Fatigue based structural design optimization implementing a generalized Frost-Dugdale crack growth law

K. KRISHNAPILLAI, R. JONES and D. PENG
CIEAM, Department of Mechanical Engineering
Monash University
Wellington Road, Clayton, Melbourne, Victoria, 3800
AUSTRALIA
http://www.monash.edu.au

Abstract: - The structural optimisation procedure proposed integrates 3D geometrical modelling, structural analysis and optimization into one complete and automated computer-aided design process. This approach includes the implementation of the generalised Frost-Dugdale model, in which recent observation has revealed a near log-linear relationship between natural log of the crack length and the fatigue life for crack growth lengths as small as a few microns. Consequently, design against fatigue failure can include the analysis of near-threshold crack propagation, i.e. growth in the low-to-mid stress intensity factor range. The research presented here uses the generalised Frost-Dugdale model in a 3D numerical fatigue based optimisation study of a 7050-T7451 Aluminium structure. Two types of optimisation techniques were considered for this investigation; a gradient-less based and an enumeration scheme. The enumeration scheme takes advantage of a cluster computer architecture to produce a solution space. As a result, this procedure illustrates that for the design of light weight structures, a fatigue based optimisation used in conjunction with visualisation of the solution space may provide a viable design methodology. Furthermore, the possibility of the application of the Generalised Frost-Dugdale model in design optimisation has been demonstrated. This procedure has the potential to be applied to structures with complex structural configurations taking into account crack propagation in the near-threshold region.

Key-Words: - Structural optimisation; Finite element; Cluster computing; Short crack growth

1 Introduction

Structural optimisation is being increasingly used to design lightweight structural components and develop localized shape rework (life extension) strategies to restore ageing structural components. Several commercial CAE packages with fully integrated CAD modelling, structural analysis and structural optimisation tools have been developed to design stronger, lighter and safer structures. Therefore, the subject of non-linear, finite dimensional optimisation is relatively mature. However, these optimum design tools do not consider either failure by fracture or fatigue, the effects of initial cracks or damage tolerance issues. It is within this area of durability that the following research has been focussed upon. Several structural optimisation algorithms in literature have been developed to account for damage tolerance issues and initial cracks, but to date few have analysed the effect of near-threshold crack propagation, i.e. short initial cracks in the low-to-mid ΔK region. This is mainly due to the fact that these structural optimisation algorithms only use Paris like laws which are applicable only in Region II (Paris region) and not the low-to-mid ΔK region. The growth rate of a small crack must be known in order to accurately estimate the fatigue life because it has been found that fatigue lives of components are controlled mainly by the propagation of a small cracks [1]. Such growth may occur at greater rates then expected, from an extrapolation of the Paris law regime to include low stress intensity values, and thus can cause component failures at a fraction of predicted life time. Therefore it is clear that the Paris Law over predicts the crack growth rate

There is also the assumption in the literature that; if the crack size is very small in comparison to the length scale associated with the cut out, the local rework or the structural detail being optimised, then the stress intensity factors will be directly related to the tangential stress around the cut out [2]. This suggests that a stress based solution will produce an identical optimised geometry to a fatigue based solution if the crack length is short. For these reasons the development of an optimisation procedure involving near-threshold crack propagation was investigated. In this context the present paper will utilise the 'Generalised Frost-Dugdale' law in a design optimisation procedure. The results were compared to Newman's law which is a Paris-like law.

2 Numerical Formulation

2.1 Crack Growth Model

2.1.1 Paris Law

Fatigue crack growth has traditionally revolved around the belief that the crack growth rate, *da/dN* can be related to the stress intensity factor range, ΔK, and/or the maximum stress intensity factor K_{max}. This correlation was first suggested by Paris et al [3] and resulted in the well know Paris equation:

$$da / dN = C(\Delta K)^m \tag{1}$$

where C and m are experimentally obtained constants, which are considered to be constant for a particular material. This relationship has had a number of modifications to account for various observations, such as R ratio ($R = K_{min} / K_{max}$), K_{max} effects [4, 5, 6] and closure effects [7, 8]. The NASA fatigue crack growth structural analysis program [7] implements Newman's law, which is a closure effect variant of the Paris law. The Paris law, and its variants are only applicable in the Paris region, Region II.

2.1.1 Generalised Frost-Dugdale Law

Recent observation has revealed, for constant amplitude loading, a near log-linear relationship between natural log of the crack length and the fatigue life for crack growth lengths as small as a few microns in the near-threshold region [9, 10, 11]. From these observations Barter et al. [9] presented a generalised Frost-Dugdale crack growth law to describe this relationship.

$$\frac{da}{dN} = C(a)^{1-\frac{m}{2}}(\Delta K_{eff})^m \tag{2}$$

where C and m are constants, and ΔK_{eff} is the effective stress intensity factor. It has been shown in [9, 11] that this relationship holds for the 7050-T7451 aluminium alloy, in which C is 1.78E-10 and m is 3.36. Thus, confirming the implementation of the generalised Frost-Dugdale law for the 7050-T7451 aluminium alloy. The NASA fatigue crack growth structural analysis program [7] has been modified to implement this law.

2.1 3D Optimisation Procedure

The structural optimisation procedure proposed integrates geometrical modelling, structural analysis and optimization into one complete and automated computer-aided design process, termed an 'Integrated Design Optimisation System'. It determines the shape of the boundary of the three-dimensional structural component under geometric constraints and structural conditions.

2.2.1 Structural Analysis

For a common Finite Element Method (FEM), the finite element modelling of a complex three-dimensional crack requires a fine numerical mesh due to the stress singularities along the crack front. This resulted in a large number of degrees of freedom, making the problem become computationally inefficient, extremely time consuming and requiring large computer resources [12]. As a result the NASA fatigue crack growth structural analysis program [7] implemented the Finite Element Alternating method (FEAM), which does not model cracks explicitly and proves to be a very efficient means for solving fracture problems involving single or multiple cracks [2, 12]. FEAM only requires the location of the crack centres in its analysis. Therefore, cracks were placed all along the design surface of the model allowing for an effective modelling of the stress intensity factor variation around the boundary surface. Using these stress intensity factor solutions the fatigue crack growth structural analysis program then uses the appropriate crack growth law to calculate the fatigue life at the crack locations. The objective of this analysis is to provide the necessary information (i.e. the fatigue life at the crack locations of the model) required by the optimisation algorithm.

The cracks specified along the boundary represented the location of the centre of the 3D semi-elliptical cracks. The crack specifications as described in [7] were implemented where the major axis (c) of the semi-elliptical cracks were in the z-direction and the minor axis (a) were normal to the design surface as illustrated in Fig. 1.

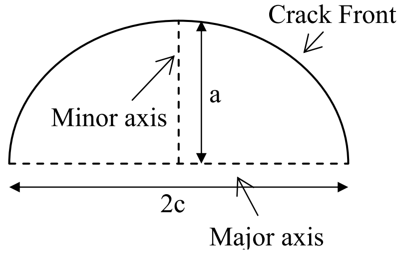


Figure 1. Semi-elliptical crack specifications

2.2.2 Step 3 – Optimisation
Using the information from the fatigue crack growth structural analysis, the optimisation algorithm finds a new set of design variables according to the objective function. In this case it is to maximise the minimum fatigue life. The optimisation models chosen in this study were the biological algorithm (gradient-less algorithm), and an enumeration technique. A new design model is then constructed and fed once more into the analysis. This procedure is repeated until a prescribed termination criterion is satisfied or the objective function reaches a constant state. The objective function in this case is to maximise the minimum fatigue life. For the biological algorithm, the design variables are represented by the geometric points that indicated the crack locations. However, for the enumeration scheme the design variables were the parameters of the equation that represented the design boundary.

2.3 Optimisation Algorithms
2.3.1 Biological Algorithm
The gradient-less algorithm attempts to satisfy iteratively the optimality criteria without calculating the gradients of the objective and constraint functions. It was shown that in the study of shape optimization with fatigue life as a design objective the fatigue life along the boundary were essentially constant. Therefore, it was hypothesised that an optimum shape would be equally fatigue critical along the boundary. According to the biological algorithm the amount to which to move a given design point (node) i on the boundary in the normal direction is expressed by equation [12-16]:

$$d_i = \frac{q_i - q_{th}}{q_{th}} s \tag{4}$$

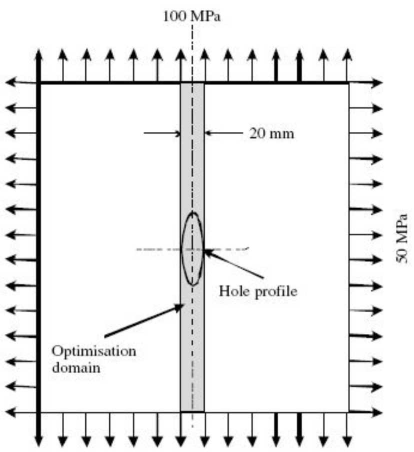
A positive value for d_i indicates material to be added and a negative value indicates material to be removed. s is an arbitrary step size scaling factor, typically a value of 1 [13]. q represents the design objective (fatigue life), and qth is a non-zero fatigue life reference value chosen depending on the type of problem under investigation. In this investigation qth represents the minimum fatigue life along the boundary, thus only allowing material removal. In a simple 3D approach described in [17] the boundary is represented by a three-dimensional surface and the displacement of the design points would be represented by a vector of the form:

$$\vec{d}_i = d_i\left(n_x\hat{i} + n_y\hat{j} + n_z\hat{k}\right) \tag{5}$$

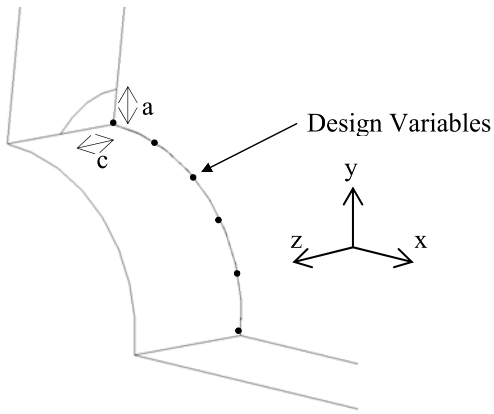
2.3.2 Enumeration Technique
The Enumeration technique calculates the objective function solutions at every point within a finite search space, or a discretized infinite search space, i.e the creation of a solution space from the parameters (design variables) in the design space. Due to the development of cluster computing it is possible to calculate a number of solutions in parallel. This allows a larger design space, a larger amount of parameters and increases the efficiency. The NIMROD/G [18] system is used in this research as it allows an automated parameter sweep generation as well as job distribution for multiple resources in simultaneously running multiple parameter combinations.
NIMROD is a package for distributed parametric modelling. It manages the execution of parametric studies across distributed computers. NIMROD/G is a Grid aware version of NIMROD and its capabilities were utilized in this research in conjunction with the Victorian Partnership for Advanced Computing (VPAC) cluster. The VPAC cluster consists of a 97 node, 194 CPU Linux cluster based on Xeon 2.8 GHz CPUs. The paper by Jones et al. [2] illustrates the advantages of using the NIMROD package in conjunction with cluster computer architecture and highlights the benefits of visualising the solution space.

3 Numerical Analysis
The problem of a through-hole in a rectangular block under biaxial loading was considered. It consisted of a 10 mm thick 7050-T7451 Aluminium alloy with a length and height of 200 mm and a centrally located hole with a radius of 10 mm. The block was subjected to a uniform biaxial tensile stress field of 100 MPa × 50 MPa as illustrated in Fig. 2a. Due to the symmetry of the model, only an eighth of the structure was modelled to increase the efficiency of the optimisation procedure (i.e. a quarter of the structure and half its thickness, illustrated with crack specifications in Fig 2b). The material properties used were Young's modulus = 71.7 GPa and Poisson's ratio = 0.30. The optimisation domain as illustrated by the shaded area in Fig. 2a constrains the hole in the x-direction by 10 mm. The objective of this problem was to obtain the optimal geometry of the centrally located through-hole.
The variation in the crack dimensions were restricted according to the USAF specifications in the USAF damage tolerance handbook [19], JSSG-2006 paragraph A3.12. This states that a 0.040 in.

(~1.0mm) initial crack size is required for holes and cut outs. Therefore, in this research, initial crack lengths greater than 1 mm were categorised as medium and large cracks, whereas initial crack lengths less than 1 mm where categorised as small cracks. The final crack size remained a constant and was chosen to be 8 mm.



(a)



(b)

Figure 2. a) Full Schematic of the 7050-T7451 aluminium alloy. b) 1/8 model with crack specs

3.1 Fatigue Crack Growth Law Comparison Study

3.1.1 Comparison Using the Biological Algorithm

The minimum fatigue life of the optimised structure for both fatigue crack growth law with each crack configuration are summarised in Table 1, along with the relative difference of the minimum fatigue life of the optimised structure between the generalised Frost-Dugdale law and Newman's law. The results indicate that when the initial crack size is larger then ~1 mm (Large and Medium category) the generalised Frost-Dugdale law predicts a larger fatigue life than the Newman law. As the initial crack size increases, the difference decreases. An opposite effect is experienced when the initial crack sizes are smaller than ~1 mm (medium/small and small category). In this case the generalised Frost-Dugdale law predicts a smaller fatigue life than the Newman law. As the initial crack size decreases, the relative difference increases. This suggests that if optimised, with the Newman law, a structure may have an over predicted life as stated previously. This is due to the fact that the Newman law is not applicable in the threshold region as explained previously. However, the generalised Frost-Dugdale law was formulated to be applied in the threshold region which suggests a more accurate life prediction for cracks less than ~1 mm. For cracks larger ~1 mm the generalised Frost-Dugdale solutions provide an alternative to the Newman solutions.

Table 2 presents the optimised geometry of the hole for both fatigue crack growth laws with each crack configuration. The hole height represents the height of the hole in a two dimensional (XY) plane as illustrated in Fig. 3.

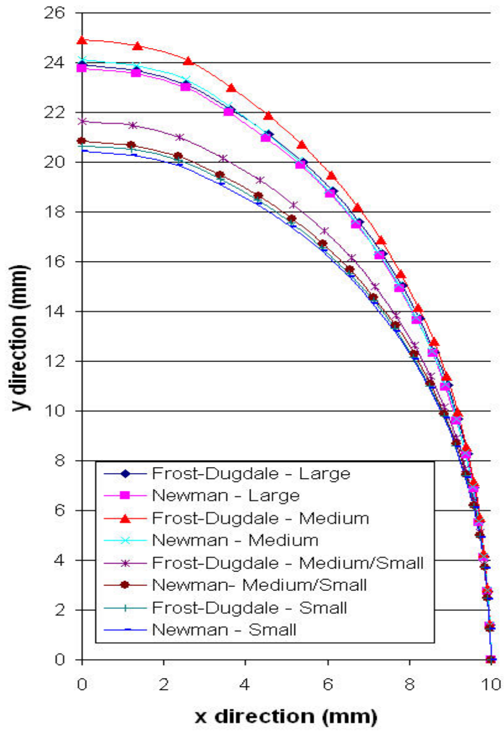


Figure 3. 2-D representation of optimised hole geometries for all cases implementing the biological algorithm.

From the results it is clear that despite the large changes in the predicted fatigue lives, the optimised hole geometry calculated for both fatigue crack growth laws produce little difference. There is a slight improvement for the generalised Frost-Dugdale law of approximately 1-3%.

Table 1 – Comparison of optimised minimum fatigue life

Categorised Crack Length	Crack Specifications (mm)				Objective Function		Difference (%)
	Initial crack length		Final crack length		Minimum Fatigue Life (Cycles)		
	c_i	a_i	c_f	a_f	Newman	G. Frost-Dugdale	
Large	6	6	8	8	2271	2451	8% increase
Medium	3	3	8	8	10560	39587	64% increase
Medium/Small	0.1	0.1	8	8	349960	326550	7% decrease
Small	0.01	0.01	8	8	8695000	3882000	55% decrease

Table 2 – Comparison of optimised hole geometry

Categorised Crack Length	Crack Specifications (mm)				Hole Geometry (mm)		Difference (%)
	Initial crack length		Final crack length		Hole Height		
	c_i	a_i	c_f	a_f	Newman	G. Frost-Dugdale	
Large	6	6	8	8	23.78	23.90	0.52% increase
Medium	3	3	8	8	24.11	24.93	3.42% increase
Medium/Small	0.1	0.1	8	8	20.84	21.65	3.89% increase
Small	0.01	0.01	8	8	20.44	20.65	1.02% increase

However, this can be considered insignificant in this case. This suggests that in the case of the 'hole in a plate' problem the crack growth law has no effect on the optimised geometry. However, the predicted lives and optimised geometry vary depending on the initial crack size. This phenomenon may not occur in other structures.

It was commented previously that the stress based solution will produce an identical optimised geometry to a fatigue based solution if the crack length is small. The papers [2, 12-17] have shown that a stress based solution for this problem is approximately a 2:1 ellipse. This means that the height of the optimised hole is 20mm. The results for the small cracks indicate that a stress based solution is indeed sufficient in this case as there is a less than 1 % difference between the fatigue based solution and stress based solution as illustrated in Table 2. This provides further evidence for the statement above due to the fact that the generalised Frost-Dugdale law is applicable in the threshold region.

3.1.2 Implementation of the Enumeration Technique

The main aim of this section was to produce a solution space that would aid the design optimisation procedure. In this case only the generalised Frost-Dugdale law was utilised for medium and small cracks simply to observe the results of the three-dimensional solution space.

The geometric representation of the design boundary for the enumeration technique was chosen so that an effective geometry could be represented by the least possible amount of parameters. Therefore, the polar equation of an ellipse was selected and the coordinates on the points along the ellipse can be given by the following equations:

$$x = \left(\frac{\cos^p \theta}{a^p} + \frac{\sin^p \theta}{b^p} \right)^{-\frac{1}{p}} \cos\theta \qquad (10)$$

$$y = \left(\frac{\cos^p \theta}{a^p} + \frac{\sin^p \theta}{b^p} \right)^{-\frac{1}{p}} \sin\theta \qquad (11)$$

It is clear from the equations that there are three variables a, b and p. These represent the design parameters for the optimisation procedure. The parameters a and b represent the maximum size of the hole in the x and y directions in the XY plane. However, due to a geometric constraint of the problem (as illustrated in Fig. 2a) the hole is constrained in the x-direction by ±10 mm. Therefore, parameter a becomes a constant value of 10 mm in the symmetrical model. The parameter p describes the curvature of the hole where p is equal to 2 for an ellipse. This resulted in a two parameter (b and p) geometric representation of the hole. All possible values of b and p were searched within a

finite design space and a graphical representation of the solution space was produced.
Table 3 indicates the design parameters (*b, p*) of the optimised ellipse equation and the objective function (minimum life). Table 4 directly compares the height parameter and the objective function with the biological algorithm. It clearly shows that for the medium and small cases both techniques produce similar results. There is a less than 1% difference between the hole height. From earlier results it was clear that a slight change in the hole height can produce a large change in the optimum life.

Table 3. Results from Enumeration Scheme for the generalised Frost-Dugdale law

Categorised Crack Length	Crack Specifications				Hole Geometry Parameters (mm)		Objective Function Minimum Fatigue Life (Cycles)
	Initial crack lengths		Final crack lengths				
	c_i	a_i	c_f	a_f	b	p	
Medium	3	3	8	8	24.21	2.0	41939
Small	0.01	0.01	8	8	20.81	2.1	4707360

Table 4. Comparison of results between optimisation techniques

Categorised Crack Length	Objective Function		Hole Geometry (mm)	
	Minimum Fatigue Life (Cycles)		Hole Height (b)	
	Biological	Enumeration	Biological	Enumeration
Medium	39587	41939	24.93	24.21
Small	3882000	4707360	20.65	20.81

This explains the large difference between the objective functions between techniques. However, the relative difference is small and can be accounted for because of the different techniques used.
Fig. 4 and 5 provide the visualisation of the solution space for medium and small cracks respectively. This allows the designer to observe the occurrence of multiple optimums. For this structure Fig. 4 & 5 clearly indicate the existence of a single maximum. Of notable interest, the biological algorithm approximated close to this optimum. The advantages of visually observing the solution space is clear. If a designer required a specific minimum life, the graphical representation allows the search of the design parameters that would satisfy the designer's requirements. Another advantage is that an optimisation algorithm can be used to narrow the search for an optimum point over a smaller design space, since a full solution space is known. When used in conjunction with cluster based computer architecture this scheme represents an alternative to existing optimisation methods. In general it should be stressed that solutions obtained are best thought of as "better", or local optima, rather than true global optimal solutions.

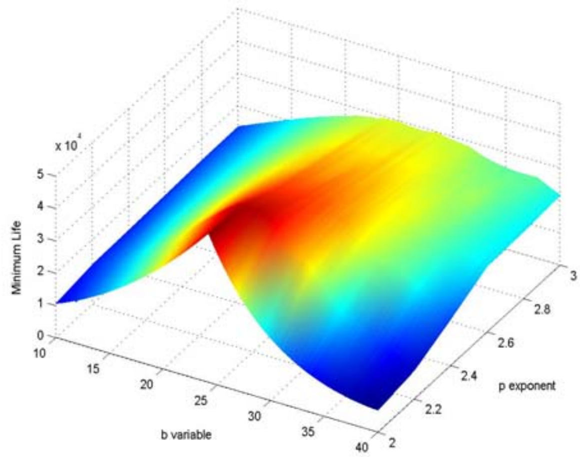


Figure 4 - Three-dimensional solution space representation for medium categorised crack length

4 Conclusion

This research presents a fatigue based optimisation procedure involving the generalised Frost-Dugdale crack growth law and has produced a comparison study with Newman's law for a 7050-T7451 Aluminium structure of a through hole in a plate. This work revealed that the optimal geometry depends very little on the fatigue crack growth law for a 'through hole in a plate' structure. However, the optimal fatigue life depends heavily on the fatigue crack growth law. For large cracks, greater than approximately 1 mm, the generalised Frost-Dugdale law produced larger fatigue lives than the Newman law depending on the initial crack length.

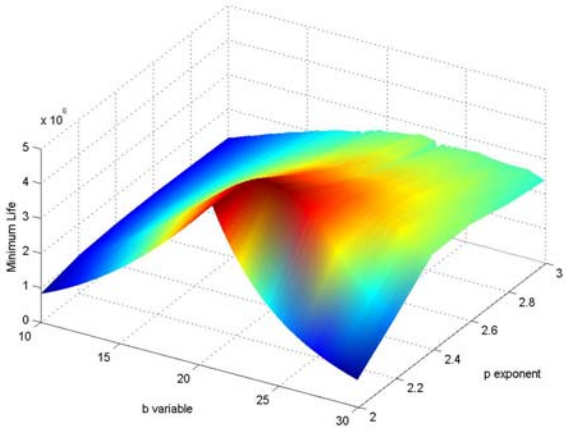


Figure 5 – Three-dimensional solution space representation for small categorised crack length

For small cracks less, than approximately 1 mm, the generalised Frost-Dugdale law produced smaller fatigue lives than the Newman law. The extent of this depended on the initial crack length.

Since Paris like laws are not applicable in the threshold region, the optimised solutions produced by the generalised Frost-Dugdale law provided an alternative optimised solution. Therefore, the optimal geometrical shape and predicted fatigue life depended on the initial crack sizes, structural geometry, boundary conditions and fatigue crack growth law. It was also discovered that for small initial cracks (less than approximately 1mm) the fatigue based solution corresponded to the stress based solution for this design model, confirming the previous statement. However this phenomenon discovered may not occur for other structures.

The applications of the NIMROD/G with the VPAC cluster computer architecture have illustrated the advantage of visualising the solution space. Such solution spaces represent a major challenge for any optimisation procedure. This method represents an alternative to the existing optimisation methods. For the design of light weight structures, a fatigue based optimisation used in conjunction with visualisation of the solution space may provide a viable design methodology. Furthermore, it has the potential to be applied to structures with complex structural configurations taking into account short crack growth.

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