Vortex Structure and Pressure Pulsations in a Swirling Jet Flow

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Abstract: - Both experimental and theoretical studies of swirling flow with structure in a form of precessing vortex core (PVC) have been carried out. LDA and acoustic technique have been employed to explore instantaneous and averaged structure of the unsteady flow. The theoretical approach is based on a model of vortex with a core in form of helical rope of circular cross-section. It is found tat in a special case of uniform vorticity distribution the Euler equations can be integrated to yield relation between the pressure and velocity fields. The model describes well the experimental velocity distributions and gives satisfactory prediction for the pressure field.

Key-Words: - Swirl flow, Vortex, Helical structure, Precessing vortex, Pressure pulsations, Velocity measurement

1 Introduction

Unsteady vortex structures are frequently observed in industrial and natural swirling flows. In technological devices these structures may be a cause of such undesirable phenomena as loud noise and strong vibrations. For instance, pressure/velocity pulsations induced by the precessing vortex rope are of especial concern for the hydroturbine's designers as this flow unsteadiness becomes particularly prominent at operation in partial or over loading regimes [1]. The first experimental work regarding the vortex precession effect was done by Vonnegut [2] who studied periodical flow oscillations in a vortex whistle, which were apparently issued from a vortex precession. A review on the history of studies of precessing vortices can be read in monograph by Gupta et al. [3], in a paper by Alekseenko and Okulov [4] and, in particular, in a recent work by Cala et al. [5].

Many researchers do not identify a PVC with the motion of large-scale vortical structures (helical vortices) but they do see a correlation between the generation of PVC and the development of counter-flow along the axis of the swirl flow [6]. As stated by Alekseenko et al. [7, 8], the existence of a counter-flow is a good indicator of a helical vortex in the flow. The lack of information about flow spatial structure in most papers may be attributed to the complexity of structure recognition in a precessing flow or explained by the lack of appropriate experimental techniques for the study of three-dimensional swirl flows [4]. Indeed, PVC can be easily observed in the plane normal to the flow axis. However, recovery of spatial geometry of the flow from a plain picture is an extremely complicated problem. In a very recent paper [5] authors used the λ_2-technique [9] to educe the quasi-steady 3D vortex structure in a swirl jet flow right away the nozzle exit. They identified additionally to primary helical-like vortex two secondary vortical structures of weaker intensity.

Here we will pay attention only to main helical vortex. As shown in [7, 8] the precessing vortex as well as PVC can be interpreted (at least in a local sense) as a helical vortex that rotates around its axis. The model of helical vortex with a core of circular cross-section was developed by Kuibin and Okulov [10] and later in [7, 8]. They deduced analytical formula describing the velocity of motion of helical vortex in boundless space as well as in cylindrical tube. As a consequence a formula for vortex precession frequency was derived.

The question on the pressure field in a swirl flow with PVC stay practically open till now. The present work covers both experimental and theoretical analysis of the pulsating pressure field induced by the precessing vortex core.

2 Experimental technique

The experiments were done with the air swirling jet issuing from the vortex chamber of a model vortex burner (Fig. 1). The working section of the set-up consists basically of a swirler device *3*, mixing chamber *1* of inner diameter 50 mm and combustion chamber *2* with diameter 110 mm and total length up to 300 mm. At mixing chamber exit a contraction is installed with diameter $d = 40$ mm.

Atmospheric air is used as a working media and it is loaded from a blower. The airflow rate is measured at a calibrated orifice connected to a U-type water manometer. All the data presented in the paper are made dimensionless using the nozzle diameter d and the flowrate-based mean velocity inside the nozzle, U_0. The Reynolds number Re = $U_0 d/\nu$ in experiments was of order 10^4–10^5. Here ν is the kinematic viscosity of the fluid.

The swirl level of the flow is controlled by changing the angle of blades position. It is characterized using the swirl number S calculated through the swirler geometry [3]

$$S = \frac{2}{3}\left[\frac{1-(D_1/D_2)^3}{1-(D_1/D_2)^2}\right]\tan\varphi\,, \tag{1}$$

where $D_1 = 90$ mm is the diameter of the central hub supporting the blades, $D_2 = 120$ mm is the external diameter of the swirler, and φ is the blade angle. During the experiments the swirl number was varied from 0 to 1.5.

The flow regime at high Re and swirl numbers was characterized by an instability leading to the vortex core precession around the geometrical center. The instantaneous pressure field induced by the precessing vortex was registered by a Brüel&Kjær condenser microphone mounted in a pressure probe made of a fine tube with outer diameter 1 mm. The signal from the microphone was amplified by amplifier and acquired by ADC board. Application of the microphone tip enabling local pressure measurement was crucial to studying the pressure filed distribution without significant flow disturbance. Since the acoustic probe produces an attenuation of the signal amplitude and also shift in the phase, knowledge of the transfer function is needed to recover actual pressure fluctuations in a local point. This was done using theoretical model of the measuring probe [11]. The theory results were verified using a test facility which was employed to determine experimental transfer function of the pressure probe [12]. Standard one component dual beam Dantec LDV system operating in a forward scattering mode has been used for the velocity measurements [5]. The flow seeding was carried out using a paraffin oil atomizer. A phase averaging was applied to LDV or local pressure signals recorded simultaneously with a reference signal tracking the precession phase angle [13].

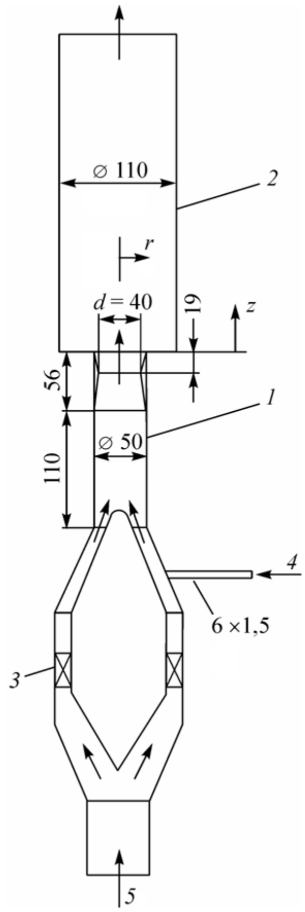

Fig. 1. Schematic diagram of experimental setup. *1* – mixing chamber, *2* – combustion chamber, *3* – blade swirler, *4* – aerosol feeding, *5* – air feeding

3 Theoretical model

Theoretical approach used in the present work is based on the Theory of Helical Vortices [7, 8]. Consider a model of vortex with a core in form of helical rope of circular cross-section with radius ε (Fig. 2). Let us denote the helix radius as a, pitch $h = 2\pi l$, and intensity of the vortex Γ. The helix is placed coaxially in a tube of radius R, and velocity at the tube axis is u_0. Suppose that the axial vorticity component ω_z is uniform inside the core and outside it the flow is potential. To solve the problem

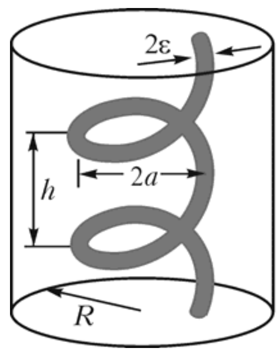

Fig. 2. The model of helical vortex

on the velocity field induced by the helical vortex of finite core size one needs in integration of the velocity induced by infinitely thin helical vortex over the core cross-section. Before this operation it is necessary to determine the shape of core cross-section. Fortunately, as was found by Fukumoto and Okulov [14], the core shape in wide range of parameters presents helical tube with circular cross-section and outside the core velocity field coincides with high accuracy with velocity induced by infinitely thin helical vortex. Thus, in the model we will take into account these outcomes and inside the vortex core we will find velocity by means of linear interpolation like in a model of Rankine vortex.

The next stage of the model construction lies in determination of the pressure field. An analytical solution of the problem for inviscid incompressible fluid is possible due to two factors: helical symmetry of the flow considered and its periodicity in time. Indeed in this case equations of motion in the cylindrical coordinate system (r, θ, z) in the Gromeka – Lamb notation has the following form

$$\frac{\partial u_r}{\partial t}+\frac{\partial}{\partial r}\left(\frac{u_r^2+u_\theta^2+u_z^2}{2}+\frac{p}{\rho}\right)=u_\theta\omega_z-u_z\omega_\theta,$$

$$\frac{\partial u_\theta}{\partial t}+\frac{\partial}{r\partial \theta}\left(\frac{u_r^2+u_\theta^2+u_z^2}{2}+\frac{p}{\rho}\right)=u_z\omega_r-u_r\omega_z, \quad (2)$$

$$\frac{\partial u_z}{\partial t}+\frac{\partial}{\partial z}\left(\frac{u_r^2+u_\theta^2+u_z^2}{2}+\frac{p}{\rho}\right)=u_r\omega_\theta-u_\theta\omega_r,$$

$$\frac{\partial(ru_r)}{\partial r}+\frac{\partial u_\theta}{\partial \theta}+\frac{\partial(ru_z)}{\partial z}=0. \quad (3)$$

Here u and ω with indices mean the velocity and vorticity components respectively, p is pressure and ρ is the fluid density. With introduced new variable $\chi = \theta - z/l$, helical symmetry allows for the relations [7, 8]:

$$u_z+\frac{r}{l}u_\theta=u_0, \quad \frac{\partial}{\partial\theta}=\frac{\partial}{\partial\chi}, \quad \frac{\partial}{\partial z}=-\frac{1}{l}\frac{\partial}{\partial\chi}, \quad (4)$$

$$\omega_r=0, \quad \frac{r}{l}\omega_z=\omega_\theta, \quad (5)$$

$$\boldsymbol{u}(r,\theta,z)=\boldsymbol{u}(r,\theta-z/l,0)=\boldsymbol{u}(r,\chi). \quad (6)$$

$\boldsymbol{u}$ denotes the velocity vector. The parameters of vortex being known, the frequency, $f = n/2\pi$, of its precession can be found with help of formulae derived in [10]. Then

$$\boldsymbol{u}(r,\chi,t)=\boldsymbol{u}(r,\chi-nt,0), \quad \frac{\partial}{\partial t}=-n\frac{\partial}{\partial\chi}. \quad (7)$$

Taking into account Eqs. (4) – (7) we reduce the system (2) to two equations

$$-n\frac{\partial u_r}{\partial\chi}+\frac{\partial}{\partial r}\left(\frac{\boldsymbol{u}^2}{2}+\frac{p}{\rho}\right)=u_\theta\omega_z-u_z\frac{r}{l}\omega_z, \quad (8)$$

$$-n\frac{\partial u_z}{\partial\chi}-\frac{1}{l}\frac{\partial}{\partial\chi}\left(\frac{\boldsymbol{u}^2}{2}+\frac{p}{\rho}\right)=u_r\frac{r}{l}\omega_z. \quad (9)$$

Finally, with use of new variable

$$u_\chi=u_\theta-\frac{r}{l}u_z,$$

stream function defined to satisfy the equation of continuity (3)

$$u_r=\frac{1}{r}\frac{\partial\psi}{\partial\chi}, \quad u_\chi=-\frac{\partial\psi}{\partial r},$$

definition of ω_z and formulae (4) we transform equations (8, 9):

$$\frac{\partial}{\partial r}\left(\frac{\boldsymbol{u}^2}{2}+\frac{p}{\rho}+nlu_z\right)=-\frac{\partial}{\partial r}\left(\psi+n\frac{r^2}{2}\right)\omega_z,$$

$$\frac{\partial}{\partial \chi}\left(\frac{\boldsymbol{u}^2}{2}+\frac{p}{\rho}+nlu_z\right)=-\frac{\partial}{\partial \chi}\left(\psi+n\frac{r^2}{2}\right)\omega_z.$$

For a uniform distribution of vorticity ω_z these equations are shown to be integrable to yield

$$\frac{p}{\rho}=const-\frac{\boldsymbol{u}^2}{2}-nlu_z-\left(\psi+n\frac{r^2}{2}\right)\omega_z. \quad (10)$$

Equation (10) allows determining pressure distribution at known parameters of helical vortex. Moreover we can predict pressure pulsations because of velocity and stream function are periodical functions of time. In particular, for pressure pulsations on a wall in the case of helical vortex in a tube we have

$$\frac{p_0}{\rho}=const-\left(nlu_z+\frac{u_\theta^2+u_z^2}{2}\right)\Bigg|_{\substack{r=R\\ \theta=0}}.$$

4 Results

The experimental system used generates very stable flow pulsations that are clearly identifiable in the velocity and pressure signals. The case with $Re = 15200$ (U_0 = 4.06 m/s) and S = 1.01 (blade angle 50°) was chosen for the analysis of the pressure field. The time-averaged velocity profiles are shown in Fig. 3. We suppose that the main precessing vortex structure looks as a helical vortex and there exists local helical symmetry of the flow. To determine the local parameters of the main structure, pitch l and value of the axial velocity at the flow axis u_0, we will analyze only the inner part of

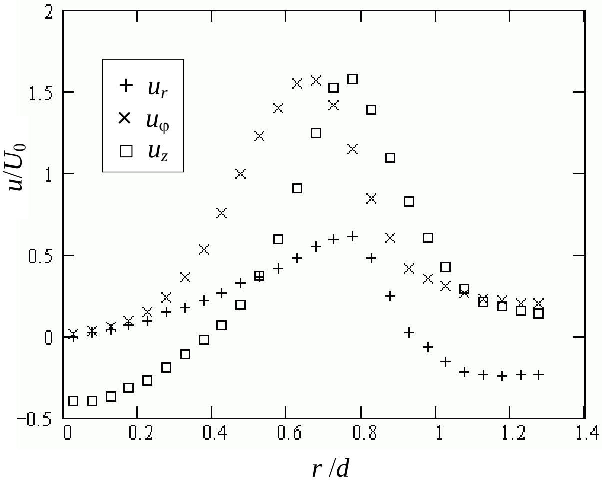

Fig. 3. Time-averaged flow structure

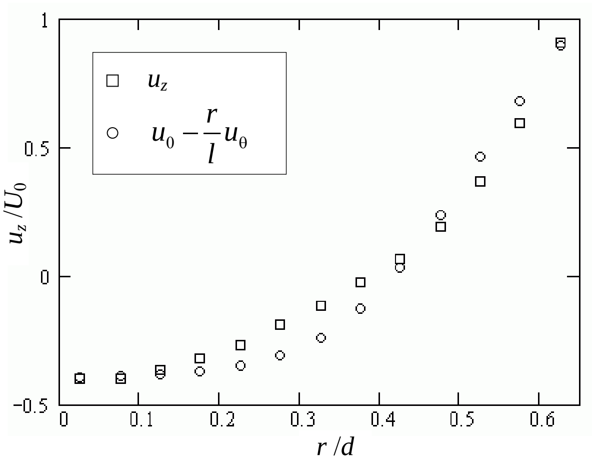

Fig. 4. Testing the local helical symmetry

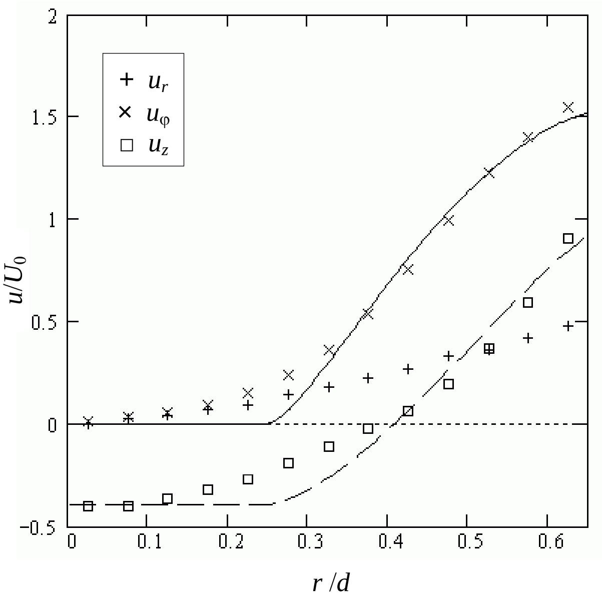

Fig. 5. Model (lines) and experimental (symbols) velocity profiles

a

b

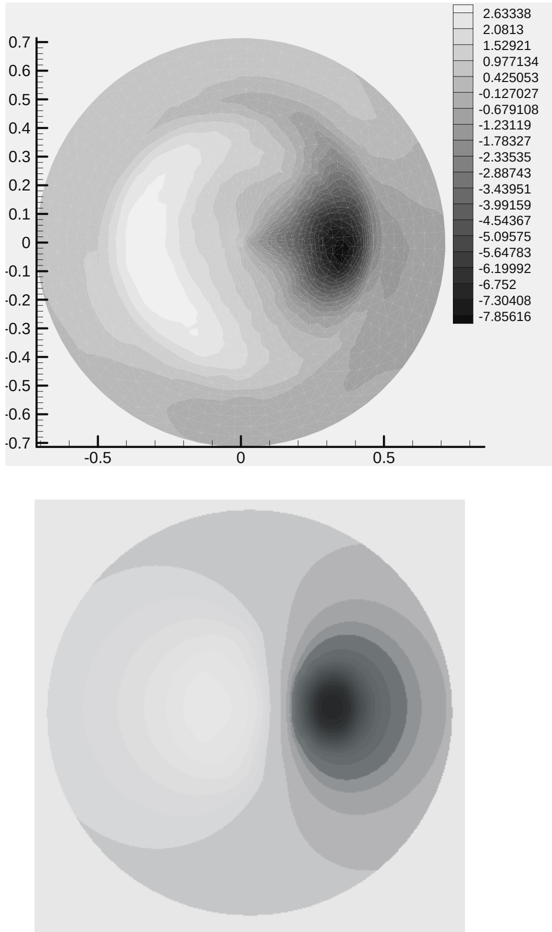

Fig. 6. The phase-averaged plot of the pressure distribution at outlet from an air vortex chamber (*a*) and pressure distribution in the model of helical vortex (*b*)

the profiles. The plot in Fig. 4. demonstrates that the condition of helical symmetry (the first formula in (4)) takes place in the inner area with parameters $l/d = -0.75$, $u_0/U_0 = -0.39$.

The final constructing of the model gives the last parameters of the helical vortex, radius of helix a and radius of core ε [7, 8]. The results are presented in Fig. 5. As seen, the flow field in the inner area is well described by the model.

Now, when we have all the vortex parameters we can use equation (10) to determine an unsteady pressure field. Figure 6 shows comparison of the experimental data on the phase-averaged pressure field with theoretical model. In the theoretical model we have very deep pressure minimum in the core and in the plot we used non-uniform levels to see small local pressure maximum in the diametrically opposite area which is observed in the experiment. The agreement can be considered as satisfactory.

One should note that in the experimental conditions considered there exists radial expansion of the vortex and it will be better to consider flow in a long cylindrical tube. Such a statement is planned in a perspective research. As for theoretical model, it is necessary to develop approach to take into account a smooth vorticity distribution.

5 Conclusion

The experimental study of the unsteady velocity and pressure fields in a swirl flow is done. A clearly pronounced coherent helical-like vortex structure with separated line in the spectrum is registered. This structure obviously relates to the phenomenon of precessing vortex core. The use of technique of phase-averaged LDA-measurements made it possible to obtain the rotating non-axi-symmetrical pressure field.

For description of these experimental data an analytical study is done. A formula of connection between the pressure and velocity field is proposed. A satisfactory correspondence between measured data and model is found.

Thus, the foundations are developed for description and prediction of pressure pulsations induced by a precessing vortex in a swirl flow.

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References:

[1] C. Nicolet, J. Arpe, F. Avellan, Identification and Modeling of Pressure Fluctuations of a Francis Turbine Scale Model at Part Load Operation. In *Proc 22nd IAHR Symposium on Hydraulic Machinery and Systems*, Stockholm, Sweden, June 29 - July 2, 2004.

[2] B. Vonnegut (1954) A vortex whistle. *J. Acoust Soc Am*, Vol. 26, pp. 18–20.

[3] A.K. Gupta, D.G. Lilley, N. Syred, *Swirl flows*. Abacus Press, Tunbridge Wells, UK, 1984

[4] S.V. Alekseenko, V.L. Okulov, Swirl flow in technical applications (review), *Thermophysics and Aeromechanics*, Vol. 3, No. 2, 1996, pp. 97–128

[5] C.E. Cala, E.C. Fernandes, M.V. Heitor, S.I. Shtork, Coherent structures in unsteady swirling jet flow, *Exp. Fluids*, Vol. 40, No. 2, 2006, pp. 267–276.

[6] P.A. Yazdabadi, A.J. Griffiths, N. Syred, Characterization of the PVC phenomena in the exhaust of cyclone dust separator, *Exp. Fluids*, Vol. 17, 1994, pp. 84–95.

[7] S.V. Alekseenko, P.A. Kuibin, V.L. Okulov, S.I. Shtork, Helical Vortices in Swirl Flow, *J. Fluid Mech.*, Vol. 382, 1999, pp. 195-243.

[8] S.V. Alekseenko, P.A. Kuibin, V.L. Okulov, *Introduction to theory of concentrated vortices*, Institute of Thermophysics, Novosibirsk, Russia, (in Russian), 2005.

[9] J. Jeong, F. Hussain, On the identification of a vortex, *J. Fluid Mech.*, Vol. 285, 1995, pp. 69–94.

[10] P.A. Kuibin, V.L. Okulov, Self-induced motion and asymptotic expansion of the velocity field in the vicinity of helical vortex filament, *Phys. Fluids*, No. 10, 1998, pp. 607–614.

[11] E.C. Fernandes, The Design of Sound Probes. In: *Proceedings of the 12th International Congress on Sound and Vibration*, Lisbon, Portugal, 2005.

[12] E.C. Fernandes, *The onset of combustion-driven acoustic oscillations*, PhD thesis, Instituto Superior Técnico, Lisbon, Portugal, 1998.

[13] S.I. Shtork, C.E. Cala, E.C. Fernandes, M.V. Heitor, Coherent Helical Structures in Swirl Flows. *Techn. Phys. Lett.*, Vol. 31, No. 8, 2005, pp. 660-662.

[14] Y. Fukumoto, V. L. Okulov, The velocity field induced by a helical vortex tube, *Phys. Fluids*, Vol. 17, No. 10, 2005, pp. 107101(1 – 19).