

Three Dimensional Transport Model Using in Soaking Process

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Abstract: - In this paper, more-dimensional models of the process of raw hide desalination and their theoretical analysis are presented. Unlike our previous one-dimensional model, the more-dimensional models are more general, but they require numerical solution. The main result of this paper is the fact that the losses of the cured hide material during the desalination process depend on the areas, where the cured hide thickness is very small, which was not allowed by our previous one-dimensional model. Both contributions proved usefulness of the application of theoretical tools of chemical engineering even in such areas as leather industry, where it had not been applied before.

Key-Words: Soaking, Cured Hide, Concentration Shock, Diffusion, Two-dimensional Model

1 Introduction

In Part I of our contribution, the change in thickness of cured hide samples in dependence on the position was not considered. The thickness of cured hide samples is considerably different in the dorsal part and in the flank region. The differences in thickness reach up to 300%. Considering this fact, we deal with the problem how are the conclusions of Part I influenced in case of a more exact mathematical-physical model of the described industrial reality.

2 Theory

2.1 Chemical engineering model

Consider a sample occupying a three-dimensional domain (Ω), which is placed into a basin of liquid. At time $\tau = 0$ the sample contains a certain amount of salt at a given concentration which is then gradually washed away into the surrounding liquid. The surrounding liquid is well stirred and we thus consider diffusion in the sample only. The concentration of salt in the surrounding liquid is considered to be a function of time only – thus there are no concentration gradients outside the sample. We suppose that the concentration on the boundary of the sample is equal to the concentration in the surrounding liquid multiplied by the porosity of the sample at each moment. The surrounding liquid (with a certain amount of dissolved salt) is allowed to leave the system and it is replaced by fresh unsalted liquid. Thus, the concentration in the surrounding liquid changes with the flux of salt out of the sample and with the outflow of the salt liquid from the system (and the

inflow of the fresh liquid). The mass of the salt is thus not conserved.

Let us denote by $c(\tau, x)$ the concentration of salt at time τ and point $x \in \Omega$ of the sample. The concentration in the surrounding liquid at time τ is denoted by $k(\tau)$. The above described system is modeled by the following system of equations. Diffusion inside the sample with a constant coefficient of diffusion $D > 0$ is described by the equation

$$\frac{\partial c}{\partial \tau}(\tau, x) = D\Delta c(\tau, x) \text{ for } x \in \Omega \text{ and } \tau \in (0, \infty) \quad (1)$$

The initial concentration is given by

$$c(0, x) = c_0 \quad \text{for } x \in \Omega \quad (2)$$

The concentration on the boundary of the sample is equal to the surrounding concentration multiplied by the porosity ε of the sample, thus

$$c(\tau, x) = \varepsilon k(\tau) \quad \text{for } x \in \partial\Omega \quad (3)$$

where $\partial\Omega$ is the boundary of the domain Ω .

The initial concentration in the surrounding liquid is given by

$$k(0) = k_0 \quad (4)$$

The change in the surrounding concentration is proportional to the flux of the substance out of the sample and the outflow of the salt liquid (and thus the inflow of the fresh liquid). This is modeled by equation

$$V \frac{dk}{d\tau}(\tau) + \dot{V}k(\tau) + D \int_{\partial\Omega} \frac{\partial c}{\partial \nu}(\tau, x) dS = 0 \quad (5)$$

Here V stands for the volume of the surrounding liquid and \dot{V} denotes the inflow of the fresh liquid (which is equal to the outflow of the salt solution). The term

$$D \int_{\partial\Omega} \frac{\partial c}{\partial \nu}(\tau, x) dS$$

represents the flux of the substance across the boundary of the sample, where ν stands for the unit normal vector to the boundary $\partial\Omega$. By the Gauss Theorem, this term can be substituted by

$$D \int_{\Omega} \Delta c(\tau, x) dx$$

In view of equation (1), equation (5) thus takes the form

$$V \frac{dk}{d\tau}(\tau) + \dot{V}k(\tau) + D \int_{\Omega} \frac{\partial c}{\partial \tau}(\tau, x) dx = 0 \quad (6)$$

2.2 Dimensionless formulation

Let us now transform the above system into dimensionless variables. Let us choose any characteristic dimension a of the sample and introduce the variables

$$X = \frac{x}{a}, \quad \left(X_1 = \frac{x_1}{a}, X_2 = \frac{x_2}{a}, X_3 = \frac{x_3}{a} \right)$$

$$Fo = \frac{D\tau}{a^2}$$

$$C(Fo, X) = \frac{c(\tau, x)}{c_0}$$

$$K(Fo) = \frac{k(\tau)}{k_0} = \frac{\varepsilon k(\tau)}{c_0}$$

The domain Ω is transformed to Ω' . The transformation of equations (1) – (4) and (6) yields

the following system:

$$\frac{\partial C}{\partial Fo}(Fo, X) = \Delta C(Fo, X) \quad \text{for } X \in \Omega' \text{ and } Fo \in (0, \infty) \quad (7)$$

$$C(0, X) = 1 \quad \text{for } X \in \Omega' \quad (8)$$

$$C(Fo, X) = K(Fo) \quad \text{for } X \in \partial\Omega' \quad (9)$$

$$K(0) = 1 \quad (10)$$

$$M_1 \frac{dK}{dFo}(Fo) + M_2 K(Fo) + \int_{\Omega'} \frac{\partial C}{\partial Fo}(Fo, X) dX = 0 \quad (11)$$

where $M_1 = \frac{V}{\varepsilon a^3}$ and $M_2 = \frac{\dot{V}}{\varepsilon a D}$

2.3 Two-dimensional case

Consider now a two-dimensional model. Let the examined sample have the shape of a prism with a trapezoidal cross section as shown in Figure 1. Denote the width of the prism by a , its length by L and let its cross section vary from $2b_1$ to $2b_2$. The sample is submerged in a liquid that takes the shape of a prism with the sides a, L and $2d$. It is important to note that in practice the dimension a is often greater than b_1 and b_2 by two orders. Let us assume that at time $\tau = 0$, the concentration of salt inside the sample is equal to the constant c_0 . The initial concentration within the surrounding liquid equals zero ($k_0 = 0$) and the whole system (*i.e.* the sample together with the surrounding liquid) does not exchange mass with its environment (thus $\dot{V} = 0$).

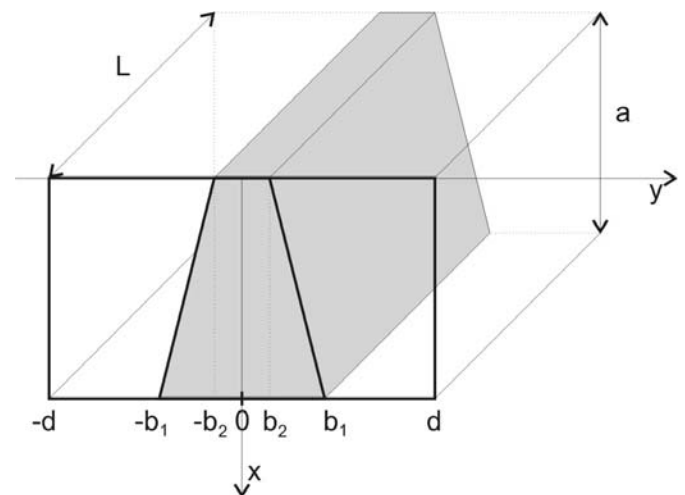


Fig. 1. Two-dimensional model

Considering the fact that the dimension a is generally by two orders of magnitude higher than the dimensions b_1 and b_2 (typical values are $L = a = 1000$ mm, $2b_1 = 25$ mm and $2b_2 = 5$ mm), it would seem possible to neglect the concentration gradients (and thus diffusion, too) in the direction of the x -axis and to restrict ourselves to a one-dimensional (cross sectional) model. However, this would lead to a serious inaccuracy. The surrounding liquid is well stirred, and consequently the upper and lower parts of the sample are in immediate contact through the surrounding liquid. This effect would be neglected by a one-dimensional model. Thus, it is necessary to preserve the two-dimensional essence of this model.

In this case, the two-dimensional domain Ω is characterized by the equations

$$0 \leq x \leq a \quad \text{and} \quad -(px + b_2) \leq y \leq px + b_2$$

where $p = \frac{b_1 - b_2}{a}$. In the dimensionless formulation, the

domain Ω is transformed into Ω' which is given by the equations

$$0 \leq X \leq 1 \quad \text{and} \quad -\left(pX + \frac{b_2}{a}\right) \leq Y \leq pX + \frac{b_2}{a}$$

For a numerical solution, we consider a sample of the parameters $a = L = 1000$ mm, $2b_1 = 25$ mm and $2b_2 = 5$ mm. The diameter of the surrounding liquid is $2d = 35$ mm, which means that the rate of the sample volume to

the volume of the surrounding liquid is $\frac{V_1}{V} = \frac{3}{4}$

For simplicity let us consider $\varepsilon = 1$. The diffusion coefficient inside the sample is assumed to be $D = 10^{-3} \text{ mm}^2 \text{ s}^{-1}$. Four hours of the washing process are modeled.

3 Results and Discussion

Fig. 2 describes the evolution of the concentration field in the upper part of the sample (thickness 5 mm), Figure 3 in the central part of the sample (thickness 15 mm) and Figure 4 in the lower part of the sample (thickness 25 mm). A concentration shock occurs in the points close to the sample surface – first, the concentration falls dramatically, then it gradually rises again due to an increasing concentration in the surrounding liquid. The washing described by Figure 4 proceeds much faster than in the lower parts (the dependence of the time necessary for washing the sample is a quadratic function of the

thickness) and after some time, one witnesses back saturation of the thin parts by the salt from the surrounding liquid, where salinity has increased meanwhile due to the washing of the lower layers. This effect cannot be captured by a one-dimensional model and it can be important for the resulting quality of the cured hide. For better understanding, the evolution of the concentration in various parts of the sample is shown in Fig 5.

Consider now a through-flow washing of a two-dimensional sample when the initial concentration in the surrounding liquid equals the initial concentration in the sample. In this case, in addition, there is fresh liquid flowing into the system at rate \dot{V} and the contaminated liquid is allowed to leave the system at the same rate. In our case, we assume such an inflow that the values of the both dimensionless constants equal

$$M_1 = 0,02 \quad \text{and} \quad M_2 = 4760$$

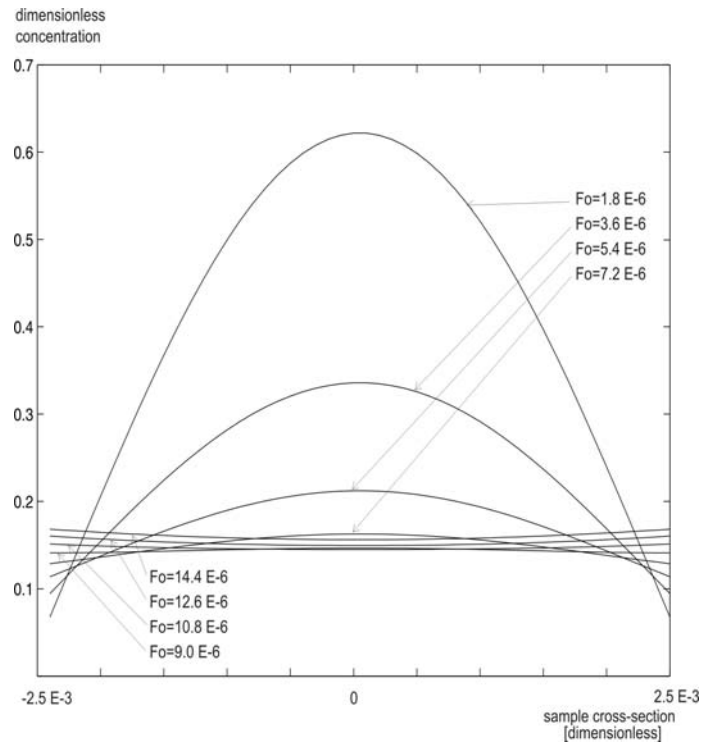


Fig. 2. Behavior of the concentration field in the upper part of the sample of the thickness 5 mm for non through-flow washing.

This time, let us simulate the whole situation for eight hours. The values L, a, b_1, b_2, d, D and N are the same as in the previous one-dimensional model.

Figures 6–8 show the evolution of the concentration field at the same points and times as Figures 2-4. Notice that in this case, no considerable concentration shocks take place.

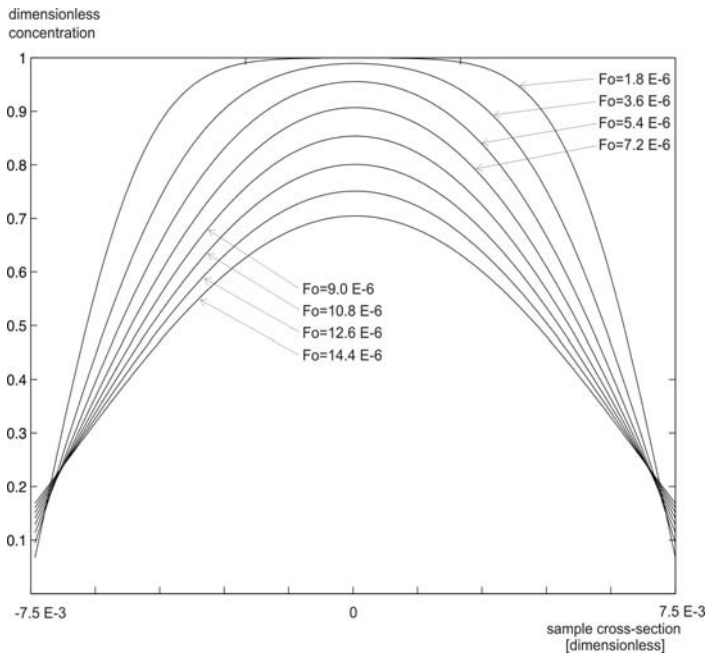


Fig. 3. Behavior of the concentration field in the central part of the sample of the thickness 15 mm for non through-flow washing.

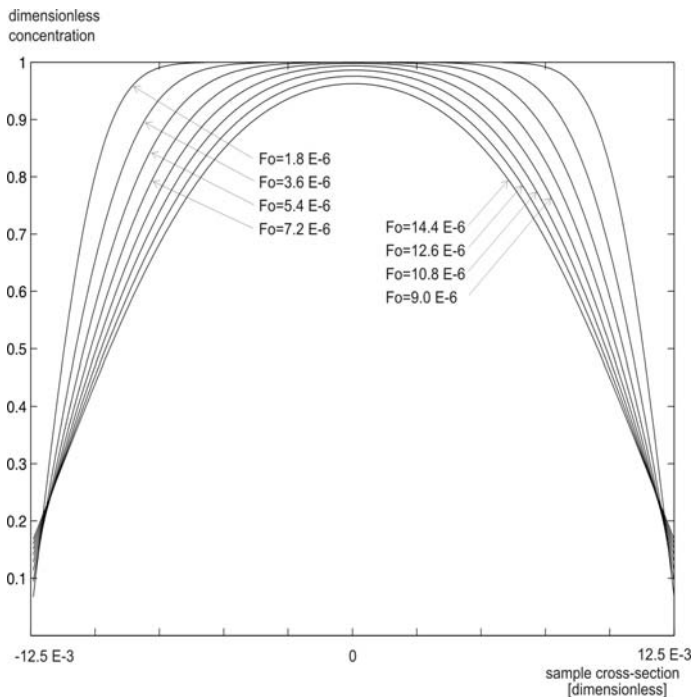


Fig. 4. Behavior of the concentration field in the bottom part of the sample of the thickness 25 mm for non through-flow washing.

Curve A in Fig. 5 describes the center of the thickest part of the sample (thickness of 25 mm), curve B the center of the middle part of the sample (thickness of 15 mm) and curve C the center of the thinnest part of the sample (thickness of 5 mm). Curve D shows a point in the thinnest part of the sample, which is close to the surface (0.1 mm under the surface).

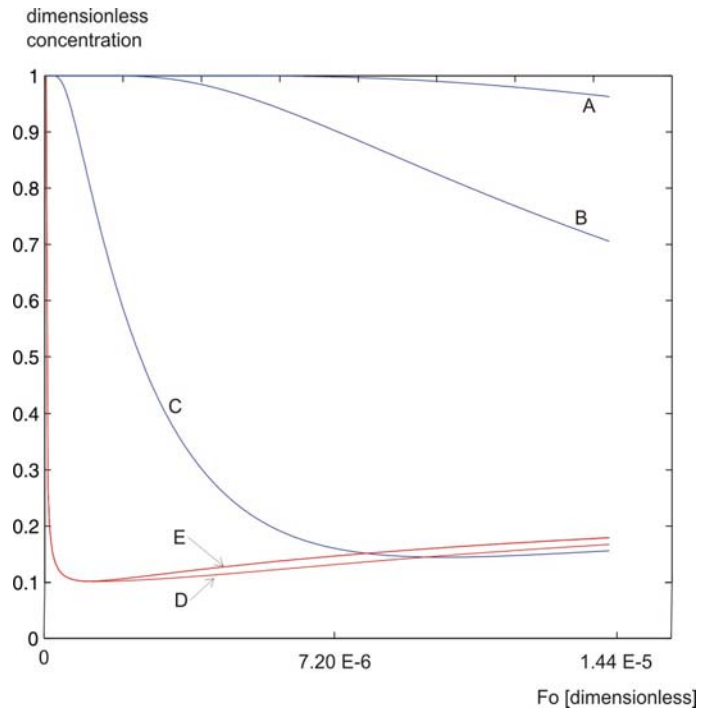


Fig. 5. Evolution of the concentration field in various parts of the sample for non through-flow washing.

Curve E shows a point in the central part of the sample, which is situated close to the surface (0.1 mm under the surface) and it coincides with the curve that shows a point 0.1 mm under the surface of the thickest part of the sample.

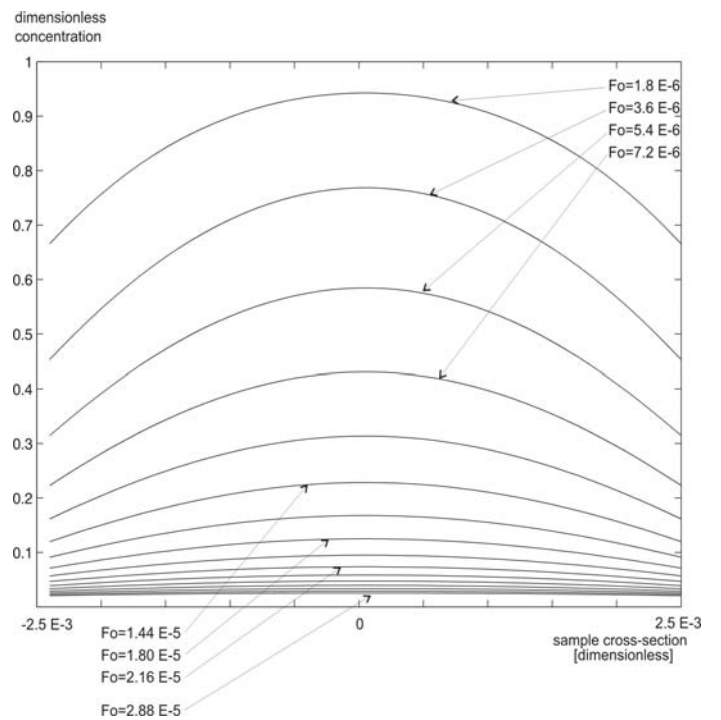


Fig. 6. Behavior of the concentration field in the upper part of the sample of the thickness 5 mm for through-flow washing.

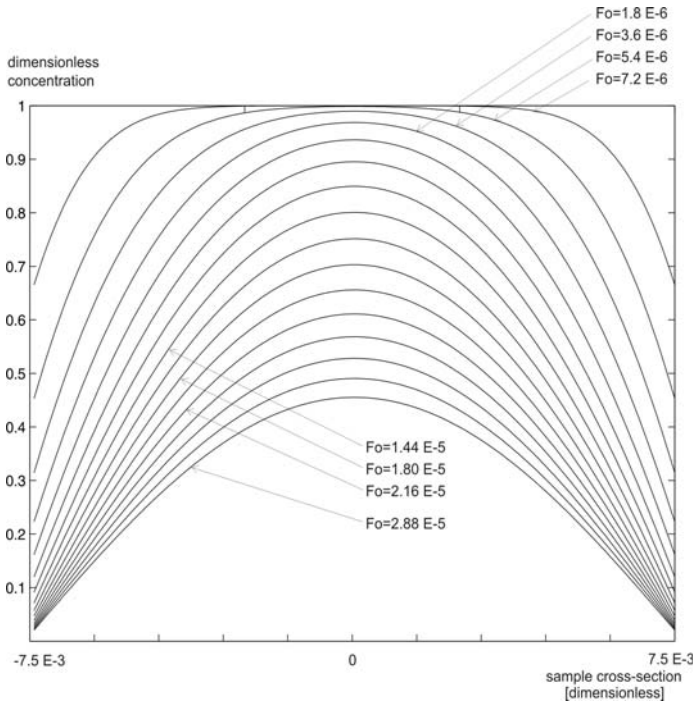


Fig. 7. Behavior of the concentration field in the central part of the sample of the thickness 15 mm for through-flow washing.

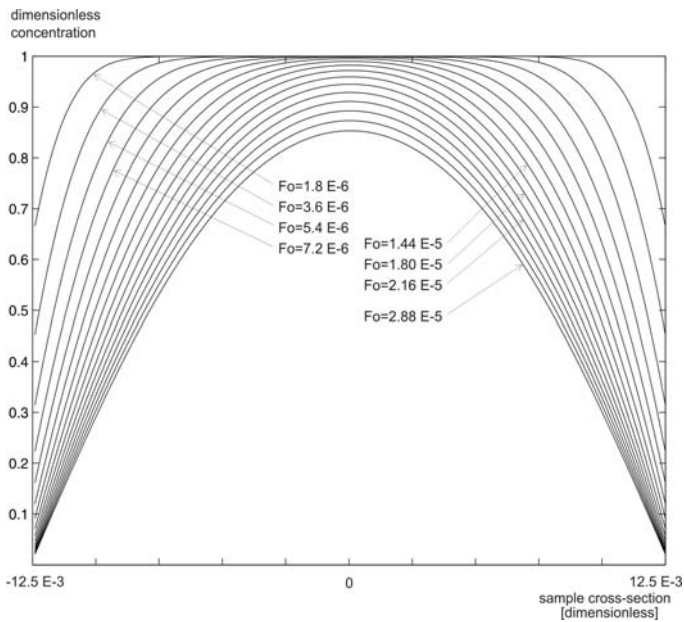


Fig. 8. Behavior of the concentration field in the bottom part of the sample of the thickness 25 mm for through-flow washing.

Curve A in Fig. 9 describes the center of the thickest part of the sample (thickness of 25 mm), curve B the center of the middle part of the sample (thickness of 15 mm) and curve C the center of the thinnest part of the sample (thickness of 5 mm). Curve D shows a point in the thinnest part of the sample, which is close to the surface (0.1 mm under the surface).

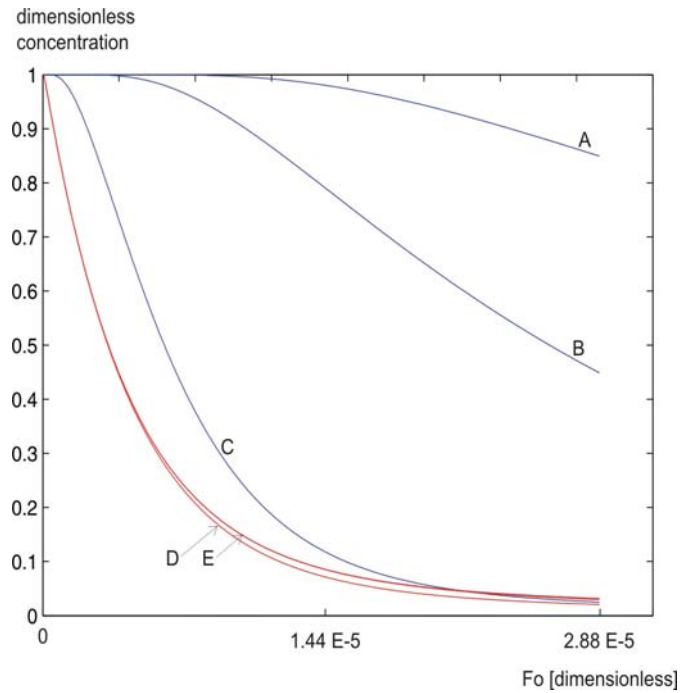


Fig. 9. Evolution of the concentration field in various parts of the sample for through-flow washing

Curve E shows a point in the central part of the sample, which is situated close to the surface (0.1 mm under the surface) and it coincides with the curve that shows a point 0.1 mm under the surface of the thickest part of the sample.

4 Conclusion

Both of our contributions proved the usefulness of theoretical tools of chemical engineering even in such areas where they have not been used yet. That is for the rationalization processes, in which raw hide is transformed into leather. The theoretical and experimental results of our contribution have shown that mathematical simulation has not only theoretical but also practical meaning, because they found the ways how to save raw hide material and such to avoid losses of this material. The second result of the theoretical calculations lead to finding of the control algorithm of the processes where the implementation of automatic control of technological processes is rational and brings desired effect.

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List of symbols:

$a, b_1,$	specific dimensions of hide	[mm]
b_2, L	sample (see the text for detail)	
c	concentration of salt in cured hide	[kg m ⁻³]
c_0	initial concentration of salt in cured hide	[kg m ⁻³]
C	dimensionless concentration of salt in cured hide	[1]
D	effective diffusion coefficient NaCl in cured hide	[m ² s ⁻¹]
Fo	Fourier number – dimensionless time	[1]
k	concentration of salt in surrounding liquid	[kg m ⁻³]
k_0	initial concentration of salt in surrounding liquid	[kg m ⁻³]
K	dimensionless concentration of salt in surrounding liquid	[1]
S	surface area of the sample	[m ²]
V	volume of surrounding liquid	[m ³]
\dot{V}	the inflow of fresh liquid	[m ³ s ⁻¹]
x	coordinate	[m]
ε	porosity of solid state	[1]
τ	time	[s]

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