and  $1.33 \times 10^{-12}$   $\text{m}^3/\text{kN}$  respectively.  
 $t$  is the accumulative time of abrasion,  $\text{sec.}$   
 $e$  is the average abrasive thickness of surface media,  $\text{m}$   
 $v_0$  is the average velocity of abrasion,  $\text{m/s}$   
 $b$  is the width of drainage structure of debris flow valley,  $\text{m}$

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