Applying the fuzzy preference relation to the software selection

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Abstract: - Software selection is not a well-defined or structured decision problem. Lai et al. [5] uses the Analytic Hierarchy Process (AHP) method to solve a Multimedia authoring systems (MAS) selection problem. However, the AHP method is difficult to obtain a convincing consistency index when the number of attributes or alternatives is increased. Hence, this paper applies the fuzzy preference relation (Fuzzy PreRa) method to solve the problems mentioned in the paper written by Lai et al. [5]. Applying the Fuzzy PreRa method to decision making is more efficient, as not only the ratio of the pairwise comparison times of the priority weight for the six factors between Fuzzy PreRa and AHP is 5:15, but also the outcome obtained by Fuzzy PreRa almost coincides with the one that produced by the AHP method.

Key-Words: - Fuzzy preference relation, Fuzzy PreRa, AHP, Multimedia authoring systems (MAS)

1 Introduction

Software selection is not a well-defined or structured decision problem. Owing to the complexity of the product and profusion of alternatives, a systematic process of selection can be formidable and expensive. Hence, Lai et al. [5] uses the AHP method to solve a MAS (Multimedia authoring systems) selection problem.

Analytic Hierarchy Process (AHP) is an important material analysis method which was proposed by Saaty [8,9] in 1977. The strength of AHP is that it organizes tangible and intangible factors in a systematic way, and provides a structured yet relatively simple solution to decision making problems [11]. Up to present, the AHP method has been applied in many different domains, including risk assessment [12], enterprise resource planning assessment [16], project management [1] and so forth.

However, additional to the fact that the AHP method must perform very complicated pairwise comparisons amongst elements (attributes or alternatives), and it is also difficult to obtain a convincing consistency index with an increasing number of attributes or alternatives. Hence, there are improved techniques such as Fuzzy AHP [4], referenced AHP [10], extended Fuzzy AHP [15], modified AHP [13], random AHP [6], chainwise paired comparisons [7] and DS/AHP [2] which have been proposed in professional journals and conferences involving various disciplines.

Herrera-Viedma et al. [3] developed a new method, the fuzzy preference relation (Fuzzy PreRa), which focuses on avoiding the inconsistent solutions in the decision making processes. The object of this paper is applying the Fuzzy PreRa method to solve the problems of in the paper written by Lai et al. [5].

The paper is organized as follows: section 2 outlines the basis and concept of the fuzzy preference relation (Fuzzy PreRa) method, section 3 contains two parts, 3.1 gives a brief review of the method of Lai et al. [5] for software selection, and 3.2 employs the example from [5] to illustrate the selection of a multimedia authoring system of Fuzzy PreRa. Finally, discussion and some concluding remarks are presented in section 4 and 5, respectively.

2 Fuzzy Preference Relation

Perfect consistency is difficult to obtain in practice, especially when measuring preferences on a set with a large number of alternatives. The Fuzzy PreRa [3] method focuses on avoiding the inconsistent solutions in the decision making processes.

The concept of this method is that if there are n attributes $X=\{x_1,...,x_n, n\geq 2\}$, then we can obtain the pairwise preference relation data $\{p_{12,}p_{23},...,p_{n-1n}\}$, from $n-1$ comparing and constructing a consistent reciprocal fuzzy preference relations P'. This method follows the one of traditional AHP method characteristics, which is preference relation satisfied transitivity property.

Herrera-Viedma et al. [3] had proof that for a reciprocal additive fuzzy preference relation $P=(p_{ij})$, the following statements are equivalent:

$$p_{ij}+p_{jk}+p_{ki}=\frac{3}{2}\forall i<j<k \quad (1)$$

$$p_{i(i+1)}+p_{(i+1)(i+2)}+\cdots+p_{(j-i)j}+p_{ji}=\frac{j-i+1}{2}$$
$$\forall i<j \quad (2)$$

According to (1) (2), therefore, we can deduce that

$$p_{ji}=\frac{j-i+1}{2}-p_{i(i+1)}-p_{(i+1)(i+2)}-\cdots-p_{(j-1)j} \quad (3)$$

and according to the additive reciprocal

$$p_{ij}+p_{ji}=1\forall i,j\in\{1,\ldots,n\} \quad (4)$$

The p_{ij} indicates the ratio of the preference intensity of alternative x_i to that of x_j. If $p_{ij}=1/2$ implies there is no difference between x_i and x_j $(x_i\sim x_j)$, $p_{ij}>1/2$ implies x_i is preferred to x_j $(x_i>x_j)$, $p_{ij}=1$ indicates that x_i is absolutely preferred to x_j.

The steps are described in the following:

1. Compute the set of preference values B as

$$B=\{p_{ij}, i<j\wedge p_{ij}\notin\{p_{12},p_{23},...,p_{n-1n}\}\} \quad (5)$$

2. Find P

$$P=\{p_{12},p_{23},...,p_{n-1n}\}\cup B$$
$$\cup\{1-p_{12},1-p_{23},...,1-p_{n-1n}\}\cup\neg B \quad (6)$$

3. The consistent fuzzy preference relation P' is obtained as $P'=f(P)$ such that

$$f:[-a,1+a]\rightarrow[0,1]$$

$$f(x)=\frac{x+a}{1+2a} \quad (7)$$

If there is a set of alternatives $X=\{x_1,...,x_n\}$, which is associated with a reciprocal multiplicative preference relation $A=(a_{ij})$, and $a_{ij}\in[1/9,9]$, then we can use equation (8) to find the corresponding reciprocal additive fuzzy preference relation $P=(p_{ij})$, and $p_{ij}\in[0,1]$.

$$p_{ij}=g(a_{ij})=1/2\cdot(1+\log_9 a_{ij}) \quad (8)$$

Thus, we professed that equation (8) is a transformation function g, and using the function g a reciprocal multiplicative preference relation matrix can be transformed into a kind of preference relation.

After the Fuzzy PreRa method was proposed, Wang and Chen [14] adapted it with the linguistic concept, which uses variables as fuzzy linguistic assessments, and proposed it as the fuzzy linguistic preference relations method.

3 Methodology

3.1 Review of MSA selection with AHP by Lai *et al*. [5]

The evaluation and selection of a MAS are important parts of multimedia development projects. In their paper, "Software selection: a case study of the application of the analytical hierarchical process to the selection of a multimedia authoring system," Lai et al. [5] applied AHP method in software selection. Decision makers judge the importance of one alternative over another can be made subjectively and converted to a numerical value using a scale of 1-9 where 1 denotes equal importance and 9 denotes the highest degree of favoritism.

There are six important factors used to measures the performance of MSA product: development interfaces (F1); graphic support (F2); multi-media data support (F3); data file support (F4); cost effectiveness (F5) and vendor support (F6). The preference relation matrix for pairwise comparison of criteria is shown in Table 1 (which represents the evaluation of member 1 of the group) [5].

Table 1
Preference relation matrix for pairwise comparison of criteria (AHP)

	F1	F2	F3	F4	F5	F6
F1	1	1/3	1/4	1/3	5	7
F2	3	1	1/3	1/4	5	6
F3	4	3	1	2	7	8
F4	4	4	1/2	1	6	8
F5	1/5	1/5	1/7	1/6	1	5
F6	1/7	1/6	1/8	1/8	1/5	1

After obtaining the pairwise comparisons, a normalized matrix was developed in Table 2 by dividing each element by the sum of its respective columns. The row entries in the last two columns of the normalized matrix table comprised the sum of six elements in the row and the average of those row elements (principal vector), respectively.

Table 2
Normalized matrix of Table 1

	F1	F2	F3	F4	F5	F6
F1	0.0881	0.0383	0.1063	0.0853	0.2066	0.2000
F2	0.2645	0.1149	0.1417	0.0645	0.2066	0.1714
F3	0.3527	0.3448	0.4255	0.5168	0.2893	0.2286
F4	0.2645	0.4598	0.2127	0.2583	0.2479	0.2286
F5	0.0176	0.0230	0.0607	0.0432	0.0413	0.1429
F6	0.0126	0.0192	0.0531	0.0322	0.0083	0.0285

Thus, the relative weights for each attribute, after calculation, are found to be: F1=0.1208, F2=0.1606, F3=0.3596, F4=0.2786, F5=0.0548 and F6=0.0256. Given by the mathematical expression shown below:

F3>F4>F2>F1>F5>F6

3.2 Fuzzy PreRa for MSA selection

This study uses fuzzy preference relation method re-computation with the six evaluation criteria in [5].

First, this study applies Eq. (8) to transform the original data $a_{12}, a_{23}, a_{34}, a_{45}, a_{56}$, which transforms the reciprocal multiplicative preference relation with an interval scale [1/9,9] into a reciprocal additive fuzzy preference relation with an interval scale $[0,1]$ from a set of $n-1$ preference values $\{p_{12}, p_{23}, ..., p_{n-1n}\}$.

In this case, there are only five comparisons required for six evaluation criteria, which are as follows:

$$p_{12} = (1 + \log_9 1/3)/2 = 0.25$$
$$p_{23} = (1 + \log_9 1/3)/2 = 0.25$$
$$p_{34} = (1 + \log_9 2)/2 = 0.6577$$
$$p_{45} = (1 + \log_9 6)/2 = 0.9077$$
$$p_{56} = (1 + \log_9 5)/2 = 0.8662$$

Then, using Eq. (1) ~ (4), we obtain the entire preference relation matrix, which is shown in Table 3. For clarity, examples for p_{21}, p_{31}, p_{42} and p_{52} are shown below:

$$p_{21} = 1 - p_{12} = 1 - 0.25 = 0.75$$
$$p_{31} = 1.5 - p_{12} - p_{23} = 1.5 - 0.25 - 0.25 = 1.00$$
$$p_{42} = 1.5 - p_{23} - p_{34} = 1.5 - 0.25 - 0.6577 = 0.5923$$
$$p_{52} = 2 - p_{23} - p_{34} - p_{45} = 0.1845$$

Table 3
Preference relation matrix for pairwise comparison of criteria (Fuzzy PreRa)

	F1	F2	F3	F4	F5	F6
F1	0.5000	0.2500	0.0000	0.1577	0.5655	0.9317
F2	0.7500	0.5000	0.2500	0.4077	0.8155	1.1817
F3	1.0000	0.7500	0.5000	0.6577	1.0655	1.4317
F4	0.8423	0.5923	0.3423	0.5000	0.9077	1.2740
F5	0.4345	0.1845	-0.0655	0.0923	0.5000	0.8662
F6	0.0683	-0.1817	-0.4317	-0.2740	0.1338	0.5000

We make note that, the primary values in Table 3 are not in the interval [0,1], but in an interval $[-a, 1+a]$, being $a > 0$, we need to transform the obtained values using Eq. (7) a transformation function which preserves reciprocity and additive consistency, that is a function $f:[-a, 1+a] \rightarrow [0,1]$.

According to Eq. (7), $a = 0.4317$, therefore, the entire preference relation matrix can be transformed as shown in Table 4 and the normalized matrix is shown in Table 5.

Table 4
After transformation matrix

	F1	F2	F3	F4	F5	F6
F1	0.5000	0.3658	0.2317	0.3163	0.5351	0.7317
F2	0.6342	0.5000	0.3658	0.4505	0.6693	0.8658
F3	0.7683	0.6342	0.5000	0.5846	0.8035	1.0000
F4	0.6837	0.5495	0.4154	0.5000	0.7188	0.9154
F5	0.4649	0.3307	0.1965	0.2812	0.5000	0.6965
F6	0.2683	0.1342	0.0000	0.0846	0.3035	0.5000

Table 5
Normalized matrix of Table 4

	F1	F2	F3	F4	F5	F6
F1	0.1506	0.1455	0.1355	0.1427	0.1516	0.1554
F2	0.1910	0.1989	0.2140	0.2032	0.1896	0.1839
F3	0.2315	0.2522	0.2925	0.2637	0.2276	0.2123
F4	0.2060	0.2185	0.2430	0.2255	0.2036	0.1944
F5	0.1400	0.1315	0.1150	0.1268	0.1416	0.1479
F6	0.0808	0.0534	0.0000	0.0382	0.0860	0.1062

Thus, the relative weights for each attribute, after calculation, are found to be: F1=0.1469, F2=0.1968, F3=0.2466, F4=0.2152, F5=0.1338 and F6=0.0608. Given by the mathematical expression shown below:

F3>F4>F2>F1>F5>F6

4 Discussion

This study applies the fuzzy preference relation (Fuzzy PreRa) to solve the problems in Lai's paper

[5]. The analyzed outcome obtained by Fuzzy PreRa almost coincides with that produced by the AHP method, shown in Table 6. Notably, the ratio of the pairwise comparison times of the priority weight for the six influential factors between Fuzzy PreRa and AHP is 5:15.

Table 6
Comparing AHP and Fuzzy PreRa compute result

	AHP	ranking	Fuzzy PreRa	ranking
F1	0.1208	4	0.1469	4
F2	0.1606	3	0.1968	3
F3	0.3596	1	0.2466	1
F4	0.2786	2	0.2152	2
F5	0.0548	5	0.1338	5
F6	0.0256	6	0.0608	6

5 Conclusion

Although the AHP method has been widely applied in many research domains, it still has some problems that need to be solved. Therefore, owing to the fact that the AHP method must perform very complicated pairwise comparison amongst elements (attributes or alternatives), and since it is difficult to obtain a convincing consistency index with an increasing number of attributes or alternatives.

Then, Herrera-Viedma et al. [3] developed the fuzzy preference relation (Fuzzy PreRa) method to improve above problem.

If we have n attributes, using the AHP method we need to count the pairwise comparison times of $n(n-1)/2$ to be able to obtain a result. However, using the fuzzy PreRa (fuzzy preference relation) method we only need to count the pairwise comparison times of $n-1$. In this case, there are six criteria, using AHP we need to compare the criteria 15 times, but using fuzzy PreRa we only need to compare them 5 times. Regarding the pairwise comparing times, fuzzy PreRa requires less comparisons than AHP by 10 times. Fuzzy PreRa is clearly faster to execute and more efficient than AHP.

Applying Fuzzy PreRa method, it is possible to assure better consistency of the fuzzy preference relations provided by the decision makers, and in such a way as to avoid the inconsistent solutions in the decision making processes.

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