On the Strong Solution to the Oseen– Problem in an Exterior domain in L^q spaces

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Abstract: - We consider the Oseen problem in an exterior domain $\Omega \subset \mathbb{R}^3$. We prove the existence of the strong solution in L^q quotient spaces for all q.

Key-Words: - Strong solution, Oseen problem, Existence, Uniqueness

1 Introduction

The classical Oseen problem in an exterior domain Ω is described by the velocity vector u and by the associated presure p as a linearized version of the incompressible Navier-Stokes equations precisely as a perturbation of v_∞ the velocity at infinity; v_∞ is generally assumed to be constant in a fixed direction, say the first axis, $v_\infty = |v_\infty|e_1$. In the next we consider $|v_\infty| = 1$ and we will write the Oseen operator $\partial_1 v$.

The first complete investigation of the existence and uniqueness of the Oseen problem in exterior domains was done by Faxén [8], who generalized the method introduced by Odqvist [20] for the Stokes problem. In the book of Galdi [19] we can found comprehensive answers on the questions of the existence and the uniqueness in homogeneous Sobolev spaces $D^{1,p}$. Existence and uniqueness results for three dimensional flows in weighted anisotropic Sobolev spaces with weights reflecting the decay properties of the fundamental solution have been proved by Farwig [5]. Then in case of anisotropic Sobolev spaces results was improved in L^p spaces with weights satisfying so called A_p condition see [13, 14]. Modified Oseen problem which appears in problem arising from flow around a rotating body was studied by Kračmar and Penel see [15], [16]. They considered one extra term $a\nabla v$ (which in case of rotating body in a fluid is exactly term $(\omega \times x) \cdot \nabla u$). They got solution in an appropriate Sobolev weighted spaces. To extend results from R^3 to an exterior domain the method of hydrodynamical potentials can be used. The application of this method to the Oseen problem without weights is well known. Moreover this method has been used in the work of Farwig [5] for solution of the Oseen equations in weighted Sobolev spaces. Applying localization procedure for the extension of anisotropically weighted estimates from the whole R^3 onto the case of an exterior domain we can find in the work of Kračmar and Nečasová [17]. Another type of weighted space with logarithmic weights or some kind of anisotropical structure we can find in work of Amrouche and his collaborators [2]-[4]. These methods are efficient for various modifications of Oseen problem connected with additional assumptions see [11], [12], [6].

In case of homogeneous Sobolev spaces the existence and uniqueness of the strong solution in L^q is proved under condition $q \le (n+1)/2$ and the extension of the existence and uniqueness results was done by Farwig [7]. The question of existence and uniqueness for $q \ge (n+1)/2$ was left out. We will interested in case of $q \ge (n+1)/2$ and we will proved the existence and uniqueness in suitable quotient spaces. We consider the Oseen problem in an exterior domain $\Omega \subset R^3$

$$\begin{array}{rcll} -\nu\Delta u + \partial_1 u + \nabla p &=& f & \text{in } \Omega \\ \text{div}\, u &=& 0 & \text{in } \Omega \\ u &=& u_* & \text{on } \partial\Omega \\ u &\to& 0 & \text{as } |x| \to \infty. \end{array} \tag{1}$$

We rely on standard notation for the various function spaces, such as $L^q(\Omega)$, $W^{m,q}(\Omega), W_0^{m,q}(\Omega)$ and their duals. Moreover, with

$$|u|_{m,q} = (\sum_{|l|=m} \int_\Omega |\mathbf{D}^l u|^q)^{\frac{1}{q}}, \tag{2}$$

for $m \ge 0, q \ge 1$, we denote by $\mathbf{D}^{m,q}(\Omega)$ the *homogeneous* Sobolev spaces :

$$\mathbf{D}^{m,q}(\Omega) := \{u \in L^1_{loc}(\Omega) : |u|_{m,p} < \infty\}, \tag{3}$$

and by $\mathbf{D}_0^{m,q}(\Omega)$ the completion of $C_0^\infty(\Omega)$ in the norm (3). The dual space of $\mathbf{D}_0^{m,q}(\Omega)$ is denoted by $\mathbf{D}_0^{-m,q'}(\Omega), 1/q+1/q'=1$. Throughout the paper, the space $\mathcal{D}(\Omega)$ is the subspace of $C_0^\infty(\Omega)$ of smooth solenoidal vector-valued functions with compact support, $\mathcal{D}_0^{m,q}(\Omega)$ denotes the closure of $\mathcal{D}(\Omega)$ in $\mathcal{D}^{m,q}(\Omega)$ and $\mathcal{D}_0^{-m,q'}(\Omega)$ the dual of $\mathcal{D}_0^{m,q}(\Omega)$, $1/q+1/q'=1$. We shall only be interested in domains with a C^2 boundary, eventually more regular boundary. For more details see [1], [19].

We introduce the following space $\tilde{D}^{2,q}$ such that

$$\tilde{D}^{2,q}(\Omega)=D^{2,q}(\Omega),\ q\geq(n+1)/2$$

$$\tilde{D}^{2,q}(\Omega)=\{u\in D^{2,q}(\Omega): |u|_{1,r}\leq\infty,$$

$$r=\tfrac{(n+1)q}{n+1-q}\},\ (n+1)/2<q<n+1$$

$$\tilde{D}^{2,q}(\Omega)=\{u\in D^{2,q}(\Omega):$$

$$\|u\|_s+|u|_{1,r}<\infty, s=\tfrac{(n+1)q}{(n+1-2q}, r=\tfrac{(n+1)q}{n+1-q}\},$$

$$1<q<(n+1)/2.$$

Denote by Σ_q the subspace of $\tilde{D}^{2,q}(\Omega)\times D^{1,q}(\Omega)$ formed by solutions v,p to (1) with $f=0$, $u^*=0$.

For fixed $r>\delta(\Omega^c)$ and $l\geq 0$, $\nu\geq 1$ we set

$$\|u\|_{\nu,R;l,q}=\|u\|_{\nu-1,q,\Omega_R}+\Sigma_{i=1}^{l+\nu}|u|_{i,q,\Omega}.$$

The aim of our article is to prove the following theorems.

Theorem 1 *Let Ω be an exterior domain of class C^{m+2}, $m\geq 0$. Given $f\in W^{m,q}(\Omega)$, $v_*\in W^{m+2-1/q,q}(\partial\Omega)$, $1<q<\infty$ there exists a unique solution to (1) such that*

$$v,p\in\tilde{D}^{2,q}(\Omega)\times D^{1,q}(\Omega)/\Sigma_q. \tag{4}$$

Moreover,

$$v\in\bigcap_{k=0}^{m}D^{k+2,q}(\Omega),\ p\in\bigcap_{k=0}^{m}D^{k+1,q}(\Omega) \tag{5}$$

and corresponding estimates (10)-(12) are satisfied.

Theorem 2 *Assume that Ω, f, v_* satisfy the assumptions of Theorem 1 with $1<q<\infty$ and let $f\in L^t$, $v_*\in W^{2-1/t,t}(\partial\Omega)$, for $1<t<2$. Then given $v_0\in R$, $A=\{A_{ij}\}$, trace $(A):=0$, $v_0\not\equiv 0$ there exists one and only one solution (v,p) to problem (1) such that $\tilde{v}_\infty(x)=v_0+Ax, w=v-\tilde{v}_\infty$, we have*

$$w\in\tilde{\mathbf{D}}^{2,t}(\Omega)\cap(\bigcap_{k=0}^{m}\mathbf{D}^{k+2,q}(\Omega)) \tag{6}$$

$$p\in\mathbf{D}^{1,t}(\Omega)\cap(\bigcap_{k=0}^{m}\mathbf{D}^{k+1,q}(\Omega)). \tag{7}$$

Moreover, w and p satisfy the following estimate

$$\begin{aligned}&\|w\|_s+|w|_{1,r}+|w|_{2,t}+\|p\|_s+|p|_{1,t}+\\&+\textstyle\sum_{k=0}^{m}(|w|_{k+2,q}+|p|_{k+1,q})+\|\frac{\partial v}{\partial x_1}\|_q\\&\leq c(\|f\|_t+\|f\|_{m,q}+\|v_*-\tilde{v}_\infty\|_{m+2-1/t,t,\partial\Omega}+\\&+\|v_*-v_\infty\|_{m+2-1/q/q,\partial\Omega}),\end{aligned} \tag{8}$$

with $r=3t/(3-t)$, $s=3t/(3-2t)$ and $c=c(N,q,t,m,\Omega)$. Further, we have

$$\begin{aligned}&\textstyle\int_{S_n}|w(x)|=O(\frac{1}{|x|^{3-r}}),\\&\textstyle\int_{S_n}|\nabla w(x)|=O(\frac{1}{|x|^{3-t}}),\\&\textstyle\int_{S_n}|p(x)|=O(\frac{1}{|x|^{3-t}}),\end{aligned} \tag{9}$$

where S_n denotes the unit sphere of R^3. Finally if $q>n+1$ and $1<t<n+1$ then

$$\lim_{|x|\to\infty}\nabla w(x)=0$$

uniformly. Also if $q>(n+1)/2$, $1<t<(n+1)/2$ then

$$\lim_{|x|\to\infty}\nabla w=0$$

uniformly.

2 Mathematical Preliminaries

Lemma 3 *Let $\Omega,\ f,v_*,\ \Omega\subset R^n$ of a class C^{m+2}, $n\geq 2$, $m\geq 0$, corresponding to $f\in W^{m,q}(\Omega)$, $v_*\in W^{m+2-1/q,q}(\partial\Omega)$, $1<q<\infty$ and let $v\in\tilde{D}^{2,q}(\Omega)$ be a solution to (1) corresponding to f and v_*. Then $v\in D^{k+2,q}(\Omega), p\in D^{k+1,q}(\Omega)\ \forall=0,1,...,m$ and if $q\geq(n+1)/2$ we have*

$$\begin{aligned}&\inf_{(h,\pi)\in\Sigma_q}\{|v-h|_{2,R;m,q}+\|p-\pi\|_{1,R;m,q}\}\leq\\&\leq c(\|f\|_{m,q}+\|v_*\|_{m+2-1/q,\partial\Omega}),\end{aligned} \tag{10}$$

if $(n+1)/2 < q < (n+1)$

$$\begin{aligned}&\inf_{(h,\pi)\in\Sigma_q}\{|v-h|_{1,r}+\|p-\pi\|_r+\\&+\|v-h\|_{2,R;m,q}+\|p-\pi\|_{2,R;m,q}\}\le\\&c(\|f\|_{m,q}+\|v_*\|_{m+2-1/q,\partial\Omega}),\end{aligned} \tag{11}$$

where $r=\frac{(n+1)q}{n+1-q}$
and if $1<q<\frac{n+1}{2}$

$$\begin{aligned}&\|v\|_s+|v|_{1,r}+\|p\|_r+\|v\|_{2,R;m,q}+\\&\|p\|_{1,R;m,q}\le c(\|f\|_{m,q}+\|v_*\|_{m+2-1/q,q,\partial\Omega}),\end{aligned} \tag{12}$$

where $s=\frac{(n+1)q}{n+1-2q}$.

Proof: see [18].

Lemma 4 *Let v, p be a solution to (1) in an exterior domain $\Omega \subset R^n$ of a class C^{m+2}, $n\ge 2$, $m\ge 0$, corresponding to $f\in W^{m,q}(\Omega)$, $v_*\in W^{m+2-1/q,q}(\partial\Omega)$, $1<q<\infty$. Assume that*

$$v\in D^{2,q}(\Omega).$$

Then $v\in D^{k+2,q}(\Omega), p\in D^{k+1}(\Omega)$ for all $k=0,1,...,m$ and for any $R>\delta(\Omega^c)$ it holds that

$$\begin{aligned}&\|v\|_{1,q,\Omega_R}+\textstyle\sum_{k=0}^m\{|v|_{k+2,q}+|p|_{k+1,q}\}\\&\le c(\|f\|_{m,q}+\|v_*\|_{m+2-1/q,q,\partial\Omega}+\|v\|_{q,\Omega_R}+\\&\|p\|_{q,\Omega_R}),\end{aligned} \tag{13}$$

where $c=c(n,m,q,R)$ and $\Omega_R=\Omega\cap B_R$, $B_R(x)=\{y\in R^n, |x-y|<R\}$.

Proof: It follows immediately from [19].

Lemma 5 *Let Ω be an exterior domain of class C^2 and set $d=dim(\Sigma_q)$. Then*

$$d=\begin{cases} n+n^2+1,\ q\ge(n+1),\\ n,\ q\in[(n+1)/2,n+1],\\ 0,\ q\in(1,(n+1)/2).\end{cases} \tag{14}$$

Proof : We give only the sketch of the proof. The proof is based on the following observations.

Observation 1: *For any $v_\infty\in R^n-0$ there is a unique (nonzero) solution $v,p\in C^\infty$ to (*) such that*

$$\lim_{|x|\to\infty}|v(x)-v_\infty|=0.$$

This solution satisfied that

$$u\in\tilde{D}^{2,q}(\Omega),\ \text{for } q\ge(n+1).$$

Proof: It follows from [19] [Theorem 7.1] [Chapter 6].

Observation 2: *For any matrix $A=\{A_{ij}\}, A_{ij}\ne 0$ with trace $(A)=0$, there is a a unique (nonzero) solution $v,p\in C^\infty(\Omega)$ to (1) such that*

$$\lim_{|x|\to\infty}|v(x)-A\cdot x|=0.$$

This solution verifies the condition $v\in\tilde{D}^{2,q}(\Omega)$, for $q\ge(n+1)$.

Proof: We make a suitable solenoidal extension of the field $V_0\equiv A\cdot x$. Let w denote a solution to the problem

$$\nabla\cdot w=\nabla\varphi\cdot V_0=g \text{ in } \Omega' w=0 \text{ at } \partial\Omega' \tag{15}$$

where $\varphi=\varphi(|x|)\in C^\infty(\Omega)$ with $\varphi=0$ for $|x|\le\rho$ and equals one for $|x|\ge R/2$, $\delta(\Omega^c)<\rho<R/2$, Ω' is a locally lipschitzian subdomain of Ω that contains the support of φ. Since $\int_{\Omega'}g=0, g\in C_0^\infty(\Omega')$ and from results properties of Bogovskii operator , we can take $w\in C_0^\infty$, vanishes near $\partial\Omega$ and equals V_0 at large distances. It implies that $D^2a\in C_0^\infty$ and from existence of generalized solution we can show the existence of generalized solution v to (1) such that $v=u+a$ and $u\in D_0^{1,2}(\Omega)$. It is not difficult to show that $D^2v\in L^q(\Omega)$ for all $q>1$. Since v does not belong to space $D^{1,r}(\Omega)$ and to space $L^s(\Omega)$, then

$$v\in\tilde{D}^{2,q}(\Omega) \text{for } q\ge(n+1),$$

which completes the proof of Observation 2.

Now, let h_i,π_i, $i=1,...n$ be the solutions to (1) of the type of Observation 1 corresponding to the three orthonormal vectors $v_{\infty i}=e_i$. Likewise, let u_{ij},τ_{ij} be n^2-1 solutions to (1) of the type of Observation 2 corresponding to the n^2-1 matrices of zero trace E_{ij}, where

$$E_{ij}=\begin{cases} e_i\times e_j,\ \text{if } i\ne j\\ e_i\times e_j-e_n\times e_n \text{ if } i=j\ne n.\end{cases}$$

It can be seen that the system constitued by the n^2+n-1 vectors $\{h_i,u_{ij}\}$ is linearly independent. Now, if v,p is a solution to (1) with $v\in\tilde{D}^{2,q}$ for some $q>1$, from [19] [Chapter 7] it follows that there exists $v_o \in R^n$ and a traceless matrix B such that as $|x| \to \infty$ we have

$$v(x) = v_0 + B \cdot x + O(1/|x|^{n-2}).$$

From the properties of the Oseen equation the proof is accomplished.

Theorem 6 *Let Ω be an exterior domain in R^3, of class $C^{m+2}, m \geq 2$. Given $f \in W^{m,q}(\Omega), v_* \in W^{m+2-1/q,q}(\partial\Omega)$, $1 < q < 2$, $v_\infty \in R^N$ there exists one and only one corresponding solution (v,p) to the Oseen problem*

$$\begin{array}{rcll} -\Delta v + R\frac{\partial v}{\partial x_1} + \nabla p &=& f, & \text{in } \Omega \\ \nabla \cdot v &=& 0, & \text{in } \Omega \\ v &=& v_* & \text{on } \partial\Omega, \\ v &\to& v_\infty, & |x| \to \infty, \end{array} \tag{16}$$

such that

$$\begin{array}{l} v - v_* \in W^{m,s_2}(\Omega) \cap \{\bigcap_{l=0}^m \mathbf{D}^{l+1,s_1}(\Omega) \cap \mathbf{D}^{l+2,q}(\Omega)\}, \\ p \in \bigcap_{l=0}^m \mathbf{D}^{l+1,q}(\Omega) \end{array}$$

with $s_1 = \frac{(N+1)q}{N+1-q}$, $s_2 = \frac{(N+1)q}{N+1-2q}$. Moreover (v,p) verify

$$\begin{array}{l} a_1\|v - v_\infty\|_{m,s_2} + R\|\frac{\partial v}{\partial x_1}\|_{m,q} + \\ \sum_{l=0}^m \{a_2|v|_{l+1,s_1} + |v|_{l+2,q} + |p|_{l+1,q}\} \\ \leq c\{R\|f\|_{m,q} + \|v_* - v_\infty\|_{m+2-1/q,q}\}, \end{array} \tag{17}$$

where $c = c(m,q,n,\Omega,R)$, $a_1 = min\{1, R^{1/2}\}$, $a_2 = min\{1, R^{1/4}\}$. However if $q \in (1, \frac{3}{2})$ and $R \in (0, B]$ for some $B > 0$, c depends solely on m, N, q, Ω and B, R denotes the Reynolds number.

Proof: see [19], [Theorem 7.1].

Lemma 7 *Let Ω be an exterior domain in R^n and let v be a solution corresponding to $f \in L^t$ with $v \in \tilde{D}^{2,q}$. Then if $1 < q < (n+1)$ and $t > (n+1)$ we have*

$$\lim_{|x| \to \infty} \nabla v(x) = 0$$

uniformly and

$$\lim_{|x| \to \infty} v(x) = \infty$$

uniformly while $1 < q < \frac{n+1}{2}$ and $t > \frac{n+1}{2}$.

Proof: see [18].

3 Proof of the main theorems

Proof of the Theorem 1 We approximate f and v_* by sequences of functions $\{f_s\} \subset C_0^\infty$ $\{v_{*k} \subset W^{m+2-1/r,r}(\partial\Omega)$ any $r \in (1,\infty)$. From [19]Chapter 7, [Theorem 4,2], for all $s \in N$ there exists a generalised solution $v_s, p_s \in D^{1,2}(\Omega) \times L^2(\Omega)$ corresponding to f_s, v_{*s}, and to $v_\infty = 0$ in the case where $n > 2$.It is easy to see that $v_s \in W^{2,q}(\Omega_R)$, $p_s \in W^{1,q}(\Omega_R)$ for all $R > \delta(\Omega^c)$. And then we get

$$\begin{array}{l} v_s \in L^t(\Omega),\ t > \frac{n+1}{n-2} \\ \nabla v_s \in L^r(\Omega),\ r > \frac{n+1}{n-1} \\ D^2 v_s \in L^q(\Omega),\ q > 1. \end{array}$$

It implies that $v \in \tilde{D}^{2,q}(\Omega)$ for all $q > 1$. The solutions (v_s, p_s) will then satisfy (10), (11), (12) depending on the values of q and n. Assume $q \geq n+1$. Given $\epsilon > 0$ from (10) and from the linearity of problem (1) for s', s'' sufficiently large we deduce

$$\inf_{(h,\pi) \in \Sigma_q} \{|v_{s'} - v_{s''} - h|_{2,q} + |p_{s'} - p_{s''} - \pi|_{1,q}\} < \epsilon.$$

This relations implies that (v_s, p_s) is a Cauchy sequence in the quotient space $\tilde{D}^{2,q}(\Omega) \times D^{1,q}(\Omega)/\Sigma_q$ and there is an element $(v,p) \in \tilde{D}^{2,q}(\Omega) \times D^{1,q}(\Omega)$ to which v_s, p_s tend to quotient norm defined by the left hand side of (3). Similarly for $q \in ((n+1)/2, (n+1))$ and $q < (n+1)/2$.

Proof of the Theorem 2 : From Theorem 1 it follows the existence of solution of (1), $v \in \tilde{\mathbf{D}}^{2,t}$, $p \in \mathbf{D}^{1,t}$ satisfying

$$\begin{array}{l} \|v\|_{s_2} + |v|_{1,s_1} + |v|_{2,t} + \|p\|_{s_1} + \\ |p|_{1,t} + |\frac{\partial v}{\partial x_1}|_t \leq c(\|f\|_t + \|v_*\|_{2-1/t,t,\partial\Omega}). \end{array} \tag{18}$$

Since $f \in W^{m,q}(\Omega), v_* \in W^{m+2+1/q,q,\partial\Omega}$, then $v \in \mathbf{D}^{k+2,q}, p \in \mathbf{D}^{k+1,q}$ see [19], [Theorem 7.1], Chapter 7 and moreover

$$\begin{array}{l} \sum_{k=0}^m (\|v\|_{k+2,q} + |p|_{k+1,q}) \leq (\|f\|_{m,q} + \\ \|v_*\|_{m+2-1/q,q,\partial\Omega} + \|v\|_{q,\Omega_R} + \|p\|_{q,\Omega_R}). \end{array} \tag{19}$$

Finally, applying the following inequalities

$$\begin{array}{l} |v|_{q,\Omega_R} \leq c_1\|v\|_{s_2} + c_2|v|_{1,s_1} + \epsilon\|D^2 v\|_{q,\Omega_R}, \\ |p|_{q,\Omega_R} \leq c_1\|v\|_{s_2} + c_2|v|_{1,q}. \end{array} \tag{20}$$

We get estimate (12). From Lemma 7, [19] [Lemma II.5.2], [Lemma VII.6.1], [Theorem VII 6.2] we get the rest of Theorem 2.

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