# Selective Harmonic Elimination PWM Based on Walsh Transform and Its Application in Dynamic Voltage Restorer 

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#### Abstract

The Selective Harmonic Elimination PWM (SHEPWM) based on Walsh transform converters nonlinear transcendental equations in the Fourier domain to linear algebraic ones. Under appropriate initial condition, the piecewise linear relation between fundamental voltage amplitude and switching angles is obtained, so voltage regulation and harmonic elimination can be realized online. Thus, the technique highly suits to the inverter of dynamic voltage restorer (DVR). In the paper, Walsh transform, construction and solution of SHE equations in Walsh domain are introduced. The sameness and the difference between switch angles distribution of SHEPWM and of Centroid PWM (CPWM) are summarized, and, according to the relationship between both PWM modes, a new method to solve SHE equation in Walsh domain is proposed. In contrast to conventional ones, the new method effectively reduces blindness of searching initial switch angles in the course of SHE equation solution and improves computation efficiency. The experimental result of 200kVA DVR using SHEPWM, which indicates that its inverter can effectively restrain all the harmonics and has good dynamic performance in all the range of output voltage, proves feasibility and validity of the proposed method.


Key-Words: - Dynamic Voltage Restorer, Inverter, PWM, Selective Harmonic Elimination PWM, Centriod PWM, Walsh function

## 1 Introduction

Dynamic voltage restorer (DVR) is an equipment used to compensate for the dynamic power quality problems such as voltage sag and swell, whose core is a controllable voltage source inverter installed in power system in series ${ }^{[1]}$. As the executor of compensation command, DVR inverter influences the performance of DVR in great degree and it should fleetly regulate output voltage at any moment and keep enough low total harmonic distortion (THD) in the whole range of output voltage.

Pulse width modulation (PWM) mode directly influences even decides the performance of an inverter. Of various PWM modes, Sine PWM (SPWM) is one most frequently used in constant voltage \& constant frequency (CVCF) inverter. However, when an inverter, controlled by SPWM, varies its output voltage amplitude in a wide range, the THD of its output voltage will vary evidently: the lower voltage amplitude is, the higher the THD. While SHEPWM can make the inverter keep a preconcerted THD, no matter how its output voltage varies. In addition, at the same switch frequency, the inverter controlled by SHPWM can raise the frequency of the lowest order of harmonic higher by $33 \% \sim 35 \%$ than the one controlled by SPWM, which
can diminish parameters and size of the filter, thereby raising dynamic performance of the inverter. Thus, for DVR inverter, SHEPWM is a better PWM mode than SPWM ${ }^{[2,3]}$.

Now, research on SHEPWM mainly concentrates on SHEPWM based on Fourier transform, whose biggest obstacle lies in that: the relationship between the amplitude of harmonics of output voltage and switch angles is depicted by nonlinear transcendental equations so that it difficult to solve the equations online. As a result, in project practice, the equation is solved off-line and many groups of data are saved to realize voltage regulation, which not only wastes storage space but reduces flexibility of the system ${ }^{[4]}$.

Asumadu and Hoft proposed SHEPWM based on Walsh transform in [6], which converters nonlinear transcendental equations in the Fourier domain to linear algebraic ones so as to obtain piecewise linear relation between fundamental voltage amplitude and switching angles. [7]~[9] improved the computing method of Walsh coefficient on the basis of [6]. But, all literatures above mentioned gave initial switching angles distribution mode relying on experiences and were short of quantitative basis in the course of solving SHE equations.

The paper proposes a new initialization method to solving SHE equation that based on CPWM
technique, which evidently improves computation efficiency. The construction and solution of SHE equations are illustrated by an unipolar inverter of 200kVA DVR.

## 2 Walsh Functions and Walsh series

Walsh functions, one class of piecewise constant basis functions, are introduced by Walsh in $19233^{[4]}$. These functions form an ordered set of rectangular waveforms taking only two amplitude values: 1 and -1 , over one normalized frequency period. Walsh functions form a complete or orthogonal set, hence, they can be used to represent signals in the same way as the Fourier series. There are three ways of ordering the Walsh functions (natural, dyadic and sequency ordering) depending on the method to generate them. Sequency-ordered Walsh functions, which are chosen in the paper, are arranged in ascending order of zero crossings. Sequency is defined as one half the number of zero crossings over the unit interval $[0,1]$ and is used as a measure of generalized frequency of waveforms ${ }^{[5]}$.

Walsh functions are either symmetrical with respect to their point and are called CAL functions, or asymmetrical and are called SAL functions expressed as followed:

$$
\begin{align*}
& W A L(2 k, t)=C A L(k, t), k=0,1,2 \ldots\left(\frac{N}{2}-1\right)  \tag{1}\\
& W A L(2 k-1, t)=\operatorname{SAL}(k, t), k=1,2 \ldots \cdot \frac{N}{2} \tag{2}
\end{align*}
$$

Similarly to the Fourier series representation, the Walsh series representation of a time function that is absolutely integrable in $[0,1]$ can be defined as:

$$
\begin{equation*}
f(t)=\sum_{m=0}^{\infty} W_{m} \cdot \operatorname{Wal}(m, t) \tag{3}
\end{equation*}
$$

where $W_{m}$ is the coefficient of the Walsh function $\operatorname{Wal}(m, t)$ and is determined by

$$
\begin{equation*}
W_{m}=\int_{0}^{1} f(t) \operatorname{Wal}(m, t) d t \tag{4}
\end{equation*}
$$

For the discrete case, the integral of (4) can be approximated by a summation expression as:

$$
\begin{equation*}
W_{m}=\frac{1}{N} \sum_{i=0}^{N-1} f(i) \operatorname{Wal}(m, i), m=0,1, \cdots, N-1 \tag{5}
\end{equation*}
$$

The discrete Walsh series representation of a time function can be easily obtained:

$$
\begin{equation*}
f(i)=\sum_{m=0}^{N} W_{m} \cdot \operatorname{Wal}(m, i) \tag{6}
\end{equation*}
$$

In Eq. (5) and (6), $\operatorname{Wal}(m, i)$ is the discrete Walsh function corresponding to $\operatorname{Wal}(m, t), f(i)$ is
the ith sampled value of $f(t)$ obtained by sampling $f(t) N$ times during the interval [0,1], and $N$ is an integer power of 2 , the greater the number given, the smaller the mean-square error between the desired waveform and the actual. Chen and Sun ${ }^{[12]}$ proposed "domain-term" ${ }^{[12]}$ to choose the $N$ for synthesizing a waveform if reasonable error is accepted ${ }^{[7,8]}$.

## 3 Inverter SHEPWM Model Based on Walsh Transform

### 3.1 SHE Equation in Walsh Domain ${ }^{[6 \sim 9]}$

The paper takes uniploar single phase inverter used in past experiment as research object. A typical output voltage waveform of the inverter is shown in Fig. 1 in which one quarter period has $M$ switching angles. These switching angles $\alpha_{1}, \alpha_{2}, \cdots, \alpha_{M}$ are used to eliminate some given lower order harmonics and control the fundamental amplitude of the inverter output voltage.


Fig. 1 PWM waveform of unipolar inverter
The discrete Walsh series expression of the output voltage waveform function $f(t)$ illustrated by Fig. 1 is:

$$
\begin{equation*}
f(i)=\sum_{n=1}^{N / 4} W_{4 n-3} \operatorname{Wal}(4 n-3, i) \tag{7}
\end{equation*}
$$

where Walsh coefficient $W_{4 n-3}$ is determined by

$$
\begin{equation*}
W_{4 n-3}=\sum_{n=1}^{N / 4} f(i) \cdot \operatorname{Wal}(4 n-3, i) \tag{8}
\end{equation*}
$$

Based on the relationship between the Fourier series and the Walsh series, the fundamental and every order of harmonics of output voltage can be expressed in Walsh domain as follow:

$$
\left[\begin{array}{c}
U_{m(1)}  \tag{9}\\
U_{m(3)} \\
\cdot \\
U_{m(2 K-1)}
\end{array}\right]=\left[\begin{array}{cccc}
B_{1,1} & B_{1,5} & . . & B_{1,4 N 1-3} \\
B_{3,1} & B_{3,5} & . . & B_{3,4 N \mathrm{~L}-3} \\
. . & . & . . & . \\
B_{2 K-1,1} & B_{2 K-1,5} & . . & B_{2 K-1,4 \mathrm{~N}-3}
\end{array}\right] \times\left[\begin{array}{c}
W_{1} \\
W_{5} \\
\cdot \\
W_{4 N 1-3}
\end{array}\right]
$$

where $N 1=N / 4, U_{m(k)}$ is the amplitude of the $k^{\text {th }}$ harmonic, $\quad B_{2 K-1,4 n-3}$ is Walsh-Fourier
transformation coefficient which can be calculated directly from the equation derived by Siemens and Kitai ${ }^{[10]}$ and $W_{4 n-3}$ is the Walsh coefficient .

Walsh coefficient $W_{4 n-3}$ can be obtained on the grounds of Eq. (8). Assumed that the inverter DC input voltage $U_{d c}=1$ and there are $M$ switching angles in a quarter of the period of waveform function, each period $[0,1]$ is averagely divided into $N$ sbuintervals and a quarter $N 1$ ones. Considering $M$ angles $\alpha_{1}, \alpha_{2}, \cdots, \alpha_{M}$ in the corresponding subintervals $l_{1}, l_{2} \ldots, l_{M}$, namely
$\frac{l_{1}-1}{N}<\alpha_{1}^{\prime}<\frac{l_{1}}{N}, \cdots \cdots, \frac{l_{M}-1}{N}<\alpha_{M}^{\prime}<\frac{l_{M}}{N}$
where $\alpha_{i}^{\prime}=\alpha_{i} / 2 \pi$, Walsh coefficent $W_{m}$ can be derived as[]

$$
\begin{equation*}
W_{m}=\sum_{i=1}^{M}(-1)^{i+1} W\left(m, l_{i}\right) \tag{11}
\end{equation*}
$$

where

$$
\begin{align*}
W\left(m, l_{i}\right) & =-4 \alpha^{\prime} \operatorname{Wal}\left(m, l_{i}\right) \\
& +\frac{4}{N}\left[\sum_{k=l_{i}+1}^{N 1} \operatorname{Wal}(m, k)+l_{i} \operatorname{Wal}\left(m, l_{i}\right)\right] \tag{12}
\end{align*}
$$

and $m=4 n-3 ; n=1,2, \ldots, N 1$

$$
\text { Note } C_{m, i}=(-1)^{i+1} \cdot \operatorname{Wal}\left(m, l_{i}\right)
$$

$$
D_{m}=\sum_{i=1}^{M}(-1)^{i+1} \cdot S\left(m, l_{i}\right)
$$

Then Wash coefficents can be expressed as:
$\left[\begin{array}{c}W_{1} \\ W_{5} \\ . \\ W_{4 N 1-3}\end{array}\right]=\left[\begin{array}{cccc}C_{1,1} & C_{1,2} & . . & C_{1, M} \\ C_{5,1} & C_{5,2} & . . & C_{5, M} \\ . & . & . . & . \\ C_{4 \mathrm{~N} 1-3,1} & C_{4 \mathrm{~N} 1-3,2} & . . & C_{4 \mathrm{~N} 1-3, M}\end{array}\right] \times\left[\begin{array}{c}\alpha_{1}^{\prime} \\ \alpha_{2}^{\prime} \\ \cdot \\ \alpha_{M}^{\prime}\end{array}\right]+\left[\begin{array}{c}D_{1} \\ D_{5} \\ \cdot \\ D_{4 N-3}\end{array}\right]$
Substituting Eq. (13) into Eq. (9), the linear equations between switching angles and every orders of harmonics amplitude can be obtained:

$$
\begin{equation*}
[U]=[B][C]\left[\alpha^{\prime}\right]+[B][D]=[P]\left[\alpha^{\prime}\right]+[Q] \tag{14}
\end{equation*}
$$

Let the amplitudes of harmonics chosen to be eliminated are zero, the linear relation between switching can be obtained:

$$
\begin{equation*}
\alpha_{i}=k_{i} \cdot U_{1}+c_{i}, i=1,2, \ldots, M \tag{15}
\end{equation*}
$$

Accoring to the angles variations with Eq.(10), the range of fundamental voltage for the $i^{\text {th }}$ angle can be gained as $V^{i}, i=1,2, \cdots M$, so the range of the fundamental voltage for all the angles must have a common range as followe:

$$
\begin{equation*}
V^{c}=V^{1} \cap V^{2} \cap \ldots \cap V^{M} \tag{16}
\end{equation*}
$$

where $V^{i}=\left[\begin{array}{ll}\frac{l_{i}-N \cdot c_{i}-1}{N \cdot k_{i}} & \frac{l_{i}-N \cdot c_{i}}{N \cdot k_{i}}\end{array}\right]$

### 3.2 Production of initial switching angles ${ }^{[11]}$

Before construction and solution of SHE equations, actual distribution mode of $M$ angles in first quarter is unknown. To construct SHE equations, a set of initial swithcing angles must be given in advance. Comparing with tranditional Newtown iteration, SHE equation in Walsh domain is less sensitive to initial condition, but, if the discrepancy between actual swithcing angles and given ones exceeds a certain degree, $V^{C}$ in Eq.(16) will be an empty set, meaning that the equations have no solution and a new initial condition have to be given.

It is discovered by research that, producing initial switching angles based on Centriod PWM (CPWM) is feasible and efficient.

CPWM, a new rough selective harmonic elimination PWM mode, is simple and can be realized easily. The procedure to gain the switching angles of CPWM can be summarized as follows:

The first step is to apply the equal areas method which divides the half period of a sinusoidal wave whose peak voltage is $V_{p}$ into $n$ time sections. The area of the $n^{\text {th }}$ sinusoidal section is evaluated by

$$
\begin{equation*}
A_{n}=V_{p} \cdot\left(\cos \beta_{n-1}-\cos \beta_{n}\right) \tag{17}
\end{equation*}
$$

where $\beta_{n}$ and $\beta_{n-1}$ are the limits of the sinusoidal section. Secondly, the centroid of the $n^{\text {th }}$ sinusoidal section is computed, whose horizontal coordinate is expressed as

$$
\begin{equation*}
\overline{x_{n}}=\frac{\sin \left(\beta_{n}\right)-\beta_{n} \cos \left(\beta_{n}\right)-\sin \left(\beta_{n-1}\right)+\beta_{n-1} \cos \left(\beta_{n-1}\right)}{\cos \left(\beta_{n-1}\right)-\cos \left(\beta_{n}\right)} \tag{18}
\end{equation*}
$$

Thridly, a rectangular pulse [ $\alpha_{2 n-1}, \alpha_{2 n}$ ] is constructed in $n^{\text {th }}$ sinusoidal section, whose peak voltage and area are equal to ones of the sinusoidal section and let

$$
\begin{equation*}
\overline{x_{n}}=\frac{\alpha_{2 n-1}+\alpha_{2 n}}{2} \tag{19}
\end{equation*}
$$

Then, the locations of switching angles, which define the commutation interval $\left[\alpha_{2 n-1}, \alpha_{2 n}\right.$ ], are established by the following equations:

$$
\begin{align*}
& \alpha_{2 n-1}=\overline{x_{n}}-m \cdot\left(\frac{A_{n}}{2 V_{p}}\right)  \tag{20}\\
& \alpha_{2 n}=\overline{x_{n}}+m \cdot\left(\frac{A_{n}}{2 V_{p}}\right) \tag{21}
\end{align*}
$$

where $m$ is the modulation ratio of the inverter.

CPWM and SHEPWM, if the number of their switching angles is equal, can restrian or eliminate the same orders of harmonics despite differnce of restrianing degree, which means their switching angles distribution discrpancy is small. It's discovered by virtue of a great deal of computation and comparison that the bigger the modulation ratio is, the smaller switching angles distribution discrpancy of both PWM modes. Let $\alpha_{1}^{C}, \alpha_{2}^{C}, . . \alpha_{M}^{C}$ and $\alpha_{1}^{S}, \alpha_{2}^{S}, . . \alpha_{M}^{S}$ represent switching angles of the first quarter in the CPWM pulses and SHEPWM pulses, respectively. Note $\Delta \alpha_{i}=\alpha_{i}^{S}-\alpha_{i}^{C}$ and $\Delta l_{i}=\frac{\Delta \alpha_{i}}{2 \pi} \cdot N$, where $i$ is the sequence number of angles, $N$ is the number of subintervals in a period. Under the circumstances that modulation ratio $m \approx 1,4 \leq M \leq 48$ and appropriate computation accuracy is guaranteed, if $M$ is even number, $\left|\Delta l_{i}\right|_{\text {max }} \approx 1$; if $M$ is odd number, $\left|\Delta l_{i}\right|_{\max } \approx 2$. And, change of $\Delta l_{i}$ with angle sequence number $i$ is illustrated as Table 1 and 2 .

Table 1 Change of $\Delta l_{i}$ with angle sequence number
( $M$ is odd)
$i \leq \frac{M-3}{2} \quad i=\frac{M-1}{2} \quad i \geq \frac{M+1}{2}$

$$
\Delta l_{i}>0 \quad \Delta l_{i} \approx 0 \quad \Delta l_{i}<0
$$

Table 2 Change of $\Delta l_{i}$ with angle sequence number

|  |  | $\quad$ ( $M$ is even) |  |
| :--- | :--- | :--- | :--- |
| $i<\frac{M}{2}$ | $i>\frac{M}{i}$ | $i=$ |  |
| $\Delta l_{i}$ | $\Delta l_{i}:$ or | $\Delta l_{i} \approx$ | $\Delta l_{i}$ |
|  |  | $\Delta l_{i}<$ |  |

According to the case above mentioned, the switching angles of CPWM on the condition that $m \approx 1$, after amended referring to the comparison relation between both PWM modes denoted by Table 1 and Table 2, can act as initial switching angles of SHEPWM based on Walsh transform. Generally, efficent solution of SHE equation can be obtained
under the initial condition. In case of solving failure, amend the switching angles of CPWM accoring to Table 1 and 2 once more and solve the SHE equation again until that efficient solution is obtained.

## 4 Application of SHEPWM in 200kVA DVR Inverter

DVR in the paper, used in $220 \mathrm{~V} / 380 \mathrm{~V}$ grid, is composed of three single phase full-bridge inverter. Single line circuit diagram of DVR connected with power system is shown in Fig.2. DC input voltage of each inverter is 330 V and rated capacity is 70 kVA . Synthetically considering capacity, filter design and other factors, 31 and below orders of harmonics are to be eliminated, thus, the number of switching angles in a quarter $M=16$.


Fig.2. Single line circuit diagram of DVR connected with power system
In order to ensure computing accuracy, according to "domain-term" ${ }^{[12]}$, the period $[0,1]$ is divided in 256 subintervals. The procedure to obtain the solutions of switching angles is realized as follows:

Firstly, calculate and produce $256 \times 256$ dimensions Walsh functions set; secondly, calculate $16 \times 64$ dimensions Fourier-Walsh transform matrix [ $B_{2 K-1,4 n-3}$ ]; thirdly, let modulation ratio $m=0.95$, produce initial switching angles according the method mentioned in last section, and convert them to the series number of subintervals as follows:
$7,8,13,15,19,23,26,31,33,39,40$, 47, 48, 55, 56, 64.

Then, according to Eq.(9)~Eq.(14), switching angles coefficient matrix $[\mathrm{P}]$ and constant matrix $[\mathrm{Q}]$ can be gained as Table 3. In the end, the piecewise linear relation between switching and fundamental voltage amplitude can be obtained, part of which is illustrated as Table 4, where $k$ is proportion coefficient and $c$ is constant.

Table 3 Switching angles coefficient matrix and constant matrix

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | Q |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -1.270 8 | 1.4643 | -2.416 0 | 2.7873 | -3.508 8 | 1966 | -4.686 3 | . 444 | -5.725 7 | 6.4835 | -6.596 5 | 7.2732 | -7.352 7 | 7.7833 | 7.8263 | 7.9992 | -0.50 |
| -3.683 5 | 4.1958 | -6.365 3 | 7.0070 | -7.824 8 | 9687 | -7.624 7 | 6.2445 | -5.443 6 | 2.4155 | -1.848 0 | -2.227 7 | 2.7867 | -6.120 0 | 6.4822 | -7.992 8 | 2.205 |
| -5.722 3 | 6.3627 | -7.989 6 | 7.821 | -6.1175 | 2.9687 | -0.098 1 | -4.523 0 | 5.9893 | -7.994 4 | 7.9222 | -4.359 9 | 3.5067 | 3.1500 | -4.025 8 | 7.9799 | -1.81 |
| -7.180 6 | 7.6740 | -6.6976 | 4.8381 | 0.2941 | -5.292 8 | 7.5556 | -6.903 3 | 5.1443 | 2.5993 | -3.852 7 | 7.9175 | -7.6171 | 0.4900 | 0.8807 | -7.960 7 | 1.7202 |
| -7.911 1 | 7.9543 | -2.964 5 | -0.489 6 | 4705 | -7.720 1 | . 8342 | 3.5018 | -6.2332 | 6.3538 | -5.140 2 | -5.980 9 | 6.9943 | -4.020 1 | 4111 | 7.9351 | -1.769 2 |
| -7.8410 | 7.1676 | 1.8429 | $-5.5701$ | 7.6601 | -1.6519 | -4.512 1 | 7.4055 | -4.829 4 | -6.5767 | 7.5419 | -0.0979 | -2.032 7 | 6.6855 | -5.283 2 | -7.903 2 | 1.8706 |
| -6.978 9 | 5.4217 | 5.9677 | -7.936 7 | 2.9580 | 6.2194 | -7.650 9 | $-2.4058$ | 6.4562 | -2.218 7 | -0.2932 | 6.0954 | -4.1789 | -7.9176 | 7.2425 | 7.8650 | -2.13 |
| -5.4141 | 2.9538 | 7.9064 | -6.447 1 | -4.005 6 | 7.2323 | -0.292 8 | -7.739 6 | 4.5003 | 7.9495 | -7.3115 | -7.8539 | 7.7824 | 7.4564 | -7.954 3 | -7.820 5 | 2.07 |
| -3.3073 | 0.0975 | 6.9578 | -1.835 | -7.869 8 | 0.2923 | 7.444 | 1.2617 | -6.657 3 | -2.767 | 5.5466 | 4.1663 | -6.549 0 | -5.405 4 | 7.2997 | 7.7699 | -1.99 |
| -0.873 8 | $-2.7622$ | 3.4772 | 3.6510 | -5.674 1 | -6.945 2 | 4.9537 | 7.8985 | -4.158 8 | -6.189 5 | 3.3013 | 2.3942 | 1.259 | 2.2080 | -5.395 6 | -7.713 2 | 2.2068 |
| 1.6386 | -5.240 9 | -1.256 8 | 7.3461 | 0.8720 | -6.523 9 | -4.3145 | -0.0971 | 6.8355 | 6.6318 | -7.8969 | -7.271 8 | 4.7906 | 1.4482 | 2.5738 | 7.6505 | -2.277 8 |
| 3.9750 | -7.0072 | $-5.5131$ | 7.4648 | 6.7208 | 1.0624 | -7.633 5 | -7.879 3 | 3.8065 | 2.0120 | 2.3841 | 7.0943 | -7.8461 | $-4.7800$ | 0.6773 | -7.5818 | 2.822 |
| 5.8995 | -7.827 1 | -7.7420 | 3.9654 | 7.3816 | 7.4467 | -0.4829 | -1.059 8 | -6.990 2 | -7.860 3 | 6.1484 | -2.007 1 | 6.0258 | 7.0771 | -3.7972 | 7.5074 | -3.000 8 |
| 7.2191 | -7.5950 | -7.1410 | -1.437 7 | 2.3721 | 5.6217 | 7.2929 | 7.6841 | $-3.4451$ | 2.9165 | $-6.7860$ | -4.443 5 | -0.4816 | -7.853 8 | 6.2508 | -7.4272 | 2.2983 |
| 7.8033 | -6.347 7 | -3.943 8 | -6.114 9 | -4.430 9 | $-2.3654$ | 5.0426 | 2.1814 | 7.1208 | 5.9929 | -1.2442 | 7.7845 | $-5.3306$ | 6.9522 | -7.620 3 | 7.3414 | -3.3876 |
| 7.5971 | -4.258 1 | 0.6699 | -7.803 1 | -7.793 7 | -7.709 3 | -4.096 2 | -7.319 1 | 3.0765 | -6.647 7 | 7.6391 | -5.720 9 | 7.8078 | -4.5742 | 7.6765 | -7.250 2 | 3.1099 |

Table 4 First order coefficient and constant of each voltage section

| voltage | 0.395~0.467 |  | 0.468~0.550 |  | 0.551~0.630 |  | 0.631~0.698 |  | 0.699~0.795 |  | 0.796~0.896 |  | 0.897~0.945 |  | 0.946~1.000 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| section | $k$ | C | $k$ | C | $k$ | C | $k$ | C | k | C | $k$ | C | $k$ | C | $k$ | C |
| $\alpha_{1}$ | -2.216 | 11.34 | -2.312 | 11.38 | -2.417 | 11.44 | -0.8333 | 10.25 | 1.174 | 10.48 | -1.311 | 10.59 | -2.151 | 11.34 | -2.722 | 11.88 |
| $\alpha_{2}$ | -0.4009 | 11.37 | -0.5342 | 11.43 | -0.6789 | 11.51 | 0.8429 | 10.37 | 0.4029 | 10.68 | 0.2261 | 10.81 | -0.8756 | 11.80 | -1.615 | 12.50 |
| $\alpha_{3}$ | -3.203 | 21.53 | -3.666 | 21.75 | -4.168 | 22.03 | -2.335 | 21.06 | -3.019 | 21.54 | -3.282 | 21.74 | -5.365 | 23.62 | -6.583 | 24.76 |
| $\alpha_{4}$ | 0.4061 | 21.61 | -0.1221 | 21.85 | -0.6944 | 22.17 | 1.088 | 21.25 | 0.1613 | 21.89 | -0.1860 | 22.16 | -3.511 | 25.17 | -5.267 | 26.81 |
| $\alpha_{5}$ | -3.960 | 32.10 | -4.517 | 32.35 | -5.118 | 32.69 | -3.406 | 31.67 | -4.688 | 32.56 | -5.115 | 32.89 | -6.704 | 34.28 | -8.313 | 35.79 |
| $\alpha_{6}$ | 1.404 | 32.18 | 0.6948 | 32.50 | -0.0653 | 32.93 | 2.140 | 31.60 | 0.0299 | 33.05 | -0.5705 | 33.52 | -2.770 | 35.41 | -5.133 | 37.62 |
| $\alpha_{7}$ | -4.321 | 42.49 | -5.215 | 42.90 | -6.141 | 43.42 | -5.389 | 42.98 | -6.137 | 43.53 | -6.844 | 44.09 | -8.791 | 45.78 | -11.40 | 48.23 |
| $\alpha_{8}$ | 3.261 | 42.26 | 1.817 | 42.93 | 0.4297 | 43.70 | 0.9870 | 43.39 | 0.0927 | 44.10 | -1.053 | 44.99 | -4.501 | 48.03 | -9.459 | 52.67 |
| $\alpha_{9}$ | -4.667 | 52.88 | -4.647 | 52.84 | -6.832 | 54.04 | -6.616 | 53.94 | -7.104 | 54.31 | -8.498 | 55.39 | -11.82 | 58.33 | -17.57 | 63.72 |
| $\alpha_{10}$ | 3.253 | 53.10 | 3.894 | 52.76 | 1.440 | 54.14 | 1.312 | 54.25 | 2.337 | 53.52 | -1.731 | 56.68 | -9.640 | 63.82 | -11.05 | 65.17 |
| $\alpha_{11}$ | -5.409 | 63.61 | -6.365 | 64.03 | -6.366 | 64.06 | -6.981 | 64.48 | -7.116 | 64.56 | -10.03 | 66.83 | -17.88 | 73.89 | -18.74 | 74.73 |
| $\alpha_{12}$ | 4.525 | 63.46 | 2.645 | 64.30 | 3.654 | 63.75 | 4.164 | 63.44 | 1.063 | 65.55 | 1.765 | 65.03 | -2.889 | 68.73 | -18.40 | 83.37 |
| $\alpha_{13}$ | -6.402 | 74.55 | -6.532 | 74.62 | -6.969 | 74.88 | -5.655 | 74.06 | -8.340 | 75.92 | -8.335 | 75.94 | -12.72 | 79.45 | -26.95 | 92.88 |
| $\alpha_{14}$ | 3.824 | 74.53 | 4.652 | 74.19 | 2.653 | 75.34 | 3.263 | 74.96 | 4.480 | 74.17 | -1.259 | 78.77 | 1.559 | 76.32 | -8.758 | 85.53 |
| $\alpha_{15}$ | -4.794 | 84.44 | -6.250 | 85.13 | -7.085 | 85.62 | -6.780 | 85.43 | -6.027 | 84.96 | -11.29 | 89.16 | -9.000 | 87.17 | -18.93 | 96.03 |
| $\alpha_{16}$ | 5.899 | 84.43 | 4.592 | 85.04 | 5.310 | 84.63 | 5.368 | 84.60 | 4.998 | 84.88 | 5.275 | 84.66 | 5.396 | 84.56 | 5.681 | 84.29 |

## 5 Experimental Result

The inverter is controlled by TMS320LF2407A DSP processor. When grid voltage fluctuates in the range of $0.1 \sim 1.5$ times of rating, after compensated by DVR, the THD of load voltage is below $2.1 \%$. With inductive character load ( $\cos \varphi=0.75$ ), when modulation ratio $m=0.1$ and $m=0.9$, the THD of inverter output voltage is $3.1 \%$ and $1.7 \%$, respectively. The waveforms and spectrums are illustrated as Fig.3. Fig. 4 denotes two transient cases: modulation ratio change abruptly from 0 to 0.5 and form 0.2 to 0.75 .

Experimental results indicate that waveform quality of the output voltage of inverter controlled by SHEPWM is good and THD is low in all the range of output voltage, though the switching frequency is
low. In addition, the inverter has comparatively high response speed.

(a) Modulation ratio is $10 \%$


Fig.3. Output voltage waveform and its frequency spectrum


Fig.4. Two transient voltage waveform

## 6 Conclusion

(1) SHEPWM base on Walsh transform avoids solving nonlinear transcendental equations and can obtain piecewise linear relation between fundamental voltage amplitude and switching angles.
(2) The initialization based on CPWM technique effectively reduces blindness of searching initial switch angles in the course of SHE equation solution and improves computation efficiency.
(3) Experimental results indicate that SHEPWM based on Walsh transform has strong ability to
eliminate harmonic and good dynamic performance in full range of output voltage.

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