Laminar Mixed Convection in the Entrance Region of Horizontal Quarter Circle Ducts

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Abstract: **-** Laminar mixed convection in the entrance region for horizontal quarter circular ducts with the curved wall on top has been investigated. The governing momentum and energy equations were solved numerically using a marching technique with finite control volume approach following the SIMPLER algorithm. Results were obtained for the thermal boundary conditions of uniform heat input axially and with uniform wall temperature circumferentially (H1 boundary condition) with Pr=0.7 and a wide range of Grashof numbers. These results include velocity, temperature distributions, at different axial locations, covering all aspects of flow, axial distribution of local Nusselt numbers and local friction factor. It was found that Nusselt numbers were close to the forced convection values near the entrance region and then decreasd to a minimum as the distance from the entrance increases and then rises due to the effect of free convection before reaching constant value (fully developed). As Grashof number increases Nusselt number and friction factor increases in both developing and fully developed regions.

Key-Words: -Horizontal-Mixed Convection-Numerical-Quarter Circle- Developing-Air

1 Introduction

The flow passages in multi passage tubes (used in compact heat exchangers) are some times of circular sector cross section. Such flow passages are relatively short, so the flow is in the developing region. The mixed convection is the dominate flow mode in most heat exchangers. So the knowledge of pressure drop and heat transfer characteristics of circular sector ducts will be useful tool for the design of heat exchangers. One geometry which has received little attention is that of quarter circle duct. So this paper is concerned with the problem of laminar mixed convection in the entrance region of horizontal quarter circular ducts with uniform heating. Most of the literature for mixed convection in the entrance region is for circular and rectangular ducts. Nguyen and Galanis [1] studied the problem of laminar mixed convection in the entrance region of horizontal circular tubes for the case of constant heat flux they found that increasing the Gr the Nu_x increases in the developing and fully developed regions. Recently Mare. at al [2] studied laminar mixed convection with flow reversal in inclined circular tubes with uniform wall temperature, they found that increasing the Gr moves the location of the flow reversal upstream. Also results for laminar mixed convection in the entrance region are obtained for vertical circular tubes [3-5], for vertical rectangular ducts [6-8], for horizontal rectangular ducts [9] and for concentric annulus [10]. As for mixed convection for circular sector ducts the majority of the studies is in the fully developed region and uses the semicircular as a cross section. Nandukumar. et al [11] studied numerically the problem of fully developed laminar mixed convection flow in horizontal semicircular ducts for H1 thermal boundary condition with the flat wall at the bottom for Pr = 0.7 and 5 corresponding to air and water respectively. They found that a dual solution of two and four vortex coexists and decreasing Pr delays the appearance of the four vortex solution. Lei and Trupp [12] solved the same problem considered in Ref [11] with the flat wall on top for Pr = 5. They reported approximately the same results of Nusselt number as for the flat wall at the bottom (Ref [11]). Chinporncharoepong et al [13] studied the effect of orientation by rotating the horizontal semicircular duct from 0° (the flat wall on top) to 180° (the flat wall on the bottom) with incremental angle of 45° for Pr =4.0. Chinponrncharoenpong. et al [14] investigated numerically the problem of laminar mixed convection in the fully developed region for circular sector ducts with sector angles from 0° to 360° and rotated with three orientation angles 0°, 90° and 180° for Pr= 4.0 and Gr up to 10^7. They found that the orientation effects are significant for circular sector ducts only at high Gr. Further a comparison of Nu_{fd} for the circular sector ducts with fixed orientation showed that the Nu_{fd} increased with increasing the sector angle until a certain sector angle after which Nu_{fd} becomes constant. For the case of laminar flow in the entrance region of circular sector ducts Soliman. et al [15] investigated the laminar flow in the entrance region of circular sector ducts with sector angles of 11.25°, 22.5°, 45° and 90°. The solution obtained by solving the axial momentum equation and using a linearization method. Lei and Trupp[16] investigated the laminar heat transfer in the thermal entrance region of circular sector ducts with sector angles of 20°, 45°, 90°,130°, 180°, 270° and 360° for H1 and H2 thermal boundary conditions. They found that the H1 thermal condition is stronger than that of H2 resulting in higher heat transfer coefficients and shorter thermal entrance lengths.

Numerical studies for the case of laminar mixed convection in the entrance region of horizontal quarter circle ducts are to the knowledge of the author, nonexistent. The present investigation therefore, concerned with buoyancy effects on laminar mixed heat transfer of simultaneously developing hydrodynamically and thermally for horizontal quarter circle ducts with the H1 thermal boundary condition. These effects are to be examined over a wide range of Gr. The calculated parameters are local Nusselt number and friction factor, as well as, the development of axial velocity and temperature profiles.

Nomenclature

D_h Hydraulic diameter
f Gravitational acceleration
Gr Grashof number
Nu_x Local Nusselt number
$\overline{P}_1$ Dimensionless cross sectional average pressure
P_2 Dimensionless cross sectional excess pressure
$\overline{q}$ Rate of heat input per unit length
r Radial coordinate
x Axial coordinate
T, T_w Dimensionless fluid and wall temperature
V,W,U Dimensionless fluid velocities in r, θ, x directions respectively
Re Reynolds number
Pr Prandtl number
U_b Dimensionless mean axial velocity
Tb dimensionless mean temperature

Subscripts
fd Fully developed

2 Mathematical Model and Solution Procedure:

The flow was assumed to be laminar and simultaneously developing hydrodynamically and thermally and H1 boundary condition was applied on the ducts wall. The fluid enters the duct with uniform velocity and a uniform temperature. The problem is analyzed for constant fluid properties and negligible viscous dissipation. The variation of density is taken into account only in the body forces (Boussinesq approximation). All terms containing the second derivative of any quantity with respect to x are neglected. The fluid pressure decomposition which is quite wide used is given by Patankar and Spalding [17]. The governing Navier- Stokes equations and the energy equation are putted in a non dimensional form by using the following dimensionless parameters:

$$R=\frac{r}{r_o},\quad X=\frac{x}{D_h PrRe},\quad V=\frac{vr_o}{\nu},\quad W=\frac{wr_o}{\nu},\quad U=\frac{u}{u_b}$$

$$T=\frac{t-t_e}{\overline{q}/k},\quad \overline{P}_1=\frac{\overline{p}_1^*}{\rho u_b^2},\quad P_2=\frac{p_2^* r_o^2}{\rho \nu^2} \qquad (1)$$

$$Re=\frac{D_h u_b}{\nu},\quad Pr=\frac{\rho\nu C_p}{k},\quad Gr=\frac{g\beta\overline{q}r_o^3}{k\nu^2}, D_h=\left(\frac{2\pi}{\pi+4}\right)r_o$$

The dimensionless initial and boundary conditions are:
At X= 0

$$\begin{aligned} &U=1 && \text{withn the fluid}\\ &U=0 && \text{at the walls}\\ &T=0 && \text{for all R and } \theta\\ &\overline{P}_1=\overline{P}_0 && \text{within the fluid}\\ &V=W=P_2=0 && \text{for all R and } \theta \end{aligned} \qquad (2\alpha)$$

At X>0

$$\begin{aligned} &U=V=W=0 && \text{at the walls}\\ &T=T_W && \text{at the walls} \end{aligned} \qquad (2b)$$

Solution is to be progressed along X until X= 0.2 for all Gr values. This distance is sufficient to insure that the fully developed region has been reached.

Some important engineering relations

have been put in dimensionless form:

$$Nu_x = \frac{hD_h}{k} = \frac{4\pi}{(\pi+4)^2}\frac{1}{(T_w - T_b)} \quad (3)$$

The product *fRe* where the friction factor *f* is defined by:

$$f = D_h(-d\overline{p}_1^* / dx)/(2\rho u_b) \quad (4)$$

the *fRe* product in non dimensional form

$$fRe = -\frac{1}{2\Pr}\frac{d\overline{P}_1}{dX} \quad (5)$$

. The numerical method employed to solve these equations is based on the SIMPLER method of Patankar [18] with the marching method of Patankar and Spalding [17]. It uses staggered grid with 30, 60, 273 increments in the radial, circumferential and axial directions. For the discretization of the governing equations (1-5), the power law scheme suggested by Patakanr [18] was used. The procedure used to calculate T_w was described by Parakash and Liu [19]. The present results of the fully developed fRe_{fd} and Nu_{fd} for the case of forced convection (Gr= 0) have been compared with the exact results obtained from Lei and Trupp [20,21]. The values are within 0.09%, 0.18% for fRe_{fd} and Nu_{fd} respectively. For fully developed mixed convection, a comparison of Nu_{fd} and fRe_{fd} has been made with the results of Chiponrncharoenpong. et al. [13] for horizontal quarter circle duct with the same orientation considered in this study and Pr = 4. 0. These results are presented in Table 1.

Table 1. Comparison of fully developed Nu_{fd} and fRe_{fd} with Ref [14] at Pr = 4.0.

Gr	Nu_{fd} Present	Nu_{fd} Ref[14]	fRe_{fd} Present	fRe_{fd} Ref[14]
1×10^6	8.268	8.274	16.907	16.905
1×10^7	13.448	13.470	20.830	20.820

Figure 1. Show a comparison between he present results and those of Lei and Trupp [16] of the axial variation of the Nu_x for the case of forced convection (Gr=0). The present results are in good agreement with the results of Ref [16]. The noticeable difference is limited to a very small region near the entrance. This is due to the fact that our model assumes simultaneous development of the temperature and velocity profiles. While Ref [16] assumes fully developed velocity profile and developing temperature profile.

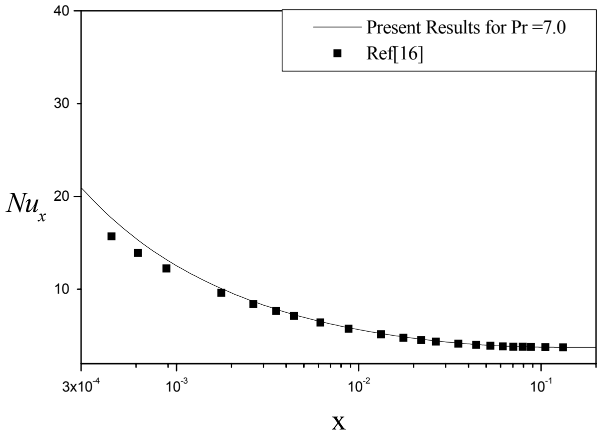

Fig 1. Variation of Nux for forced convection (Gr= 0)

3 Results and Discussion:

The numerical investigations were carried out with Pr = 0.7 and a wide range of Gr. The following sections represent the development of secondary flow patterns, axial velocity and temperature profiles and the overall development of overall quantities like Nu_x and *fRe*.

3.1 Development of secondary flow pattern:

Figure 2 shows the development of secondary flow pattern for the case of Gr = 10^7. In station (I) the secondary flow has a maximum cross- stream velocity of 39.3. The fluid moves from the retarding areas to the accelerating areas. While in station (II) the maximum cross- stream velocity has decreased to value of 27.8. In station (III) the maximum cross – stram velocity has been increased to a value of 201.7 and the flow consists of two secondary flow cells due to the effects of free convection. Also a strong upward flow has been created near the flat and curved walls. In station (IV) the two cells are fully established and the maximumcross- stream velocity is reduced to a value of 180.3. Finally in station (V) the flow is reduced to the fully developed state and the secondary flow pattern remains essentially the same from this station to the exit of the duct.

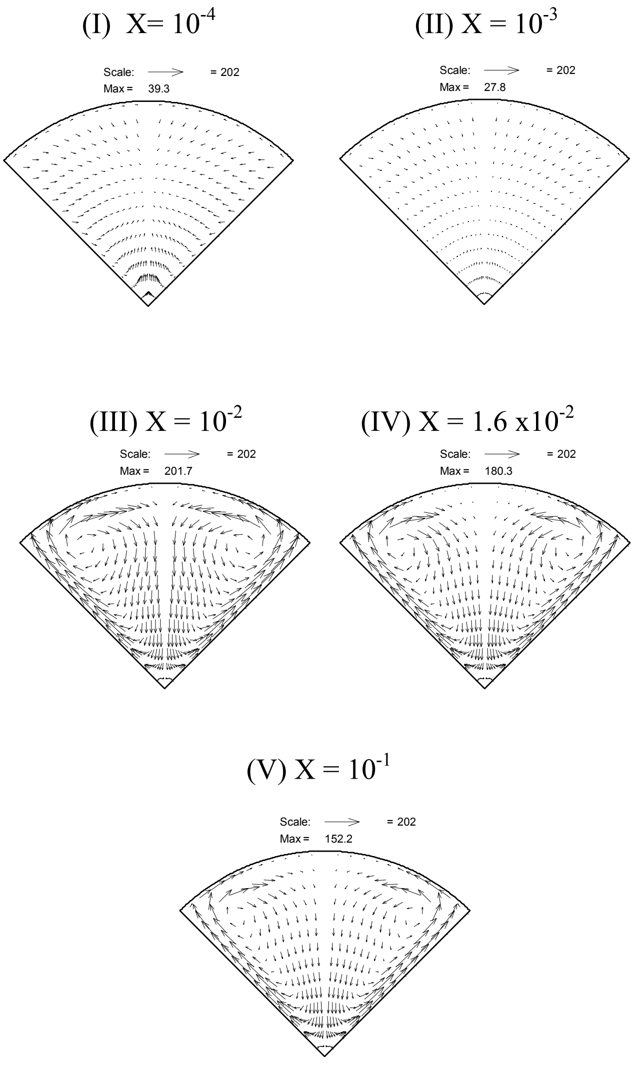

Fig. 2. Secondary flow pattern for different axial stations at Gr = 10^7

3.2 Development of axial velocity:

The isovels for the case of Gr = 107 is shown in Fig 3. In the absence of buoyancy effects, the isovels are seen to be similar to those of pure forced convection. The velocity is constant in the most part of the flow domain with sharp changes very near the duct walls as shown in stations (I) and (II). As the flow proceeds further downstream in station (III) the effect of free convection begins to appear by distorting the isovels particularly in the upper part of the duct due to the high intensity of the secondary flow near the upper corners of the duct. In station (IV) the isovels are distorted along the upper and lower part of the duct, while the maximum velocity confines at the lower part of the duct. In station (V) the fully developed state is reached and the maximum velocity is shifted further downwards toward the lower corner of the duct.

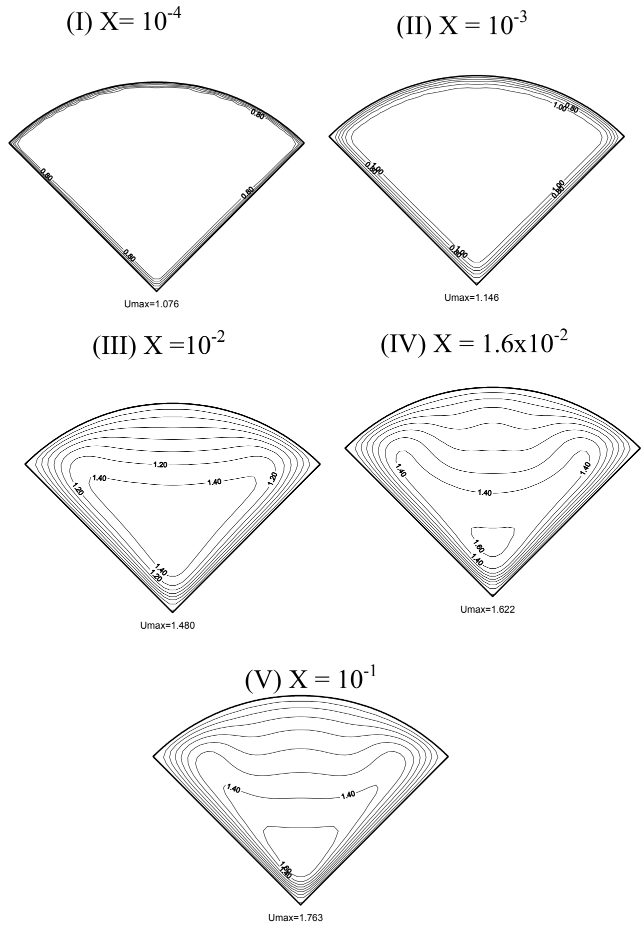

Fig 3 Axial velocity contours for different axial stations at Gr = 10^7.

3.3 Development of temperature:

Figure 4 shows the isotherms for the case of Gr = 10^7. It is clear that, in station (I) and (II), the fluid temperature is nearly uniform except at very near the wall indicating that the heat did not reach the core of the fluid in the duct cross section. In station (III), the effect of the secondary flow becomes important, especially near the upper corners. Further downstream in station (IV), the isotherms are shown to fill the whole fluid in the cross section indicating that the heat penetrated to the core of the fluid, at this station the temperature contours are clearly distorted due to the significant effect of the secondary flow. These effects are shown to shift the minimum temperature near the bottom of the cross section. At station (V) the flow becomes fully developed and the minimum temperature is still confined at the bottom.

3.4 Local Nusselt number and friction factor:

Figure 5 shows the development of the Nu_x for different Gr. The Nu_x is close to the forced convection values at smaller values of X due to the absence of the free convection. As X increases Nu_x decreases to a minimum due to the free convection effects, and then rises up to the fully developed value due to the generation of strong currents of the secondary flow. The increase in Gr enhances Nu_x in the developing and fully developed regions. The corresponding values of Nu_x at stations (IV) and (V) at Gr = 10^7 are 9.03 and 9.622 respectively which are 169% and 253% higher than those of forced convection at the same axial locations. Figure 6 shows the development of *fRe* for different Gr. Generally the results show similar behavior to that of Nu_x which was discussed above. However, the minimum location of *fRe* is not Cleary shown because the effect of free convection on the development of Nu_x is higher than that of *fRe*.

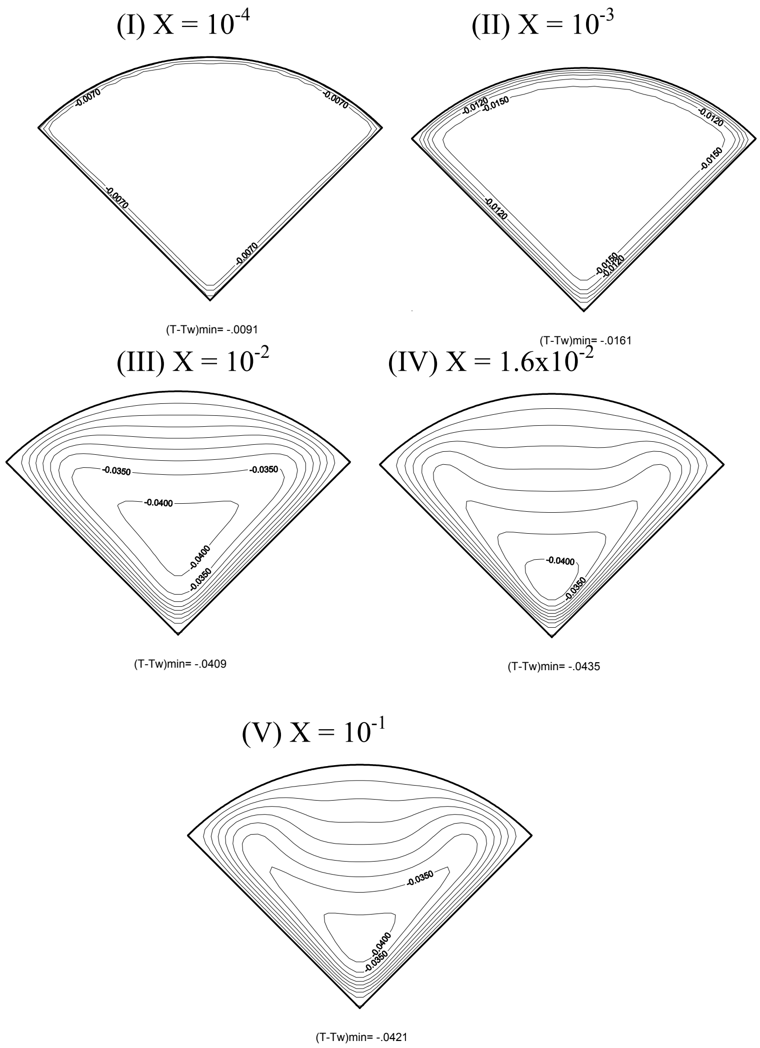

Fig4 (T-Tw) Temperature contours for different axial stations at Gr= 10^7.

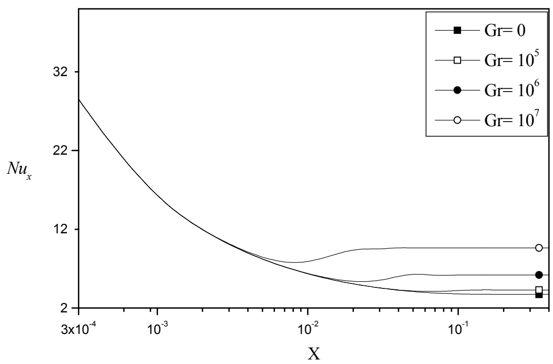

Fig 5 Effect of Gr on the axial development of Nu_x

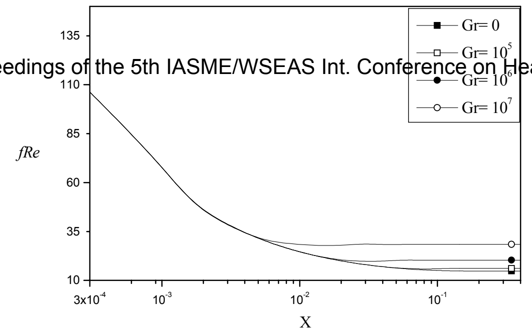

Fig 6 Effect of Gr on the axial development of *fRe*

4 CONCLUSIONS:

The main conclusions from this study is that the Nu_x increases in the developing and fully developed regions with Gr and also the thermal entrance lengths decreases with the increase of Gr. Also that *fRe* increases in the developing and fully developed regions with the increase of Gr.

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