Magnetic Field Effects on Coupled Heat and Mass Transfer by Mixed Convection along a Vertical Surface Embedded in a Porous Medium by Integral Methods

CHING-YANG CHENG
Department of Mechanical Engineering
Southern Taiwan University of Technology
1, Nantai Street, Yungkang 71005
TAIWAN

Abstract: - This work uses the integral method to study the heat and mass transfer by mixed convection from vertical plates with constant wall temperature and concentration in porous media saturated with an electrically conducting fluid in the presence of a transverse magnetic field. The approximate integral solutions are found to be in reasonable agreement with the similarity solutions. Results are plotted for the local Nusselt number, the local Sherwood number, and the ratio of the thermal boundary layer thickness to the concentration boundary layer thickness. Increasing the buoyancy parameter tends to increase the local Nusselt number and the local Sherwood number, while increasing the magnetic parameter decreases the local Nusselt number and the local Sherwood number.

Key-Words: - Magnetic field, Porous medium, Integral method, Mixed convection

1 Introduction

Many researchers have studied the flows of heat and mass transfer by natural convection or by mixed convection of Newtonian fluids in porous media because of their importance in geophysical and geothermal applications, such as the surface mass transfer on bed rock generated by chemical reaction, the underground disposal of nuclear wastes where the failure of canisters may cause the spread of radioactive materials, and the spreading of chemicals in saturated soil.

For natural convection heat and mass transfer, Khan and Zebib [1] studied the double-diffusive instability of the double boundary-layer structure near a vertical surface in temperature and concentration stratified porous media. Bejan and Khair [2] studied the natural convective flows near a vertical surface driven by temperature and concentration gradients in fluid saturated porous media. Lai and Kulacki [3] have examined the natural convection boundary layer flow along a vertical surface with constant heat and mass flux including the effect of wall injection. Nakayama and Hossain [4], and Singh and Queeny [5] employed the integral method to obtain the analytic solution of couple heat and mass transfer due to buoyancy along a vertical surface in fluid saturated porous media with constant wall temperature and concentration. Cheng [6] studied the influence of a magnetic field on the coupled heat and mass transfer by natural convection from vertical surfaces with constant wall temperature and concentration by the integral method.

For mixed convection heat and mass transfer, Lai [7] studied the coupled heat and mass transfer by mixed convection from a vertical plate in a saturated porous medium. Yih [8] examined the influence of transpiration on coupled heat and mass transfer in mixed convection over a vertical plate embedded in a saturated porous medium.

The flows of electrically conducting fluids over surfaces in the presence of a transverse magnetic field are of much importance because of various geophysical and industrial applications [6, 9-10]. Aldoss et al. [9] studied the mixed convection from vertical surfaces in a porous medium under the effect of a magnetic field. Chamkha [10] studied the hydromagnetic flow and heat transfer of a heat-generating fluid over a surface embedded in a porous medium.

Motivated by the works mentioned above, the present work uses the integral method to study the problem of laminar boundary layer heat and mass transfer by mixed convection from a vertical surface with constant wall temperature and concentration in porous media saturated with an electrically conducting fluid under the influence of a transverse magnetic field. The results obtained in the present work are compared with the similarity solutions obtained by previous studies to check the accuracy of the integral method. With the closed form analytical solution, the practicing engineer can easily and quickly obtain the heat and mass transfer rates by hydromagnetic mixed convection from a vertical surface in saturated porous media.

2 Problem Formulation

Consider the mixed convection heat and mass transfer along a vertical impermeable plate with constant wall temperature and concentration embedded in a porous medium saturated with an electrically conducting fluid subject to a uniform transverse magnetic field. The flow is laminar, steady-state and two-dimensional, and the properties of the fluid are assumed to be constant and isotropic. The fluid and the porous medium are in local thermodynamic equilibrium. The applied transverse magnetic field is assumed to be uniform, and the magnetic Reynolds number is so small that induced magnetic field can be neglected. Furthermore, the external electric field is assumed to be zero and the electric field due to polarization of charges is neglected.

The x-coordinate is measured from the leading edge of the vertical plate and y-coordinate is measured normal to the plate. The surface is maintained at a temperature T_w different from the porous medium temperature T_∞ sufficiently far from the surface of the plate. Moreover, the concentration of a certain constituent in the solution that saturates the porous medium varies from C_w on the fluid side of the vertical surface to C_∞ sufficiently far from the surface of the plate.

With introducing the boundary layer and Boussinesq approximations, the equations governing the conservation of mass, momentum, energy and constituent for Darcy flow along a vertical surface in a porous medium saturated with an electrically conducting fluid can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\left(1 + \frac{K\sigma B_0^2}{\varepsilon\mu}\right) = u_\infty + \frac{\rho g K}{\mu}\left[\beta_t\left(T - T_\infty\right) + \beta_c\left(C - C_\infty\right)\right] \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

The boundary conditions are defined as follows:

$$y = 0;\ T = T_w,\ C = C_w,\ v = 0 \tag{5}$$

$$y \to \infty;\ C \to C_\infty,\ T \to T_\infty,\ u \to u_\infty \tag{6}$$

In the above equations, u and v are the volume-averaged velocity components, T and C are temperature and concentration, respectively. Properties μ, ρ, σ, and B_0 are the solution viscosity, density, electrical conductivity, and magnetic induction, respectively. K and ε are the permeability and the porosity of the porous medium, respectively. β_t and β_c are the coefficient of thermal expansion and the coefficient of concentration expansion, respectively. The thermal diffusivity α is defined as the thermal conductivity of the fluid saturated porous medium, divided by the specific heat capacity of the fluid alone. The mass diffusivity D is the diffusivity of the constituent of interest measured through the fluid-saturated porous medium. The uniform velocity of he external flow is denoted by u_∞.

Integrating Eqs. (3) and (4) about y from 0 to ∞ and using Eq. (1), we then get the following integral equations:

$$\frac{\partial}{\partial x}\int_0^\infty u\theta\, dy = -\alpha\left.\frac{\partial\theta}{\partial y}\right|_{y=0} \tag{7}$$

$$\frac{\partial}{\partial x}\int_0^\infty u\phi\, dy = -D\left.\frac{\partial\phi}{\partial y}\right|_{y=0} \tag{8}$$

where $\theta = \dfrac{T - T_\infty}{T_w - T_\infty}$ and $\phi = \dfrac{C - C_\infty}{C_w - C_\infty}$.

To satisfy the boundary conditions, Eqs. (5) and (6), the profiles of dimensionless temperature and concentration is assumed to be the following functions:

$$\theta = 1 - 2\frac{y}{\delta_t} + 2\left(\frac{y}{\delta_t}\right)^3 - \left(\frac{y}{\delta_t}\right)^4 \tag{9}$$

$$\phi = 1 - 2\frac{y}{\delta_c} + 2\left(\frac{y}{\delta_c}\right)^3 - \left(\frac{y}{\delta_c}\right)^4 \tag{10}$$

It should be noted that the temperature and concentration profile functions defined in Eqs. (9) and (10) also satisfy the compatibility conditions and the smoothness conditions:

$$y=0;\ \frac{\partial^2\theta}{\partial y^2}=0,\ \frac{\partial^2\phi}{\partial y^2}=0 \tag{11}$$

$$y\to\infty;\ \frac{\partial\theta}{\partial y}\to 0,\ \frac{\partial\phi}{\partial y}\to 0,\ \frac{\partial^2\theta}{\partial y^2}\to 0,\ \frac{\partial^2\phi}{\partial y^2}\to 0 \tag{12}$$

Eq. (11) is obtained by evaluating Eqs. (3) and (4) at $y=0$. Using Eqs. (2), (9) and (10), and integrating Eqs. (7) and (8) about y from 0 to ∞, we can get two ordinary differential equations for the thermal boundary-layer thickness δ_t and the concentration boundary-layer thickness δ_c. The solutions of two ordinary differential equations can be given by

$$\delta_t=\delta_t^*\frac{x}{\sqrt{Pe}} \tag{13}$$

$$\delta_c=\delta_c^*\frac{x}{\sqrt{Pe}} \tag{14}$$

In Eqs. (13)-(14), $Pe=u_\infty x/\alpha$ is the Peclet number and write the unknowns δ_t^* and δ_c^* as

$$\delta_t^*=2\left(1+M^2\right)^{1/2}\left[0.3+0.1825Ra/Pe+NF(\Delta)Ra/Pe\right]^{-1/2} \tag{15}$$

$$\delta_c^*=2\left[\left(1+M^2\right)/Le\right]^{1/2}\left[0.3+0.1825NRa/Pe+\Delta F(\Delta)Ra/Pe\right]^{-1/2} \tag{16}$$

In Eqs. (15)-(16), $Ra=K\rho\beta_t g(T_w-T_\infty)x/(\alpha\mu)$ is the Darcy-modified Rayleigh number $\Delta=\delta_t/\delta_c$ is used to represent the ratio of the boundary layer thickness, $Le=\alpha/D$ is the Lewis number, $N=\beta_c(C_w-C_\infty)/[\beta_t(T_w-T_\infty)]$ is the buoyancy ratio, $M^2=\left(K\sigma B_0^2\right)/(\varepsilon\mu)$ is the square of the magnetic parameter, and the function $F(\Delta)$ can be expressed as

$$F(\Delta)=\frac{3}{10}-\frac{2}{15}\Delta+\frac{3}{140}\Delta^3-\frac{1}{180}\Delta^4\ \text{for}\ \Delta<1 \tag{17}$$

$$F(\Delta)=\frac{3}{10\Delta}-\frac{2}{15\Delta^2}+\frac{3}{140\Delta^4}-\frac{1}{180\Delta^5}\ \text{for}\ \Delta>1 \tag{18}$$

Rates of heat and mass transfer from the wall to the fluid are represented by the local Nusselt number and the local Sherwood number given by

$$Nu=\frac{2}{\delta_t^*}\sqrt{Pe} \tag{19}$$

$$Sh=\frac{2}{\delta_c^*}\sqrt{Pe} \tag{20}$$

Table 1. Comparison of the local Nusselt number for $M=0$, $Le=1$ and $N=0$

Ra/Pe	$Nu/\sqrt{Pe}$		
	Exact (a)	Present (b)	Relative error $(b-a)/a$
0	0.5642	0.5477	-0.02925
1	0.7206	0.6947	-0.03594
10	1.5163	1.4579	-0.03851
30	2.4981	2.4034	-0.03790
50	3.1909	3.0703	-0.03779
100	4.4763	4.3074	-0.03773
500	9.9555	9.5692	-0.03880

where $Nu=hx/k$ and $Sh=h_m x/D$, where h and h_m are the local heat transfer coefficient and the local mass transfer coefficient, respectively.

3 Problem Solution

In order to verify the accuracy of our present integral mrethod, we compare the present results for the Nusselt number with those obtained by Hsieh et al. [11] for mixed convection heat transfer along a vertical surface with constant wall temperature embedded in a fluid-saturated porous medium ($N=0$, $Le=1$, and $M=0$), as shown in Table 1. It is clearly seen from the table that the present integral results are in acceptable agreement with the similarity solutions reported by Hsieh et al. [11]. The absolute value of maximum percentage error found in Table 1 is about 3.88 %.

Fig. 1 depicts the variation of the local Nusselt number $Nu/\sqrt{Pe}$ with the buoyancy parameter Ra/Pe for various values of the magnetic parameter M. The results show the local Nusselt number dereases with a increase in the magnetic parameter and increases with higher values of the buoyancy parameter. The reason for such a behavior is that incresing the buoyancy parameter implies increasing the buoyancy force, accelerating the flow and thus increasing the heat transfer between the fluid and the wall. Enhancing the magnetic field tends to decrease the fluid flow velocity and thus decreases the local Nusselt number.

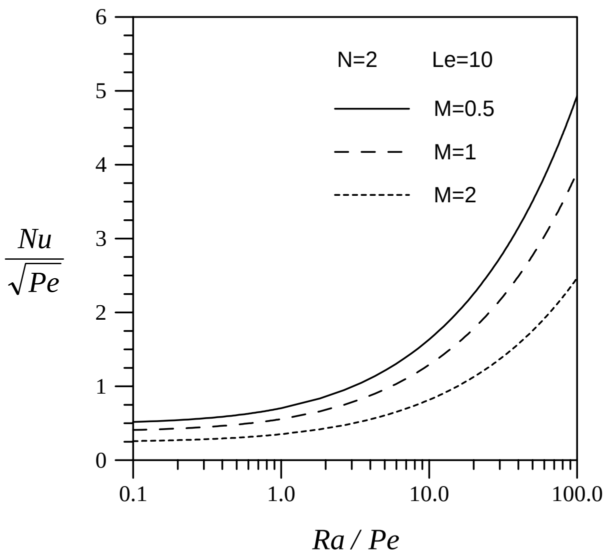

Fig. 1. Variation of local Nusselt number with buoyancy parameter for various values of magnetic parameter.

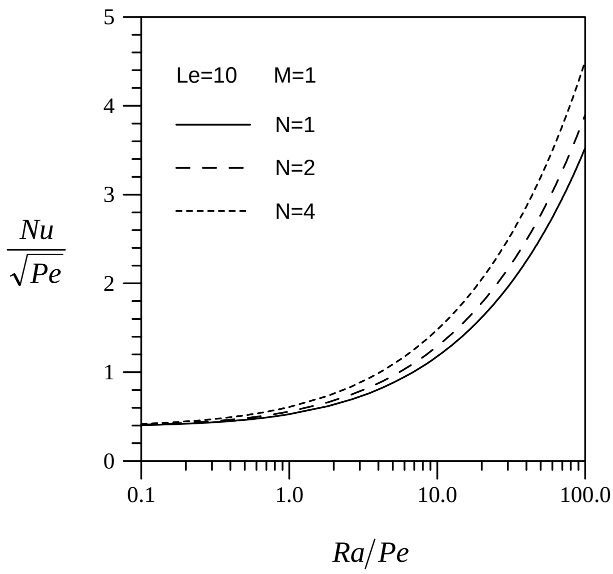

Fig. 3. Effect of buoyancy ratio on the local Nusselt number.

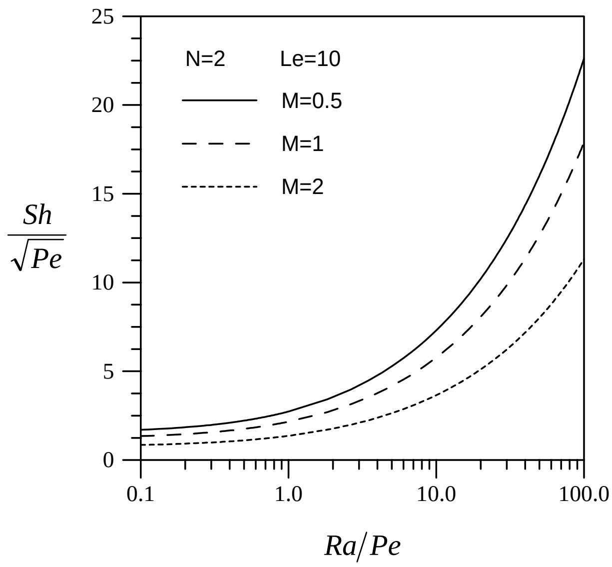

Fig. 2. Variation of local Sherwood number with buoyancy parameter for various values of magnetic parameter.

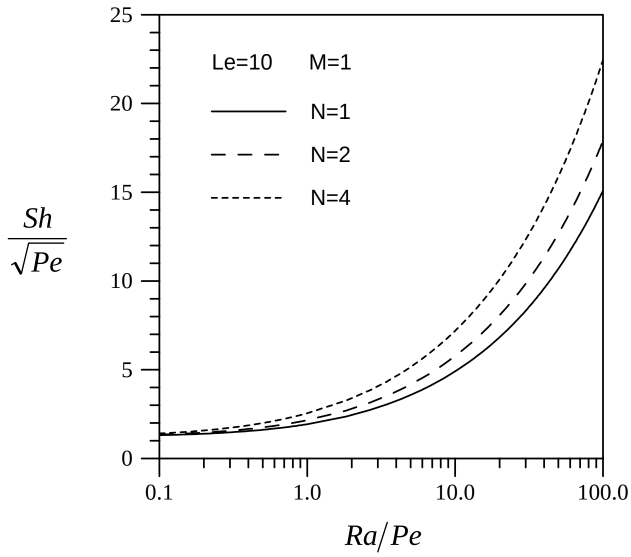

Fig. 4. Effect of buoyancy ratio on the local Sherwood number.

Fig. 2 plots the local Sherwood number $Sh/\sqrt{Pe}$ as a function of the buoyancy parameter Ra/Pe for various values of the magnetic parameter M. It is clearly shown that the applied magnetic field tends to decrease the local Sherwood number. Moreover, increasing the buoyancy parameter increases the local Sherwood number. That is because increasing the buoyancy parameter tends to increase the buoyancy force, accelerating the flow and thus increasing the mass transfer between the fluid and the wall. The decrease in the fluid flow velocity due to applied magnetic field results in the decrease of the local Sherwood number.

The influence of the buoyancy ratio N on the local Nusselt number $Nu/\sqrt{Pe}$ is depicted in Fig. 3. It is clearly shown that the local Nusselt number increases as the buoyancy ratio N is increased. Increasing the buoyancy ratio N leads to an increase in the buoyancy force, accelerating the flow and thus increasing the heat transfer between the surface and the fluid.

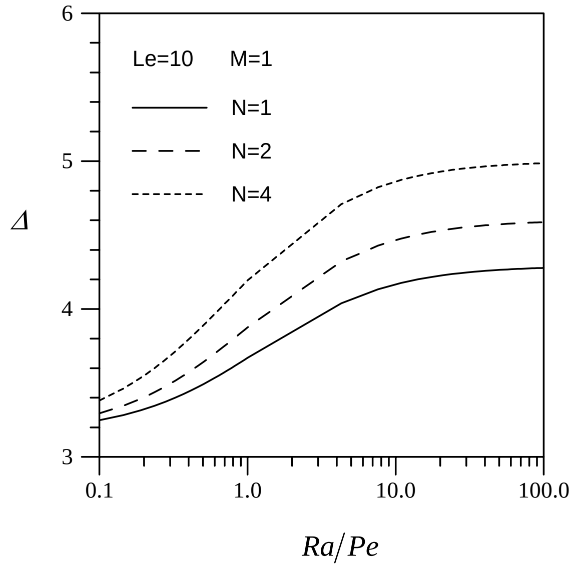

Fig. 5. Effect of buoyancy ratio on the ratio of the thermal boundary layer thickness to the concentration boundary layer thickness.

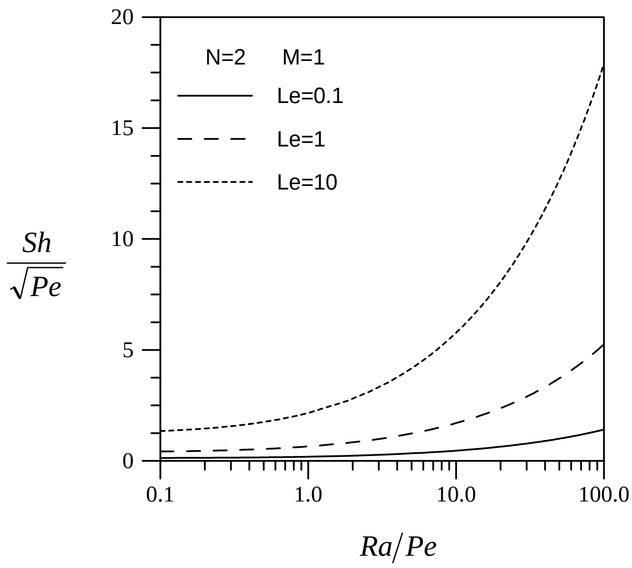

Fig. 7. Effects of Lewis number on the local Sherwood number.

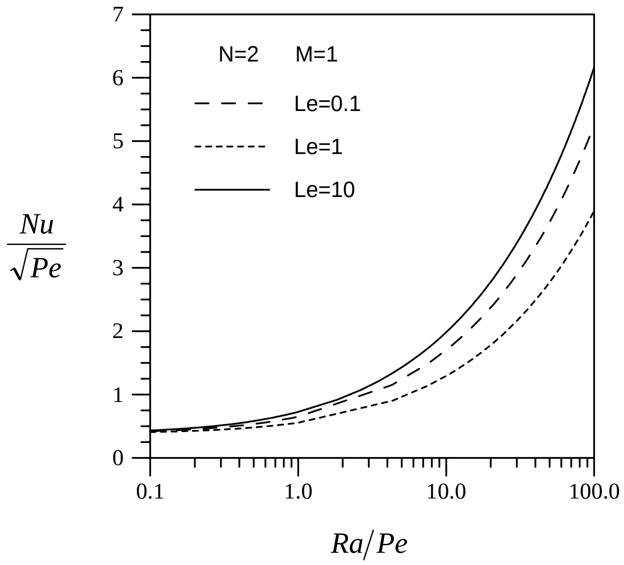

Fig. 6. Effects of Lewis number on the local Nusselt number.

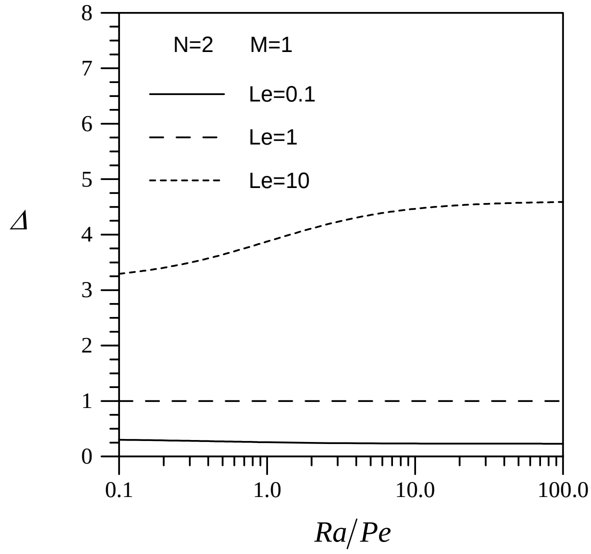

Fig. 8. Effect of Lewis number on the ratio of the thermal boundary layer thickness to the concentration boundary layer thickness.

Fig. 4 shows the influence of the buoyancy ratio N on the local Sherwood number $Sh/\sqrt{Pe}$. The local Sherwood number $Sh/\sqrt{Pe}$ tends to increase as the buoyancy ratio N is increased. That is because an increase in the buoyancy ratio tends to accelerate the flow, thinning the concentration boundary layer and thus enhancing the mass transfer near the wall.

The effect of the buoyancy ratio N on the ratio of the thermal boundary layer thickness to the concentration boundary layer thickness Δ is depicted in Fig. 5. It is clearly shown in this figure that an increase in the buoyancy ratio tends to increase the ratio of the thermal boundary layer thickness to the concentration boundary layer thickness.

Fig. 6 shows the effect of the Lewis number Le on the local Nusselt number $Nu/\sqrt{Pe}$. Comparing the three cases in Fig. 6, we can conclude that as the Lewis number Le is increased, the local Nusselt number $Nu/\sqrt{Pe}$ tends to decrease for aiding flow ($N > 0$). The influence of the Lewis number Le on the local Sherwood number $Sh/\sqrt{Pe}$ is shown in Fig. 7. The results show that the local Sherwood number tends to increase as the Lewis number is increased.

Fig. 8 depicts the effect of the Lewis number Le on the ratio of the thermal boundary layer thickness to the concentration boundary layer thickness Δ. The ratio of the thermal boundary layer thickness to the concentration boundary layer thickness increases with increasing buoyancy parameter for the case of $Le > 1$, and decreases with increasing buoyancy parameter for the case of $Le < 1$, while it is found to be independent of the buoyancy ratio for the case of $Le = 1$.

4 Conclusion

An integral approach has been used to study the coupled heat and mass transfer by mixed convection from a vertical plate with constant wall temperature and concentration embedded in porous media saturated with an electrically conducting fluid in the presence of a transverse magnetic field. The approximate solutions obtained from the present integral method are in reasonable agreement with the exact similarity solutions. This work presents the closed-form analytic expressions with which engineers can easily and rapidly calculate physical characteristics of heat and mass transfer for any value of buoyancy ratios, Lewis numbers, buoyancy parameters, and magnetic parameters. Moreover, increasing the buoyancy parameters increases the local Nusselt number and the local Sherwood number, while an increase in the magnetic parameter tends to decrease the local Nusselt number and the local Sherwood number. The ratio of the thermal boundary layer thickness to the concentration boundary layer thickness increases with increasing buoyancy parameter for the case of $Le > 1$, and decreases with increasing buoyancy parameter for the case of $Le < 1$, while it is found to be independent of the buoyancy ratio for the case of $Le = 1$.

Acknowledements

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