Effect of Horizontal and non Horizontal Heat and Mass Gradients on Thermosolutal Convection in a Partially Porous Cavity

RAHLI OMAR AND BOUHADEF KHEDIDJA

Laboratoire TPMP,
Faculté de Génie Mécanique et Génie des Procédés, USTHB
BP 32 El Alia, Bab Ezzouar, Algiers, Algeria

Abstract: - The present paper consists of analyzing the flow regimes and the effects of different dimensionless parameters on double-diffusive natural convection in an air filled rectangular enclosure divided by a saturated porous medium. The heat and mass convection is analysed by solving numerically the governing equations traduced by mass, momentum, energy and concentration balance equations.
For the range of parameters considered in this study, it is demonstrated that the thermophysical and geometrical properties as, Darcy number Da, aspect ratio A, Lewis number Le and cavity inclination angle φ, considerably affect the heat and mass transfer. For example, the results show that for high values of the permeability the exchanges are more intense and tend towards maximum limiting values. Otherwise, the influence of the aspect ratio is traduced by the existence of optimal values for the heat and mass transfers.

Keywords: Thermosolutal diffusion – Natural convection - Porous medium - Darcy-Brinkman formulation.

1 Introduction

Double–diffusion occurs in a wide range of scientific fields, such as oceanography, astrophysics, geology, biology and chemical processes ; so, the authors interest more and more for the heat and mass transfer developed in enclosures or cavities.

About these cases of fluid flows generated by combined temperature and concentration gradients, the studies of double-diffusive natural convection have centred chiefly their analyses on the limit cases of dominating thermal buoyancy force or concentration buoyancy force. The considered spaces are enclosures comprising a fluid only **[1-6]** or completely occupied by porous medium **[7-8]**.

The studies on double-diffusive natural convection in cavities containing simultaneously a fluid and porous medium are limited at case of horizontal temperature and concentration gradients **[9-11]**.

In this study, we consider the problem of steady, laminar, two-dimensional double-diffusive natural convection inside an inclined rectangular enclosure filled with air and divided by a saturated porous medium. Imposed conditions of temperature and concentration are on the left and right sides and at the bottom of porous region.
This kind of flow and heat and mass transfer problems find application in many engineering technological areas such as geothermal reservoirs, petroleum extraction, chemical catalytic reactors, prevention of water pollution, nuclear reactor, underground diffusion of nuclear wastes and other contaminants, and porous material regenerative heat exchangers.

The aim of the numerical work presented in this paper, is to shed light on an entirely new class of flows in this kind of cavities. We analyze the thermal and solutal exchanges generated in the case of co-operating thermal and concentration buoyancy effects.
We also interest to the influence of different thermophysical and geometrical parameters on heat and mass transfer, and on the regime of flow which occurs.

2 Physical model

We consider a steady laminar two-dimensional double-diffusive natural convection inside an inclined cavity divided by a porous medium with an aspect ratio A= H/L. The temperature T_1 and T_2 and concentration C_1 and C_2 are uniformly and respectively imposed on the two opposing walls (left and right), while the bottom of porous region is submitted at temperature T_1 and concentration C_1. The other walls are assumed adiabatic and impermeable to mass transfer. The schematic of the system under consideration is shown in fig.1.

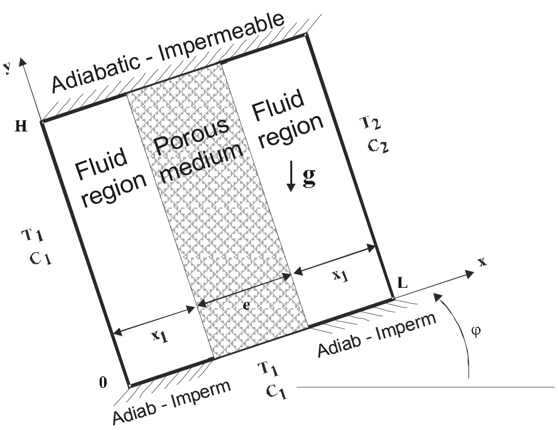

Fig. 1: Physical model

3 Mathematical formulation

In order to simplify the modelling of the problem, the equations governing the principles of conservation are formulated by adopting following simplifying assumptions:

- The fluid is considered Newtonian and incompressible.
- The Boussinesq approximation is applied, that is :

$$\rho(T,C) = \rho_{ref}\,[1 - \beta_T\,(T-T_{ref}) - \beta_c\,(C-C_{ref})] \qquad (1)$$

- The flow is supposed to be laminar and permanent.
- The movement of the fluid in the porous partition is governed by the Darcy-Brinkman model to take in account the Darcien and viscosity effects, whereas in the fluid compartments, it is described by the Navier-Stokes equations.
- The energy and mass equations are also transformed and averaged to consider the porous nature of the matrix.

3.1 Governing equations

Taking into account the assumptions quoted previously, the equations governing the phenomenon are written:

Continuity equation

$$\vec{\nabla}\cdot\vec{V} = 0 \qquad (2)$$

Momentum equation

$$\frac{\rho_f}{\varepsilon^2}\,\vec{V}\cdot\vec{\nabla}\cdot\vec{V} = -\vec{\nabla}P + \rho_f\,\vec{g} - \frac{\mu_f}{K}\,\vec{V} + \mu_{eff}\nabla^2\vec{V} \qquad (3)$$

Energy equation

$$\left(\rho\,c_p\right)_f\,\vec{V}\cdot\vec{\nabla}\cdot T = k_{eff}\,\nabla^2 T \qquad (4)$$

Diffusion equation :

$$\vec{V}\cdot\vec{\nabla}\cdot C = D_{eff}\,\nabla^2 C \qquad (5)$$

3.2 Boundary conditions

Velocity conditions:

x=0 and $0 \le y \le H$: $u = v = 0$
x=L and $0 \le y \le H$: $u = v = 0$
y=0 and $0 \le x \le L$: $u = v = 0$
y=H and $0 \le x \le L$: $u = v = 0$

Thermal and solutal conditions:

x=0 and $0 \le y \le H$:
$$T = T_1 \ , C = C_1$$
x=L and $0 \le y \le H$:
$$T = T_2 \, , C = C_2$$
$x \in [0, x_1]\, U\, [x_1 + e, L]$ and $y = 0$:
$$\left.\frac{\partial T}{\partial y}\right|_{(x,0)} = 0 \, , \left.\frac{\partial C}{\partial y}\right|_{(x,0)} = 0$$
$x \in [x_{1,}\, x_1 + e]$ and $y = 0$:
$$T = T_1 \, , C = C_1$$
$0 < x < L$ and $y = H$:
$$\left.\frac{\partial T}{\partial y}\right|_{(x,H)} = 0 \, , \left.\frac{\partial C}{\partial y}\right|_{(x,H)} = 0$$

3.3 Non-dimensional equations

In order to generalize the results, the system of equations which governs the physical phenomenon is put in adimensional form introducing the following variables:

$$X = \frac{x}{L};\ Y = \frac{y}{L};\ U = \frac{uL}{\alpha}\ ;\ V = \frac{vL}{\alpha};$$

$$\theta = \frac{T - T_{ref}}{T_1 - T_2};\ S = \frac{C - C_{ref}}{C_1 - C_2}\ ;\ P = p\frac{L^2}{\left(\frac{\rho_f \cdot \alpha^2}{\varepsilon^2}\right)}$$

$$T_{ref} = (T_1 + T_2) / 2 \quad ; C_{ref} = (C_1 + C_2) / 2$$

3.4 Heat and mass transfer coefficients

The local heat and mass transfers are represented, respectively, by the Nusselt and Sherwood numbers. In dimensionless form, one obtains the following final expressions:

On the left and the right walls:

$$Nu(Y) = -\left.\frac{\partial \theta}{\partial X}\right|_{x=0 \text{ and } 1} ; Sh(Y) = -\left.\frac{\partial S}{\partial X}\right|_{x=0 \text{ and } 1} \quad (7)$$

On the bottom wall:

$$Nu(X) = -\left.\frac{\partial \theta}{\partial Y}\right|_{x=0 \text{ and } A} ; Sh(X) = -\left.\frac{\partial S}{\partial Y}\right|_{x=0 \text{ and } A} \quad (8)$$

There after, the mean exchanges in the cavity are calculated by using the averaged Nusselt and Sherwood expressions on the right wall of the cavity.

4 Numerical procedure

The governing dimensionless equations are discretized using the control volume formulation developed by Patankar **[13]**. The power law scheme was used to calculate both the heat and mass fluxes across the boundaries of each control volume.

Taking into account the structure of the domain, and after tests on the sensitivity to the grid, we chose a grid of 60x60 nodes, not uniform in the axial direction, with a more significant concentration of the nodes in the vicinity of the walls and the fluid/porous partition interfaces. On the other hand, a sufficiently fine regular grid was adopted in the y direction.

The momentum equations are coupled between them by the intermediary of the pressure field. This difficulty of coupling velocity and pressure comes from the absence of explicit equations which control the pressure field. For this reason, one uses non direct methods as that based on algorithm SIMPLER.

The algebraic equations obtained are solved using a numerical algorithm based on an iterative line-by-line method (TDMA) and relaxation parameters. The process was repeated until convergence.

Concerning the test of convergence, we choose the criterion which relates to the residues of mass, momentum, temperature and concentration. These residues are defined by:

$$R_{\phi_P} = \sum a_{nb}\phi_{nb} + b - a_p\phi_P \quad (9)$$

Mathematically, it is traduced by the following inequality:

$$\sum_{\Omega} \left| R_{\phi_P} \right| < \xi \quad (10)$$

where Ω represents the field of calculation and ξ is a very weak value characterizing the relative error on the solution obtained, taken in our case lower than 10^{-6}

5 Results

In this section, numerical results for the streamlines, temperatures and concentrations for various values of Darcy and Lewis numbers, aspect ratio A, and enclosure inclination angle φ, are reported. In addition, representative results for the average Nusselt number Nu_m and the average Sherwood number Sh_m at various conditions are presented and discussed.

In all of these results, the Prandtl number Pr is considered at the value of air (Pr = 0.71), the conductivity ratio (R_k) the buoyancy ration N and the viscosity ratio (R_v) were fixed at value of 1, while the porous matrix thickness (E) is 33%. The other parameters are chosen as follow:

$$10^{-6} \leq Da \leq 1\ ,\ 0.1 \leq A \leq 10\ ,\ 0.01 \leq Le \leq 5,$$

$$10^{+3} \leq Ra \leq 10^{+7},\ 0° \leq \varphi \leq 90°$$

4.1 Influence of permeability

The permeability has a considerable effect on the transfers (fig. 2). For Rayleigh numbers Ra $\leq 10^{+5}$, the influence of the Darcy number highlights three modes of transfers:

- a pure diffusive mode for the low Darcy numbers.
- a convective intermediate mode for moderate Darcy number.
- a convective mode, for high Darcy numbers (Da > 10^{-2}) characterized by a boundary layer evolution type and an independency of the permeability.

This enables us to conclude that the critical value of the permeability which connects the intermediate convective mode to the boundary layer type mode, depends on the value of the Rayleigh number.

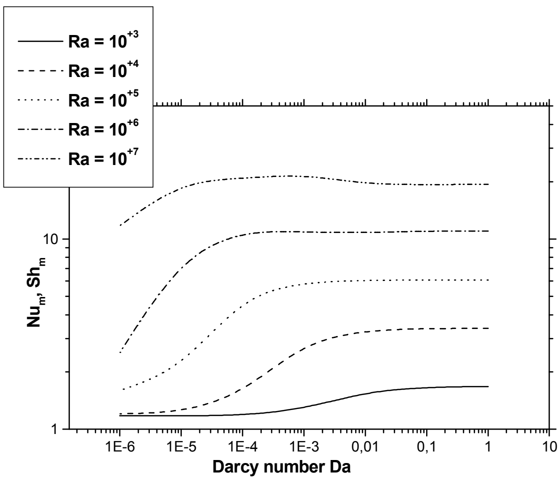

Fig. 2: Average heat and mass transfer coefficients versus Darcy number.
Pr = 0.71, Le =1, φ = 0, ε = 1, A =1, E = 0.33

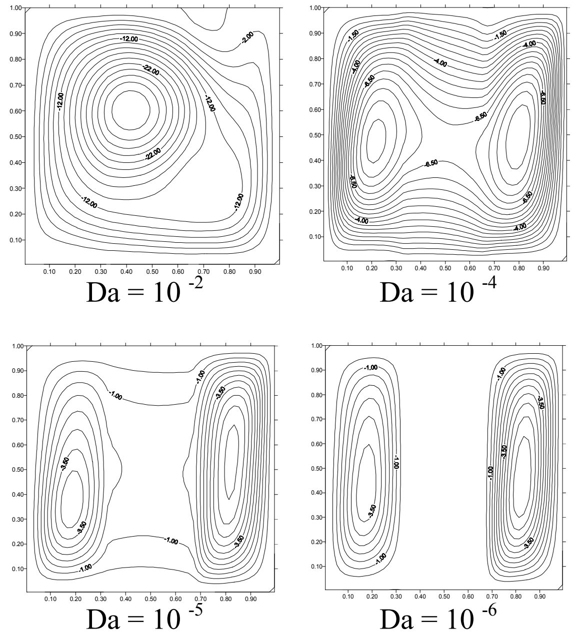

Fig. 3: Stream functions for Ra=10^{+5}, Le=1, φ = 0, ε = 0.6, A =1, E = 0.33

Figure 3 presents the streamlines for different values of the Darcy number for Ra = 10^{+5}. When the permeability is high (Da $\geq 10^{-2}$), the fluid flow is characterized by a one cell movement. In this case, the resistance to the flow is weak and this explains the non dependence with the permeability, of the asymptotic values of the exchanges (Fig.2). For smaller permeabilities the flow structure becomes two-cellular and for very weak values of Darcy (Da<10^{-5}) the porous matrix behaves more and more like a solid wall which divides the cavity in two enclosures, each of them presenting one cell movement.

4.2 Aspect ratio effect

The evolution of the heat and mass transfers, with the aspect ratio, is presented on fig. 4. One can quote that the exchanges augment with the aspect ratio A until a critical value of this parameter which value is about A=0.6 in this case. After this value, the transfers become decreasing when A is more increasing. Thus the results permit us to put in evidence an optimal value for the exchanges.

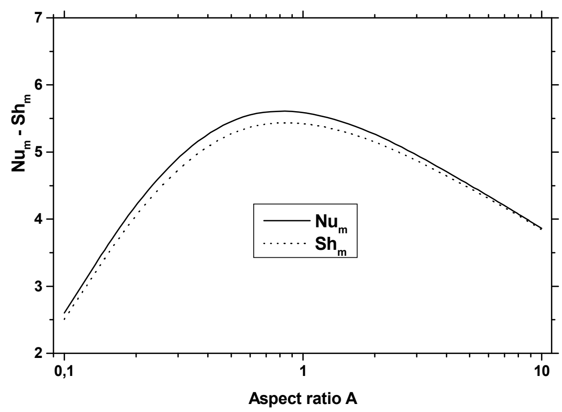

Fig. 4: Average Nusselt and Sherwood numbers versus the aspect ratio A for Da = 10^{-3}, Le = 1, Ra = 10^{+5}, ε = 0.6, φ =0

4.3 Effect of enclosure inclination angle

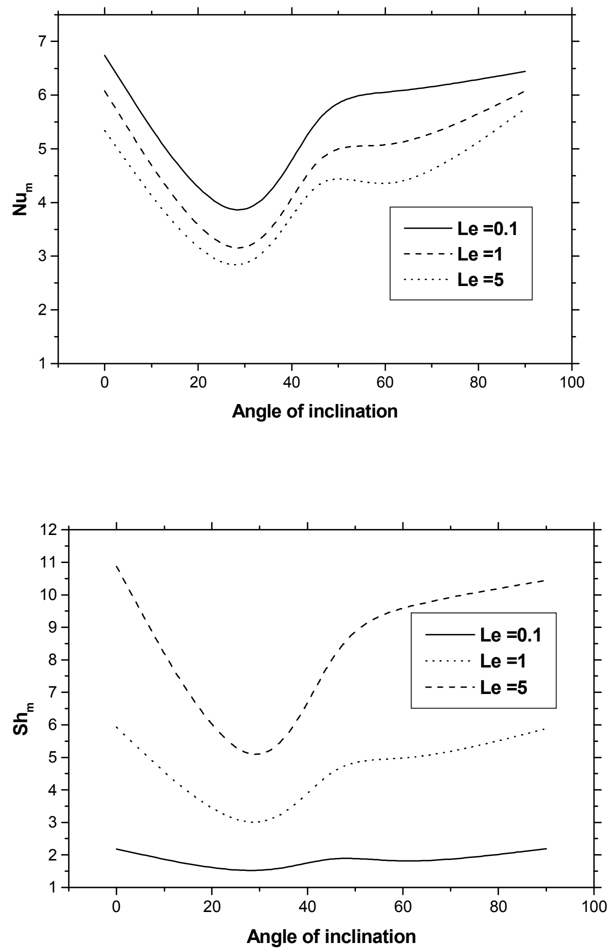

Fig. 5: Average Nusselt and Sherwood numbers function of enclosure inclination angle. Da = 10^{-3}, Ra = 10^{+5}, ε = 0.6, E=0.33

The influence of the angle of inclination φ of the cavity on the transfers is represented on fig. 5 for several values of the Lewis number. It is noted that the effect of this angle is relatively important ; thus, it is noticed that the transfer of heat starts with a significant value with φ= 0 and decreases until reaching its minimal value which corresponds to an angle φ of about 30°. Beyond this value, the average Nusselt number becomes again growing in a rather significant way at first; this increase attenuates when the inclination exceeds 50°, where one notices the appearance of a second minimum towards φ = 60° when Le≥ 1.

For values of the Lewis number lower than the unit (Le=0.1), one notes that the average Nusselt number is maximum for a zero angle of inclination. On the other hand, it is an inclination of 90° which gives a maximum of heat transfer when the Lewis number is significant (Le = 5).

For the particular value Le=1, the figure 6-a shows that the heat transfer is identical for the inclinations of 0° and 90°.

Concerning the mass transfer, the figure 6-b shows that the Sherwood number presents qualitatively the same evolution according to the angle of inclination. One notes, indeed a rather high value at the beginning (φ = 0), then a regular decrease until a minimal value at φ ≈ 30°. The mass transfer starts again to increase gradually until an inflection towards an angle of about 50°.

In contrary to the heat transfer, the solutal exchange coefficient is more significant since the Lewis number is high. For a high value of Le, the transfer is maximum for a null angle of inclination; whereas for a low Lewis number, the exchanges are almost identical at φ = 0 and φ = 90°.

5 Conclusion

The main motivation of this work falls under the general context of comprehension and control of the phenomena which appear in thermosolutal convection in spaces containing simultaneously a fluid and a porous medium. This numerical study permits to determine a certain number of results corresponding to a selected range of several characteristic non-dimensional parameters.

We, initially, highlighted the influence of the permeability (effect of the Darcy number, Da) on the heat and mass transfers. In addition to the conductive mode, this approach made it possible to identify two modes of convective flows: one intermediate and the other in boundary layer. With very low permeability, the porous partition behaves like a solid.
The rise in the Lewis number increases the mass transfer; on the other hand, the coefficient of heat exchange is much more sensitive to the variation of the permeability than to the increase in the Lewis number.
The influence of the inclination of the cavity compared to the horizontal position revealed the existence of critical values for either thermal or mass transfer.
On the same way, the investigation concerning the effect of the aspect ratio, permitted to find an optimal value of this ratio for which the exchanges are maximized.

References

[1] Han H. and Kuehn T. H, Double diffusive natural convection in a vertical rectangular enclosure. *II Numerical study, Int. J. Heat Mass Transfer,* Vol. 34, N°2, pp. 461-471, 1991.

[2] T.F. Lin, C. Huang & T.Chang, Transient binary mixture naturel convection in a square enclosure, Int. J. Heat Mass Transfer, vol. 33, pp 1315-1331 (1993).

[3] Hyun J. M. and Lee J. W., Double Diffusive Convection in a Rectangule with Cooperating Horizontal Gradients of Temperature and Concentration, *Int. J. Heat Mass Transfer,* Vol. 33, N°8, pp. 1605-1617, 1990.

[4] Jiang H. D., et Ostrach S. And Kamotani Y., Unsteady thermosolutal transport phenomena due to opposed buoyancy forces in a shallow enclosures, *Int. J. Heat Transfer,* Vol.113, pp. 135-140, 1991.

[5] Lin T.F., Huang C. and Chang T., Transient binary mixture naturel convection in a square enclosure, *Int. J. Heat Mass Transfer,* vol. 33, pp 1315-1331 (1993).

[6] Trevisan V.O., and Bejan A., Combined with heat and mass transfer by natural convection in a vertical enclosure, *Int. J. Heat Transfer,* vol. 109, pp 104-112 (1992).

[7] Alavyoon F., Masuda Y. and Kimura S.,On natural convection in vertical porous enclosures due to opposing fluxes of heat and mass at the vertical walls, *Int. J. Heat Mass Transfer,* Vol.37, pp. 195-206,1994.

[8] Chamkha A. J. and Al-Naser H., Double diffusive convection in an inclined porous enclosure with opposing temperature and concentration gradients. *Int. J. Therm. Sci*, vol. 40, pp 227-244 (2001).

[9] Bennacer R., Tobbal A., Beji H.and Vasseur P., Double diffusive convection in a vertical enclosure filled with anisotropic porous media, *Int. J. Therm. Sci.* vol. 40, pp. 30-41(2001).

[10] Goyeau B., Songbe J. P. and Gobin D., Numerical study of double-diffusive natural convection in a porous cavity using the Darcy-Brinkman formulation, *Int. J. Heat Mass Transfer,* Vol.39, pp. 1363-1378,1996.

[11] Tong T. W. and Subramanian E., Naturel convection in rectangular enclosures partially filled with a porous medium*, Int. J. Heat and Fluid Flow,* vol. 7, pp 2-10 (1988).

[12] Mhrazi M., Daoudi S. and Daguennet M., Thermosolutal natural convection in a vertically layered fluid-porous medium heated from the side, *Energy Conversion & Management,* Vol.41, pp. 1065-1090, 2000.

[13] Patankar S.V., Numerical heat transfer and fluid flow, *Hemisphere, Mc Graw-Hill,* New York (1980). (Book)