Fuzzy models of the dynamic systems for evolution of populations

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Abstract. - The fuzzy models of evolution of isolated population and also coexistence of two populations are built on the basis of fuzzy logic using the algorithm of Takagi-Sugeno. The partial cases of degeneration of new models in the simplest models are considered.

Key-Words: - fuzzy models, population, dynamic systems

1 Introduction

The modeling of the dynamic systems of evolution of populations is deeply enough studied and described by the methods of differential equations, which are built on the basis of laws of conservation of mass, energies and other. Besides the simple linear models of evolution and coexistence of populations, such as Malthus models and Verhulst model, plenty of models based on quazylinear and nonlinear equations are built [1, 5, 8, 9, 11]. The solutions of such problems in a greater or smaller measure are studied and investigated on stability in other branches of science, in particular in theoretical physics, hydromechanics and others.

The dynamic systems are less studied using of fuzzy logic, but now this direction of researches develops rapidly. We will calls certain works devoted to researches of dynamics of processes by construction of fuzzy models [2, 3, 4, 6, 12, 13]. It is necessary to take into consideration that properties of dynamics of populations are mostly studied on the basis of statistical data and that is why application of fuzzy logic is reasonable. Approaches, at which outputs data appears poorly formalized, allow to presents statement and solution of problem by a natural language, here they are universal and effective. At the same time, there is some subjectivity in forming of fuzzy rules and electing of function of membership that can be partly corrected by correction of the system parameters in the process of work.

On this time a few algorithms of obtaining of fuzzy conclusions, which are based of knowledge of specialists of subject domain as the aggregate of predicate rules, are exist, such as, algorithms of Mamdani, Tsukamoto, Takagi-Sugeno, Larsen [7, 16]. There are also different approaches in adduction to un-fuzzy, in particular, centroid approximation, approximations of the first and middle maximum, criterion of maximum, and others. Practical applications of methodology of Takagi-Sugeno to construction of the fuzzy systems are in article [14] presented, and in [15] the fuzzy dynamic systems obtained on the basis of linear function of membership. In this article the generalization of the systems is done in the case of arbitrary function of membership for the models of evolution of isolated population and coexistence of two populations.

2. Review of biological models.

Models of dynamics of one population. The simplest model of population' growth (model of Malthus) is given by equation

$$\frac{dx}{dt} = ax \qquad (1)$$

where x is quantity of population, t is time, a is the difference of coefficientsof birth-rate and dying off. The solution of equation (1) submits to the exponential law, here at $a > 0$ and $t \to \infty$ $x(t) \to \infty$, and at $a < 0$ and $t \to \infty$ $x(t) \to 0$. Equation (1) is the equation of natural growth, application of such model is limited as growth of population can take place only on condition of unlimited resources of environment. The next model of logistic growth satisfies condition of boundedness of quantity of population

$$\frac{dx}{dt} = a(x)x \qquad (2)$$

where coefficient $a(x)$ depends on the quantity of population. So, Verhulst proposed linear dependence $a(x) = \alpha - \beta x$, then

$$\frac{dx}{dt} = (\alpha - \beta x)x . \quad (3)$$

The solution of equation (3) is $x(t) = \frac{\alpha / \beta}{1 - C\exp(-\alpha t)}$, thus $C = 1 - \frac{\alpha}{\beta x(0)}$, so at $t \to \infty$ the quantity of population sent to the finite value $x(t) \to \alpha / \beta$. We will specify, also, the law of dynamics of population under the condition of co-operative interests

$$\frac{dx}{dt} = x\big(b[1 - \exp(-ax)] - D(x)\big) \quad (4)$$

thus a birth-rate submits to the law of Poisson $1 - \exp(-ax)$, and $D(x)$ is arbitrary function of dying off.

Models of coexistence of populations. The simplest model of coexistence of two kinds (model of Volterra) is

$$\frac{dx}{dt} = (\alpha - \beta y)x$$
$$\frac{dy}{dt} = (-\gamma + \delta x)y \quad (5)$$

where x is quantity of victim population, y is quantity of predator population, α, γ are coefficients of natural increase, β, δ are coefficients of interspecific coexistence.

The model of mutual interspecific competition of two kinds generalized the model of of Volterra

$$\frac{dx}{dt} = (a - a_{11}x - a_{12}y)x ,$$
$$\frac{dy}{dt} = (b - a_{21}x - a_{22}y)y \quad (6)$$

which can be the generalized by Kolmogorov' model

$$\frac{dx}{dt} = f(x,y)x ,$$
$$\frac{dy}{dt} = g(x,y)y \quad (7)$$

where, $f(x,y)$ and $g(x,y)$ are of functions differentiable on both the variables, the signs of parts derivative determine the relations between populations.

Model of Volterra of coexistence n of kinds is in the form

$$\frac{dx_i}{dt} = \left(a_i - \sum_{j=1}^{n} \gamma_{ij} x_j\right)x_i \qquad i = \overline{1,n} . \quad (8)$$

General equations of balance [10], which describe the streams of mass and energy between producers, substrates and consumers

$$\frac{dx_i}{dt} = \left(F_x^i - D_x^i\right)x_i - \sum_{j=1}^{n} u_{ij} y_j + \sum_{k=1}^{p} w_{ik} z_k + R_x^i$$
$$i = \overline{1,m} ,$$
$$\frac{dy_j}{dt} = \left(F_y^j - D_y^j\right)y_j - \sum_{r=1}^{n} v_{jr} y_r \quad j = \overline{1,n} , \quad (9)$$
$$\frac{dz_k}{dt} = \sum_{j=1}^{n} V_{kj} y_j - \sum_{i=1}^{m} W_{ki} x_i \qquad k = \overline{1,p} ,$$

where x_i ($i = \overline{1,m}$), y_j ($j = \overline{1,n}$), z_k ($k = \overline{1,p}$) are biomasses of producers, substrates and consumers F_x^i, F_y^j, D_x^i, D_y^j are coefficients of natural increase and dying off; $u_{ij}, v_{ij}, w_{ik}, W_{jk}, V_{jk}$ are speeds of consumption, transformation and production of biomasses; R_x^i is coefficient of transformation of light.

3. Fuzzy models of population' evolution.

Construction of the simplest fuzzy model of evolution of population is based on some statistical or experimental data about the parameters of the system. In particular, such data can be got quantity of population for the separate segments $[x_0, x_1]$, $[x_{i-1}, x_{i+1}]$ $(i = \overline{1, n-1})$ and $[x_{n-1}, x_n]$ the proper values to the coefficient of its growth a_j with the function of membership $\mu_j(x)$ ($j = \overline{0,n}$), which is considerate of weighting coefficient. The sum of weighting coefficients for every value x must satisfy the condition $\sum_{j=0}^{n} \mu_j(x) = 1$.

From models of population evolution (1) -(4) based on the theory of ordinary differential equations, follows, that, consequently, it is possible to get the system of fuzzy rules Π_j ($j = \overline{0,n}$) for the coefficient of population' growth

Π_0: If $x \in [x_0, x_1]$, then $\frac{\frac{dx}{dt}}{x} = a_0$.

Π_1: If $x \in [x_0, x_2]$, then $\frac{\frac{dx}{dt}}{x} = a_1$.

. . . .

Π_i : If $x \in [x_{i-1}, x_{i+1}]$, then $\frac{\frac{dx}{dt}}{x} = a_i \qquad i = \overline{1, n-1}$.

(10)

. . . .

Π_{n-1}: If $x \in [x_{n-2}, x_n]$, then $\frac{\frac{dx}{dt}}{x} = a_{n-1}$.

Π_n : If $x \in [x_{n-1}, x_n]$, than $\frac{\frac{dx}{dt}}{x} = a_n$.

Let's give the function of membership, which determines the degree of validity of the rule Π_j or value a_j as the coefficient of growth of population, in the form

$$\mu_0(x) = \begin{cases} 1 & x = x_0 \\ 1 - \varphi\left(\frac{x - x_0}{x_1 - x_0}\right) & x \in (x_0, x_1] \\ 0 & x \notin [x_0, x_1] \end{cases} ,$$

. . . .

$$\mu_i(x) = \begin{cases} \varphi\left(\frac{x - x_{i-1}}{x_i - x_{i-1}}\right) & x \in [x_{i-1}, x_i] \\ 1 - \varphi\left(\frac{x - x_i}{x_{i+1} - x_i}\right) & x \in (x_i, x_{i+1}] \\ 0 & x \notin [x_{i-1}, x_{i+1}] \end{cases} \quad (11)$$

$i = \overline{1, n-1}$,

. . . .

$$\mu_n(x) = \begin{cases} \varphi\left(\frac{x - x_{n-1}}{x_n - x_{n-1}}\right) & x \in [x_{n-1}, x_n] \\ 0 & x \notin [x_{n-1}, x_n] \end{cases} ,$$

where a function $\varphi(\alpha)$ is determined on an segment $[0,1]$, thus $\varphi(0) << 1$ (or $\varphi(0) = 0$) $\varphi(1) = 1$. In obedience to individual exits from rules Π_j ($j = \overline{0, n}$) it is possible to define value of conclusion as the weighted-mean value accordingly the algorithm of Takagi-Sugeno [16]

$$\frac{\frac{dx}{dt}}{x} = \frac{\sum_{j=0}^{n} \mu_j(x) a_j}{\sum_{j=0}^{n} \mu_j(x)}. \qquad (12)$$

Substitution of formulae (11) in expression (12) for $\mu_j(x)$ gives the simplified expressions on separate intervals

$$\frac{\frac{dx}{dt}}{x} = \left[1 - \varphi\left(\frac{x - x_{i-1}}{x_i - x_{i-1}}\right)\right] a_{i-1} + \varphi\left(\frac{x - x_{i-1}}{x_i - x_{i-1}}\right) a_i,$$

$$x \in [x_{i-1}, x_i], \quad i = \overline{1, n},$$

or

$$\frac{dx}{dt} = \left[a_{i-1} + \varphi\left(\frac{x - x_{i-1}}{x_i - x_{i-1}}\right)(a_i - a_{i-1}) \right] x \qquad (13)$$

$$x \in [x_{i-1}, x_i], \quad i = \overline{1, n}.$$

If all values to the coefficient of population' growth are equal each other, then the fuzzy model degenerates in the Malthus model (1) for arbitrary function of membership.

Partial case, when the function of membership is determined as linear dependence

$\varphi(\alpha) = \alpha$

is considered in article [15].

For the linear function of membership the model of Verhulst (3) turns out from an fuzzy model (13) at $\frac{a_1 - a_0}{x_1 - x_0} = \frac{a_2 - a_1}{x_2 - x_1} = \ldots = \frac{a_n - a_{n-1}}{x_n - x_{n-1}} = -\beta$ and $a_0 = \alpha$.

4. Fuzzy models of co-operations and coexistence of population.

For construction of fuzzy model of coexistence of two populations it is needed to define the value of coefficients of growth of populations in some region of values of quantity of these populations. We will designate a_{jr} and b_{jr} ($j = \overline{0, n}$, $r = \overline{0, m}$) as the value of coefficients on rectangles $[x_0, x_1;\ y_0, y_1]$, $[x_0, x_1;\ y_{k-1}, y_{k+1}]$, $[x_{i-1}, x_{i+1};\ y_0, y_1]$, $[x_{i-1}, x_{i+1};\ y_{k-1}, y_{k+1}]$, $[x_{n-1}, x_n;\ y_{m-1}, y_m]$ ($i = \overline{1, n-1}$, $k = \overline{1, m-1}$).

Let's formulate the system of fuzzy rules Π_{jr} for the coefficients of growth of populations

Π_{00} : If $(x, y) \in [x_0, x_1; y_0, y_1]$,

then $\frac{\frac{dx}{dt}}{x} = a_{00} \quad \frac{\frac{dy}{dt}}{y} = b_{00}$.

Π_{0k} : If $(x, y) \in [x_0, x_1; y_{k-1}, y_{k+1}]$,

then $\frac{\frac{dx}{dt}}{x} = a_{0k} \quad \frac{\frac{dy}{dt}}{y} = b_{0k}$

Π_{i0} : If $(x, y) \in [x_{i-1}, x_{i+1}; y_0, y_1]$,

then $\frac{\frac{dx}{dt}}{x} = a_{i0} \quad \frac{\frac{dy}{dt}}{y} = b_{i0}$

Π_{1k} : If $(x,y)\in[x_0,x_2;y_{k-1},y_{k+1}]$,

then $\frac{\frac{dx}{dt}}{x}=a_{1k}$ $\frac{\frac{dy}{dt}}{y}=b_{1k}$

Π_{i1} : If $(x,y)\in[x_{i-1},x_{i+1};y_0,y_2]$,

then $\frac{\frac{dx}{dt}}{x}=a_{i1}$ $\frac{\frac{dy}{dt}}{y}=b_{i1}$

. . . .

$\Pi_{i,k}$: If $(x,y)\in[x_{i-1},x_{i+1};y_{k-1},y_{k+1}]$, (14)

then $\frac{\frac{dx}{dt}}{x}=a_{ik}$ $\frac{\frac{dy}{dt}}{y}=b_{ik}$

. . . .

Π_{nk} : If $(x,y)\in[x_{n-1},x_n;y_{k-1},y_{k+1}]$,

then $\frac{\frac{dx}{dt}}{x}=a_{nk}$ $\frac{\frac{dy}{dt}}{y}=b_{nk}$

Π_{im} : If $(x,y)\in[x_{i-1},x_{i+1};y_{m-1},y_m]$,

then $\frac{\frac{dx}{dt}}{x}=a_{im}$ $\frac{\frac{dy}{dt}}{y}=b_{im}$

Π_{nm} : If $(x,y)\in[x_{n-1},x_n;y_{m-1},y_m]$,

then $\frac{\frac{dx}{dt}}{x}=a_{nm}$ $\frac{\frac{dy}{dt}}{y}=b_{nm}$.

We will give the function of membership $\lambda_{jr}(x,y)$ ($j=\overline{0,n}$, $k=\overline{0,m}$) which determines the degree of validity of the rule Π_{jr} or values a_{jr} and b_{jr} as coefficients of growth of populations as product of functions

$\lambda_{jr}(x,y)=\mu_j(x)\eta_r(y)$ (15)

thus the function of membership $\mu_j(x)$ is given by formulae (11), and function $\eta_r(y)$ analogously in a form

$$\eta_0(y)=\begin{cases}1 & y=y_0\\ 1-\psi\left(\frac{y-y_0}{y_1-y_0}\right) & y\in(y_0,y_1]\\ 0 & y\notin[y_0,y_1]\end{cases},$$

. . . .

$$\eta_r(y)=\begin{cases}\psi\left(\frac{y-y_{r-1}}{y_r-y_{r-1}}\right) & y\in[y_{r-1},y_r]\\ 1-\psi\left(\frac{y-y_r}{y_{r+1}-y_r}\right) & y\in(y_r,y_{r+1}]\\ 0 & y\notin[y_{r-1},y_{r+1}]\end{cases}$$

$$r=\overline{1,m-1}, \tag{16}$$

. . . .

$$\eta_m(y)=\begin{cases}\psi\left(\frac{y-y_{m-1}}{y_m-y_{m-1}}\right) & y\in[y_{m-1},y_m]\\ 0 & y\notin[y_{m-1},y_m]\end{cases},$$

where a function $\psi(\alpha)$ is determined on an segment $[0,1]$, thus $\psi(0)<<1$ (or $\psi(0)=0$) $\psi(1)=1$. In obedience to individual exits from rules Π_{jr} ($j=\overline{0,n}$, $r=\overline{0,m}$) it is possible to define the values of conclusion, as the weighted-mean values

$$\frac{\frac{dx}{dt}}{x}=\frac{\sum_{r=0}^{m}\sum_{j=0}^{n}\lambda_{jr}(x,y)a_{jr}}{\sum_{r=0}^{m}\sum_{j=0}^{n}\lambda_{jr}(x,y)}$$

$$\frac{\frac{dy}{dt}}{y}=\frac{\sum_{r=0}^{m}\sum_{j=0}^{n}\lambda_{jr}(x,y)b_{jr}}{\sum_{r=0}^{m}\sum_{j=0}^{n}\lambda_{jr}(x,y)} \tag{17}$$

or taking into account formulae (11), (15) and (16) for $(x,y)\in[x_{i-1},x_i;y_{k-1},y_k]$ ($i=\overline{1,n}$; $k=\overline{1,m}$)

$$\begin{aligned}\frac{\frac{dx}{dt}}{x}&=\left[1-\varphi\left(\frac{x-x_{i-1}}{x_i-x_{i-1}}\right)\right]\left[1-\psi\left(\frac{y-y_{k-1}}{y_k-y_{k-1}}\right)\right]a_{i-1\,k-1}+\\&+\varphi\left(\frac{x-x_{i-1}}{x_i-x_{i-1}}\right)\left[1-\psi\left(\frac{y-y_{k-1}}{y_k-y_{k-1}}\right)\right]a_{i\,k-1}+\\&+\left[1-\varphi\left(\frac{x-x_{i-1}}{x_i-x_{i-1}}\right)\right]\psi\left(\frac{y-y_{k-1}}{y_k-y_{k-1}}\right)a_{i-1\,k}+\\&+\varphi\left(\frac{x-x_{i-1}}{x_i-x_{i-1}}\right)\psi\left(\frac{y-y_{k-1}}{y_k-y_{k-1}}\right)a_{i\,k},\end{aligned} \tag{18}$$

$$\frac{\frac{dy}{dt}}{y}=\left[1-\varphi\left(\frac{x-x_{i-1}}{x_i-x_{i-1}}\right)\right]\left[1-\psi\left(\frac{y-y_{k-1}}{y_k-y_{k-1}}\right)\right]b_{i-1\,k-1}+$$
$$+\varphi\left(\frac{x-x_{i-1}}{x_i-x_{i-1}}\right)\left[1-\psi\left(\frac{y-y_{k-1}}{y_k-y_{k-1}}\right)\right]b_{i\,k-1}+$$
$$+\left[1-\varphi\left(\frac{x-x_{i-1}}{x_i-x_{i-1}}\right)\right]\psi\left(\frac{y-y_{k-1}}{y_k-y_{k-1}}\right)b_{i-1\,k}+$$
$$+\varphi\left(\frac{x-x_{i-1}}{x_i-x_{i-1}}\right)\psi\left(\frac{y-y_{k-1}}{y_k-y_{k-1}}\right)b_{i\,k}\quad .$$

After transformations we get formulae for $(x,y)\in[x_{i-1},x_i;y_{k-1},y_k]$

$$\frac{dx}{dt}=x\left[a_{i-1\,k-1}+\varphi\left(\frac{x-x_{i-1}}{x_i-x_{i-1}}\right)\left(a_{i\,k-1}-a_{i-1\,k-1}\right)+\right.$$
$$+\psi\left(\frac{y-y_{k-1}}{y_k-y_{k-1}}\right)\left(a_{i-1\,k}-a_{i-1\,k-1}\right)+$$
$$\left.+\varphi\left(\frac{x-x_{i-1}}{x_i-x_{i-1}}\right)\psi\left(\frac{y-y_{k-1}}{y_k-y_{k-1}}\right)\left(a_{i\,k}-a_{i-1\,k}-a_{i\,k-1}+a_{i-1\,k-1}\right)\right]$$
$$, \qquad (19)$$
$$\frac{dy}{dt}=y\left[b_{i-1\,k-1}+\varphi\left(\frac{x-x_{i-1}}{x_i-x_{i-1}}\right)\left(b_{i\,k-1}-b_{i-1\,k-1}\right)+\right.$$
$$+\psi\left(\frac{y-y_{k-1}}{y_k-y_{k-1}}\right)\left(b_{i-1\,k}-b_{i-1\,k-1}\right)$$
$$\left.+\varphi\left(\frac{x-x_{i-1}}{x_i-x_{i-1}}\right)\psi\left(\frac{y-y_{k-1}}{y_k-y_{k-1}}\right)\left(b_{i\,k}-b_{i-1\,k}-b_{i\,k-1}+b_{i-1\,k-1}\right)\right]$$
.

It is possible to get models which are based on the theory of differential equations from an fuzzy model (19). In particular, if the multipliers of function of membership $\lambda_{jr}(x,y)$ are linear functions and are equal, the model of Volterra (5) turns out from an fuzzy model (19) at

$$\frac{a_{1r}-a_{0r}}{x_1-x_0}=\frac{a_{2r}-a_{1r}}{x_2-x_1}=\ldots=\frac{a_{nr}-a_{n-1\,r}}{x_n-x_{n-1}}=-\beta\,,$$
$$a_{00}=\alpha\,,$$
$$\frac{b_{j1}-b_{j0}}{y_1-y_0}=\frac{b_{j2}-b_{j1}}{y_2-y_1}=\ldots=\frac{b_{jm}-b_{j\,m-1}}{y_m-y_{m-1}}=\delta\qquad ,$$
$$b_{00}=-\gamma\,.$$

Analogously it is possible to get a model (6) et al. This algorithm allows to generalize a model (9) to build the general fuzzy models of coexistence of producers, substrates and consumers, at the case of such parameters are known: experimental and statistical data of coefficients of natural increase and dying off, speeds of consumption, transformation and production of biomass and coefficient of transformation of light for the fixed values of quantity of all populations of the multiple system.

5. Conclusions.

The fuzzy logic extends possibilities of construction of new fuzzy dynamic models which take into account stochastic processes and are based on statistical and experimental information. In particular, on the basis of fuzzy logic after the algorithm of Takagi-Sugeno the fuzzy models of evolution of isolated populations and, also, models of coexistence of two populations are created. In partial case the new models degenerate in the known models of Malthus and Volterra.

1. Bazykin A.D. *Mathematical biophysics of coexistence populations*. Moscow, 1985. (in Russian)
2. Borisov A.N., Alekseev A.V., Krumberg O.A. *Modells of decision-making on the base of lingvistic variable*. Riga: Zinatne, 1982. 256 p. (In Russian)
3. *Genetic algorithms, artifical neuron network and problems of virtual reality* / Voronovsky G.K., Makhotilo K.V., Petrashov C.N. Sergeev S.A.- Kharkov: Osnova, 1997. – 112 p. (in Russian)
4. Dli M.I., Kruglov V.V., Osokin M.V. *Local approximation models of social-economic systems and processes*. Moscow: Nayka, Phizmathlit, 2000. (In Russian)
5. Kolmogorov A.M. Qualitative analysis of mathematical models of populations. *Problems of Cybernetics*. Vol. 25. 1972, pp.100-106. (In Russian)
6. Kruglov V.V., Dli M.I. *Intellectual information's systems: computer support systems of fuzzy logic and fuzzy output*. Moskow.: Phizmathlit, 2002. (In Russian)
7. Kruglov V.V., Dli M.I., Golunov R.Yu. *Fuzzy logic and artifical neuron networks* (In Russian)
8. Kurdjumov S.P., Malinetsky G.G., Synergetics of theory of self-organization. Ideas. *Methods. Perspectives. Mathematics, Cybernetics.* Moscow, 1973. (In Russian)
9. Lajtfoot E. *Phenomenon of transfer in the living systems*. Moscow, 1983. (In Russian)
10. Ljashenko I.M., Mukojed A.P. *Design of biological and ecological processes*. Kyiv: VPC „Kyiv university", 2002. (in Ukrainian)
11. Svirezhov Yu.M. *Nonlinear systems, dissipative systems and catastrophes in ecology*. Moscow, 1987. (In Russian)

12. Tenenjov V.A., Jakimovich B.A., Paklin N.B. Fuzzy modeling of dynamical systems // *Information technologies in innovation projects Proc. IV of Inernational. sc.-techn. conference.* (Izhevsk, 29-30 maj 2003). Vol. 4. Izhevsk: IzhSTU, 2003, pp.96-98. (In Russian)
13. Tenenjov V.A., Jakimovich B.A. *Methods of analisis and modeling of systems*. Izhevsk: IzhSTU, 2002. (In Russian)
14. Mastorakis N.E. General fuzzy systems as extensions of the Takagi-Sugeno methodology // Proc. of *the 4-th WSEAS Int. Conf on Fuzzy Sets and Fuzzy Systems*, Udine, Italy, March 25-27, 2004.
15. Mastorakis N.E. Modeling dynamical systems via the Takagi-Sugeno fuzzy model. Proc. of *the 5-th WSEAS Int. Conf on Fuzzy Sets and Fuzzy Systems,* Udine, Italy, March 25-27, 2005
16. Takagi T, Sugeno M. *Fuzzy identification of systems and its applications to modeling and control.* IEEE Trans. Syst., Man, Cybern. Vol.15, 1985, pp. 116-132.