Transient Heat Conduction for Micro Sphere

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Abstract: - The extreme miniaturization of devices has led to increasing concerns about the steady and transient thermal behavior. The conventional Fourier analysis leads to some error. Structures with dimensions in micrometers or nanometers can be fabricated with modern film deposition and patterning techniques. It is proven neither Fourier nor non-Fourier wave model provide sufficient details for the transient response in casting sand. Recently a two-step process to describe the thermal energy exchange between the solid and the gaseous phases in short times is introduced for thermal conduction. In solids, heat is carried by electrons and lattice waves, whose quanta are phonons. In dielectrics and semiconductors, the phonon contribution is dominant. The present study develops heat transfer regime map for a transient symmetrical sphere without source term with a prescribed constant temperature at the surface. The solution is carried out by finite difference scheme and results are discussed. The boundary between the Fourier, non-Fourier and dual-phase-lag model is determined. It is shown that at the short time both the non-Fourier and dual-phase-lag produce a temperature jump at the interface. The comparison suggests that both models can be useful tool in dealing with transient heat conduction problems from nano to macro .

Key-Words: - Fourier, non-Fourier, dual-phase-lag, heat transfer regime.

1 Introduction

Recent applications involving very low temperatures near absolute zero, a heat source, such as a laser or microwave with extremely short duration or very high frequency, a very high gradients, and extremely short time, non-homogeneous material, like sand and glass bids, materials with inner structures and slow thermal responses, like processed meat or biological tissues, micro and nano scale applications, like electronic devices account for the phenomena involving the finite propagation velocity of the thermal wave, the classical Fourier heat flux model should be modified. While Fourier's law considers infinite speed temperature for heat propagation with its parabolic form, the famous hyperbolic equation,given by Cattaneo and Vernotte, has removed the paradox of instantaneous heat propagation[1,2].They suggested independently a modified heat flux model as:

$$\mathbf{q}(t+\tau_q, r) = -k \nabla T(t, r) \qquad (1)$$

Where k is the thermal conductivity, q is the heat flux, t is the time, r is the radius, T is the temperature and τ is the thermal relaxation time.

Where τ_q is the time needed to accumulate energy for significant heat transfer between structural elements. The first-order Taylor expansion of **q** in Eq. (1) with respect to t, and eliminating of **q** between the equations of energy conservation in the form of :

$$-\nabla q = \rho c_p \frac{\partial T}{\partial t} \qquad (2)$$

Where ρ is the density and C_p is the thermal capacity.

Which leads to the classical hyperbolic heat conduction equation (CV):

$$\nabla^2 T = \frac{1}{\alpha}\frac{\partial T}{\partial t} + \frac{\tau_q}{\alpha}\frac{\partial^2 T}{\partial t^2} \qquad (3)$$

Where α is the Thermal diffusivity. An enhanced version of hyperbolic equation is the dual phase lag model (DPL) which discussed by Tzou [3]. The dual-phase-lag model allows either the temperature gradient to precede the heat flux vector or vice-versa in the transient process. Mathematically, this can be represented by:

$$\mathbf{q}(t+\tau_q, r) = -k \nabla T(t+\tau_T, r) \qquad (4)$$

where, τ_T is interpreted as a measure of the conduction that occurs along microscopic paths not captured by classical approach during non-equilibrium. Following the same procedure as stated for obtaining of the hyperbolic heat transfer equation , for the DPL effect yields:

$$\nabla^2 T + \tau_T \frac{\partial}{\partial t}\left(\nabla^2 T\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t} + \frac{\tau_q}{\alpha}\frac{\partial^2 T}{\partial t^2} \qquad (5)$$

Where:

$$\tau_q = \frac{\alpha}{C_E^2} \quad , \quad \tau_T = \frac{\alpha_e}{C_E^2} \qquad (5\text{-a})$$

Where α_e is the thermal diffusivity of the electron gas and C_E is thermal wave speed.

here, two distinct relaxation times appear. For crystal dielectrics τ_q is the relaxation time of momentum non-conserving processes in a phonon system and τ_T is from the same order of ordinary processes conserving the momentum. In metals τ_q has the same relaxation behavior as wave conduction of electrons and τ_T represents the phonon-electron interactions. This equation reduces to corresponding relation for the hyperbolic model by setting τ_q to zero. Also, it reduces to the relation for Fourier conduction of the classical approach by setting both τ_q and τ_T to zero (i.e. a homogeneous material).

The wave nature of heat propagation has been topic of many investigations, especially in terms of analytical and numerical solutions. Most studies related to one dimensional hyperbolic heat conduction in a semi-infinite or a slab and few in cylindrical or spherical form, to mention some; the physical meaning of the relaxation time in the non-Fourier equation for non homogeneous inner structure materials has been considered by Kaminski [4].The heat transfer regime maps for microstructures relating a geometric length scale to temperature was developed by Flik *et al* [5]. Majumdar illustrated the analogy between micro scale conduction in dielectrics and radiative transfer, and presented a regime map for diamond films[6] . Ozisik *et al* [7]. investigated the propagation and reflection of thermal waves, and the semi-infinite plate with oscillatory surface and the result of the thermal shock wave was elaborated by Tzou [8]. A clear and exhaustive theory as a base for further works was given by Ozisik *et al* [9]. A symmetrical sphere within time varying heat source at constant physical properties was analyzed by Pourmohamadian *et al* [10]. It was shown for time decaying heat source, the non-Fourier and Fourier effect was not significant. The model was generalized from the macroscopic dual-phase lag concept (DPL) which considers the lag of two macroscopic phases: temperature gradients and heat flux by Tzou [11]. The model covers a wide rang of physical responses from microscopic to macroscopic scales in both space and time. The difference between the equilibrium and the noneqilibrium temperatures under the effect of the DPL heat conduction was studied by Al-Nimr *et al* [12]. Recently, the DPL model of heat conduction was used by Antaki which to offer a new interpretation for the evidence of non-Fourier conduction in the experiments with the processed meat[13]. It was shown that the DPL model provides a more comprehensive treatment of the heterogeneous nature of the meat compared to the interpretation that used CV heat conduction model. In the present study, a symmetric spherical solid with the constant physical properties is analyzed for both CV and DPL and FO model. The solutions are carried out by finite difference method and compared with analytical Fourier heat conduction solution.

2 ANALYSIS

The basic formulation of the DPL model in a single-phase medium for a symmetrical sphere body subjected to surface temperature at r=R is :

$$\nabla^2 T + \tau_T \frac{\partial}{\partial t}(\nabla^2 T) = \frac{1}{\alpha}\frac{\partial T}{\partial t} + \frac{\tau_q}{\alpha}\frac{\partial^2 T}{\partial t^2} \qquad (6)$$

Where:

$$\nabla^2 T = \frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} \qquad (7)$$

With the following initials and boundary conditions:

$$T=T_i \quad , \quad \frac{\partial T}{\partial t} = 0 \quad \text{at} \quad t=0 \qquad (8\text{-a})$$

$$T=T_s \quad \text{at} \quad r=R \quad \text{and} \quad \frac{\partial T}{\partial r} = 0 \quad \text{at} \quad r=0 \qquad (8\text{-b})$$

To solve this problem, above equation should be discretized. The discretization can be done in many ways using Finite Difference Method (FDM) or other numerical schemes. In this work a FDM adopted and domain divided into fine meshes. Then, the finite form of the described equation can be written after defining the scheme of difference approximation for time and space. Using the central difference scheme for approximating the time and space derivative, one obtains the following equation:

$$\left(\frac{1}{\Delta r^2}-\frac{1}{r_i\Delta r}+\frac{\tau_T}{2\Delta t\Delta r^2}-\frac{2\tau_T}{4r_i\Delta r\Delta t}\right)T_{i-1}^{j+1}+\left(\frac{-2}{\Delta r^2}-\frac{2\tau_T}{2\Delta t\Delta r^2}-\frac{1}{2\alpha\Delta t}-\frac{\tau_q}{\alpha\Delta t^2}\right)T_i^{j+1}$$
$$+\left(\frac{1}{\Delta r^2}+\frac{1}{r_i\Delta r}+\frac{\tau_T}{2\Delta t\Delta r^2}+\frac{2\tau_T}{4r_i\Delta t\Delta r}\right)T_{i+1}^{j+1}=\left(-\frac{2\tau_q}{\alpha\Delta t^2}\right)T_i^j+$$
$$\left(\frac{\tau_T}{2\Delta t\Delta r^2}-\frac{2\tau_T}{4r_i\Delta r\Delta t}\right)T_{i-1}^{j-1}+\left(\frac{-2\tau_T}{2\Delta t\Delta r^2}-\frac{1}{2\alpha\Delta t}+\frac{\tau_q}{\alpha\Delta t^2}\right)T_i^{j-1}$$
$$+\left(\frac{\tau_T}{2\Delta t\Delta r^2}+\frac{2\tau_T}{4r_i\Delta r\Delta t}\right)T_{i+1}^{j-1} \quad (9)$$

The geometry and physical conditions of the cases are considered as follows:

R=0.005m, $\alpha = 0.3\times10^{-6}\, m^2/s,\ \tau_q = 8.94s$,

$\tau_T = 4.48s$, T_i=10 °C, T_s=37 °C.

Which of the physical properties are those of sand is considered and for meat :

$\alpha = 0.1396\times10^{-6}\, m^2/s,\ \tau_q = 14s,\ \tau_T = 0.056s$.

.For convergence, ΔT=0.0001 sec and $\Delta r = 2.94\times10^{-4}$ m is chosen.

3 Results and Discussion

The numerical computation is preformed in order to display the temperature profile arising from a surface heat at r=R. First, the effect of numerical accuracy is investigated. Table 1 compares the numerical computation with the analytical solution as indicated in Carslaw *et al.* [14]. which is:

$$\frac{T(r,t)-T_s}{T_i-T_s} = 2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n\pi r}\sin(n\pi r)e^{-\alpha n^2\pi^2 r^2 t} \quad (10)$$

Good agreement between the numerical solution and analytical solution are shown and the error is approximately 1.6e-5 for the indicated time and space interval.

The simulation results of temperature profile as time increases at nodes which are closer to the surface are shown in Fig.1-3 for FO, CV and DPL model respectively. Higher and wavy shape temperature is obtained for CV as expected due to the nature of the equation. The DPL model predicted similar behavior as FO model but lowers in the interior nodes due to its time delayed nature.

The ratio of temperature gradient to heat flux relxation time is varied from 0.004 to 0.95 for DPL model and is shown in fig.4. It shows temperature prediction as time increases. As the temperature gradients approaches to zero, here 0.004, the relation reduces to the CV model and by setting close to one, here 0.95, the solution reduces to the FO model which is obviuos. Similar observation was shown experimentally by Antaki for processed meat [13]. Next, Fig.5 shows that the DPL prediction approximately captures the first rapid changes in temperature change in the medium at starting time and then sudden decrease follows. The jump in temperature is higher for small relaxation times ratio and lowers as this ratio approaches to one.

In order to judge the difference between FO, CV and DPL temperatures, Fig.6 at position R=0.95 is ploted at different times. From the present results, it is seen that the CV solution predicts a jump in temperature and then converges to FO values as time increases. This is due to the finite speed of heat propagation phenomenon. The DPL model describes similar trend as of FO model and this is the lagging response in transient condition and conveges close to the FO values as time increases. The difference between FO and DPL reflects the delayed time of the micro-structural interaction effect relative to the fast transient inertia as stated by Tzou [1997].

Fig.7 compares temperature profiles after one second at interior nodes. The temperature for CV and DPL models are higher than the FO model because of the presence of time delays. The CV model drops sharply and the converges to the FO. The DPL drops sharply at front and then follows the same trend as the FO model.

4 Tables and Figures

Radius	Numerical Solution		Exact Solution	
	t= 0.5 s	t= 1s	t= 0.5 s	t= 1s
0.99	35.9947	36.3365	35.9967	36.3371
0.98	33.9618	34.9904	33.9674	34.9924
0.95	29.8967	32.2527	29.9087	32.2572

Table 1: Comparison between numerical and analytical results for FO model

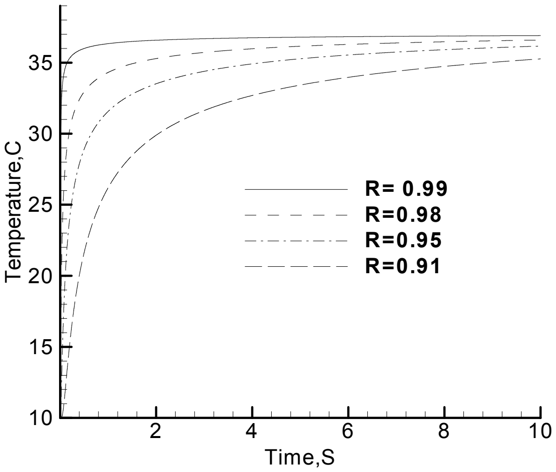

Fig.1: Temperature profile as time increases at interior nodes for FO .

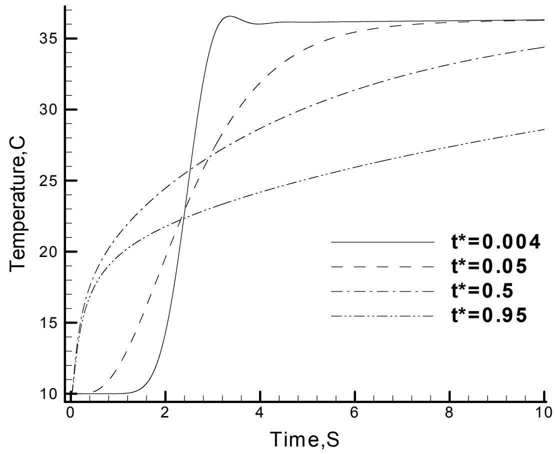

Fig. 4: Comparison of temperature profile at different relaxation times ratio at R=0.95.

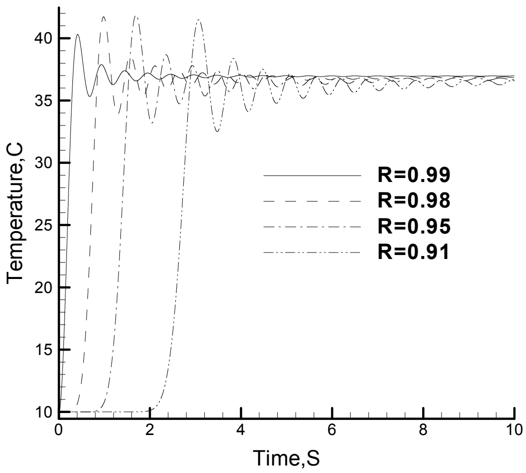

Fig. 2: Temperature profile as time increases at interior nodes for CV .

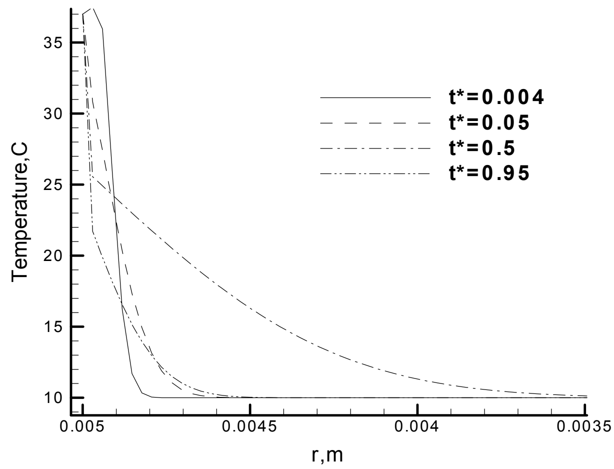

Fig. 5: Comparison of temperature profile at different relaxation times ratio after 1 sec.

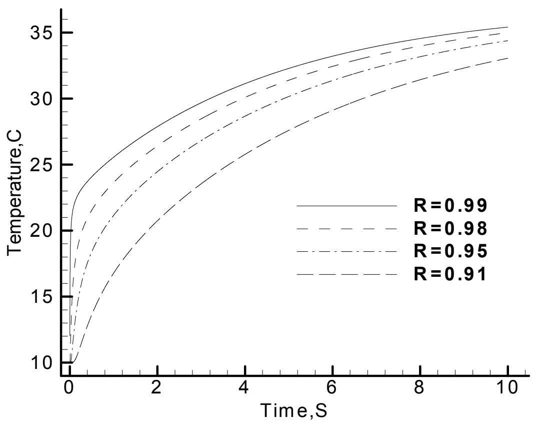

Fig. 3: Temperature profile as time increases at interior noded for DPL.

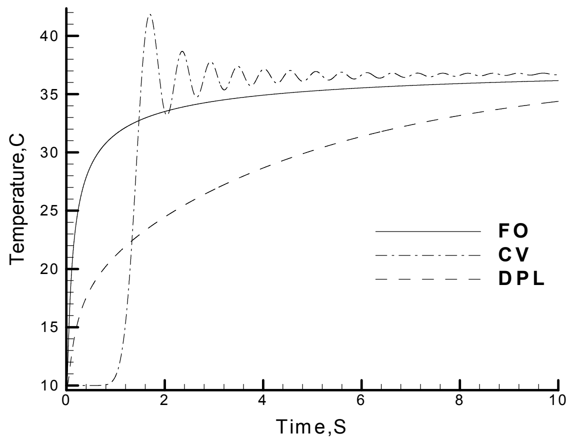

Fig. 6: Comparison of temperature profile as time increases at R=0.95.

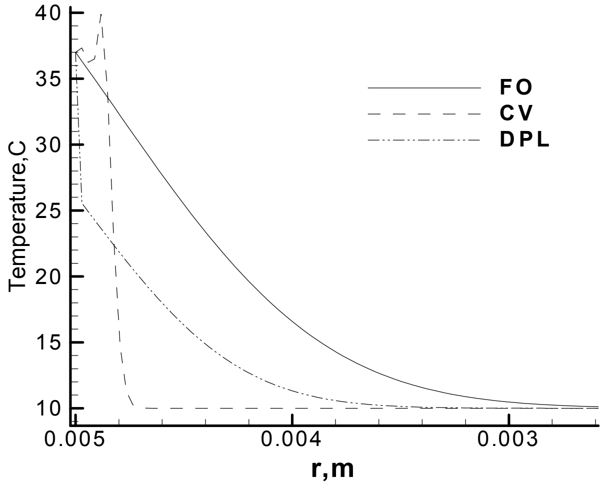

Fig. 7: Comparison of temperature profile at different radius after 1 sec.

4 Conclusion

Based on the generalized concept of dual-phase-lag (DPL) in both the heat flux vector (τ_q) and the temperature gradient (τ_T) for heat conduction in a symmetrical sphere has been solved numerically with FDM scheme and simulation results are discussed. The simulation shows the effect of FO, CV and DPL. It is shown that CV predicts higher temperature and DPL lower at specified node by comparing with FO model, but all model due to time increase, are approaching the same value and similar trend in the temperature profile. The two phase lags play important roles for nano to macro structures. This needs to be determined experimentally and then a well tabulated properties needs for engineering materials under various conditions.

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