Analysis of Laminar-to-Turbulent Threshold with *Water-γAl_2O_3* and *Ethylene glycol-γAl_2O_3* Nanofluids in Free Convection

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Abstract: - In the present work, a theoretical model based on the integral formalism approach for both laminar and turbulent external free convection is extended to Newtonian nanofluids. By using an empirical model for computing nanofluid viscosity, a close attention is focused first on the way the heat transfer is modified by increasing the volume fraction of nanoparticles and then to the transitional threshold between laminar and turbulent regimes. Using this viscosity model, a systematic degradation in the heat transfer is observed using nanofluids while compared to the base-fluid. Nevertheless, some interesting major conclusions are drawn and show that the heat transfer depends strongly on the flow regime: results have shown that the Ethylene glycol-γAl_2O_3 mixture gives a better heat transfer enhancement than the water-γAl_2O_3 mixture in the laminar regime while the reversed trend is found for a turbulent flow. Moreover, for such Newtonian nanofluids, increasing the volume fraction of metallic oxide particles delays the occurrence of the flow transition to turbulence.

Key-Words: - Heat transfer, Water-γAl_2O_3, Ethylene glycol-γAl_2O_3, Free convection, Laminar-to-turbulent transition, Transition threshold.

1 Introduction

The application of additives to base liquids in the sole aim to increase the heat transfer coefficient is considered as a powerful mean for thermal systems. Nanofluids, prepared by dispersing nanometer-sized solid particles, have been extensively studied since one decade due to the observation of an interesting increase in thermal conductivity compared to that of the base fluid [1,2]. Consequently, many researches have focused on the way to increase the thermal conductivity parameter by modifying the particle volume fraction, the particle size/shape or the base fluid [3-5]. In a recent work [6], numerical results have eloquently shown that the use of Newtonian nanofluids for the purpose of the heat transfer enhancement in natural convection was not obvious, as such enhancement is dependent not only on nanofluids effective thermal conductivities but also on their viscosities as well. In fact, the effect due to the kinematic viscosity had been found to be dominant in the external natural convection heat transfer. In this same work, the authors have used two different viscosity models, namely the one proposed by Brinkman [7] currently used in literature for natural convection flows [1,8,9] and that recently proposed by Maïga et al. [10,11] to provide a better modeling of such nanofluids. The examination of these two models led to some contradictory conclusions. That means that an exact determination of the heat transfer parameters is not warranted before the question of the choice of an adequate and realistic effective viscosity model is resolved. It is worth mentioning that this viewpoint is also confirmed in a recent work [12] for the forced convection, in which the authors indicated that the assessment of the heat transfer enhancement potential of nanofluid is difficult and closely dependent on the way the nanofluid properties are modeled.

Another feature concerning the use of nanofluids in convective flows is that, unlike forced convection, there is a striking lack of theoretical and experimental data in natural convection. The aim of the present study is to enhance the discussion on the use of nanofluids and also to add more information on the natural convection heat transfer. This paper concerns especially with the transitional events between laminar and turbulent convective flows, and focuses on the effects due to nanofluids on the transition threshold. Such effects, to our knowledge, have not yet been studied.

Our theoretical analysis is restricted to Newtonian nanofluids and based on a macroscopic modelling under the assumption of constant thermophysical properties. Two mixtures are considered, namely Water-γAl_2O_3 and Ethylene Glycol-γAl_2O_3. Results are presented only for particle volume fractions up to 4%, as no experimental data could be found in the literature concerning the Newtonian rheological behaviour of these nanofluids for higher particle volume fractions.

2 Mathematical modelling

The physical system considers a steady free convection boundary layer along a vertical wall heated with a uniform heat flux density. Both laminar and turbulent regimes, as well as the resulting transition threshold have been analyzed in the present approach. The theoretical model, based on the integral formalism, assumes sufficiently small temperature gradients across the boundary layer, so that thermophysical properties of the nanofluids are assumed to be constant except for the density variation in the buoyancy force, which is based on the incompressible fluid Boussinesq approximation. Since the solid particles have reduced dimension (30-50nm) and are easily fluidized, these particles can be considered to have a fluid-like behaviour [1, 10]. One may expect that the classical theory for single-phase fluids can be extended to nanofluids.

2.1 Laminar modelling

The complete theoretical development associated with such integral formalism has been previously presented in details in [13,14] for the free convection laminar regime and in [15] for the turbulent regime analysis. Therefore, for the sake of brevity, only the main results on the development are presented here.

One of the earliest attempts in the laminar case using an integral formalism for Pr > 0.6 and suggesting the thermal to velocity layer thickness ratio Δ to be dependent only on the Prandtl number were those presented in [13,14]. The resulting relation was found to be:

$$\Delta_{nf}^7 - \frac{799}{126}\Delta_{nf}^6 + \frac{225}{14}\Delta_{nf}^5 - \frac{134}{7}\Delta_{nf}^4 + \frac{20}{3}\Delta_{nf}^3 + \frac{10}{9Pr_{nf}} = 0 \qquad (1)$$

It is worth noting that the subscripts *p*, *bf*, *nf* and *r* refer respectively to the particles, the base-fluid, the nanofluid and the ratio 'nanofluid/base-fluid' of the parameter considered.

Because the thermal situation is that of a uniform heat flux density problem, the modified Rayleigh number is used and defined as follows:

$$Ra_x^* = \frac{g\,\beta\,\varphi_w\,x^4}{\lambda\,\nu^2}Pr \qquad (2)$$

or expressed from the base-fluid one, as:

$$Ra_{nf}^* = Ra_{bf}^*\,\frac{\beta_r\,Pr_r}{k_r\,\nu_r^2} \qquad (3)$$

In such a way, calculations lead to the following expression of the local Nusselt number in the laminar domain:

$$Nu_{nf}\big|_{LAM} = \left(Ra_{bf}^*\right)^{\frac{1}{5}}\left[\frac{2}{27\left(9\Delta_{nf}-5\right)\Delta_{nf}^4 Pr_{nf}}\frac{\beta_r Pr_r}{k_r\nu_r^2}\right]^{\frac{1}{5}} \qquad (4)$$

2.2 Turbulent modelling

Assuming that the turbulent boundary layer starts from the leading edge of the wall, the time-averaged integral forms of the boundary layer equations for the conservation of momentum and energy can be directly extended to nanofluids as:

$$\begin{cases}\dfrac{\partial}{\partial x}\displaystyle\int_0^{\delta} U^2 dy = g\beta\int_0^{\Delta\delta}\Theta dy - \left(\nu+\nu_t\right)\left(\dfrac{\partial U}{\partial y}\right)_{y=0} \\ \dfrac{\partial}{\partial x}\displaystyle\int_0^{\Delta\delta}\Theta U dy = -\left(a+a_t\right)\left(\dfrac{\partial\Theta}{\partial y}\right)_{y=0}\end{cases} \qquad (5)$$

where ν_t and a_t denote respectively the eddy diffusivity of both momentum and heat.

Since there are no results in the literature dealing with the growth of viscous and thermal layers, the present turbulent theory has been established under the assumption that the laminar boundary layer results could be extended to the turbulent regime, so that relation (1) still remains valid.

Deriving the Colburn analogy and using convenient wall shear stress under adequate velocity and temperature profiles [15, 16] within the boundary-layers give, after calculation, the turbulent Nusselt number is defined as:

$$Nu_{nf}\big|_{TURB} = 0.0631\left(Ra_{bf}^*\right)^{\frac{2}{7}}\left[\frac{\sqrt{\Pi_\Delta Pr_{nf}}}{\Delta_{nf}}\left(1+\frac{0.0823}{\Pi_\Delta Pr_{nf}^{\frac{10}{15}}}\right)\frac{k_r\nu_r^2}{\beta_r Pr_r}\right]^{-\frac{2}{7}} \qquad (6)$$

where Π_Δ is a function of Δ, the boundary-layer ratio, given by:

$$\Pi_\Delta = \Delta_{nf}^{\frac{8}{7}}\left(\frac{7}{72}-\frac{7}{60}\Delta_{nf}+\frac{21}{253}\Delta_{nf}^2-\frac{14}{435}\Delta_{nf}^3+\frac{7}{1332}\Delta_{nf}^4\right) \qquad (7)$$

2.3 Transition threshold

Because of the lack of a physical criterion to define the transitional region, the present theoretical criterion used for transition to turbulence is similar to that chosen in [17] corresponding to the location where laminar and fully developed turbulent flows interact together. And, since the transition is not a mathematical event, the most this analysis can do is to predict the order of magnitude of this transition.

Thus, at the mathematical transition point, by equalizing equations (4) and (5) and reporting the result into the expression (3), the critical Rayleigh number can be obtained, which has been found dependent only on the nanofluid Prandtl number. Indeed, it is recalled that the boundary-layer parameters Δ_{nf} and Π_{Δ} are also only Prandtl number dependent. The expression for the critical Rayleigh number is given by:

$$Ra^{*}_{nf}\Big|_{c} = 2.30\ 10^{11}\left[\frac{\Pi_{\Delta}^{5}}{\left(9\Delta_{nf}-5\right)^{7}\Delta_{nf}^{38}\ Pr_{nf}^{2}}\left(1+\frac{0.0823}{\Pi_{\Delta}Pr_{nf}^{\frac{10}{15}}}\right)^{10}\right]^{\frac{1}{3}} \quad (8)$$

3 Thermophysical properties

Introducing the particle volume fraction ϕ, the thermophysical properties of the nanofluid, namely the density, heat capacity and volumetric expansion coefficient, have been computed using classical relations developed for a two-phase mixture [1,18] :

$$\left(\rho\beta\right)_{nf} = \left(1-\phi\right)\left(\rho\beta\right)_{bf} + \phi\left(\rho\beta\right)_{p} \quad (9)$$

$$\left(C_{p}\right)_{nf} = \left(1-\phi\right)\left(C_{p}\right)_{bf} + \phi\left(C_{p}\right)_{p} \quad (10)$$

$$k_{nf} = k_{bf}\frac{k_{p}+2k_{bf}-2\phi\left(k_{bf}-k_{p}\right)}{k_{p}+2k_{bf}+\phi\left(k_{bf}-k_{p}\right)} \quad (11)$$

On the other hand, the dynamic viscosity is computed from the relations recently proposed by Maïga et al. [10,11] to provide a better modelling of the considered nanofluids, namely :

$$\frac{\mu_{nf}}{\mu_{bf}} = 123\phi^{2}+7.3\phi+1 \quad (12)$$

for water-γAl_2O_3 nanofluid,

$$\frac{\mu_{nf}}{\mu_{bf}} = 306\phi^{2}-0.19\phi+1 \quad (13)$$

for ethylene glycol- γAl_2O_3 nanofluid.

Because the theoretical modelling is based on the boundary layer equations in which the effective boundary layer thickness (thermal to dynamical layer thickness ratio) is assumed to be only Prandtl number dependent, we have drawn in Fig.1 the evolution of the effective Prandtl number with the particle volume fraction.

It can be seen that the increase of the Prandtl number is greater for the water-γAl_2O_3 mixture than for the ethylene glycol-γAl_2O_3 nanofluid in the particle loading range, to reach ~ 28% for a 4% volume fraction.

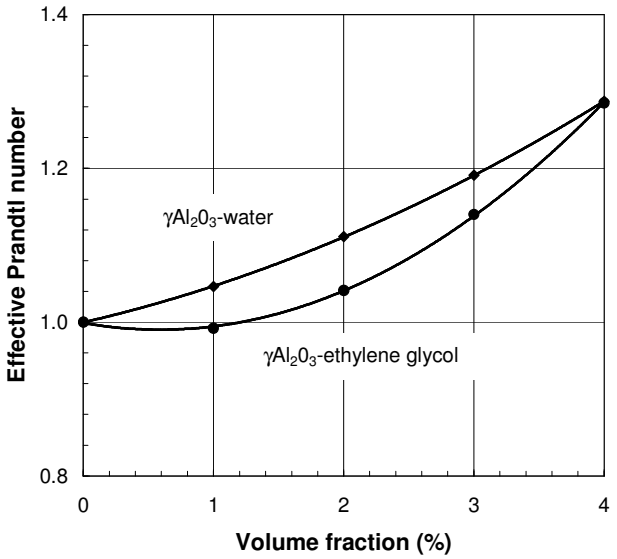

Fig. 1 Evolution of the effective Prandtl number with the particle volume fraction

The numerical results are curve-fitted with the following regressing laws (represented in Fig.1 as solid line curves):

$$\frac{\mathrm{Pr}_{nf}}{\mathrm{Pr}_{bf}} = 82.1\phi^{2}+3.9\phi+1 \quad (14)$$

for water-Al_2O_3 nanofluid and

$$\frac{\mathrm{Pr}_{nf}}{\mathrm{Pr}_{bf}} = 254.3\phi^{2}-3\phi+1 \quad (15)$$

for ethylene glycol- γAl_2O_3 nanofluid.

4 Results and Discussion

We present in Fig. 2 and Fig. 3 the evolutions of the local Nusselt number with the base-fluid Rayleigh number in the particle loading range, for the two nanofluids under study. The evolutions are built by using Eq. (4) and Eq. (6) respectively for the laminar and turbulent flow regimes. On the two graphs, a discontinuous line is used to represent the laminar-to-turbulent threshold. It is worth to recall that this threshold corresponds in fact, under a mathematical criterion, to the intersection points of the two functions (4) and (6). It can be seen that increasing the base-fluid Rayleigh number, the heat transfer parameter increases more steeply as the flow becomes turbulent ($Nu_{nf} \propto \left(Ra^{*}_{bf}\right)^{2/7}$ in the turbulent regime and $Nu_{nf} \propto \left(Ra^{*}_{bf}\right)^{1/5}$ in the laminar one).

About the particle volume fraction, what is significant is that, under the conditions adopted in this study, the use of nanofluids to enhance the heat transfer seems illusory. The higher the volume fraction is, the lesser the heat transfer enhancement. This result appears to be consistent with that from a previous published work [19], in which the authors mentioned that unlike conduction or forced convection, a systematic and definite deterioration in natural convective heat transfer has been found while using nanofluids.

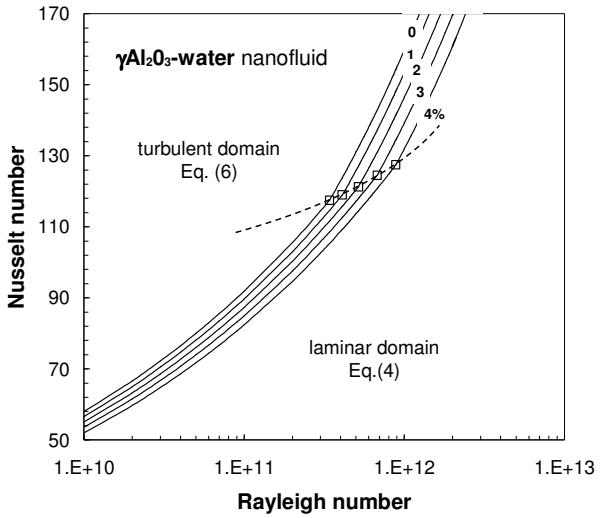

Fig. 2 Nusselt number for Water-γAl_2O_3 versus the base-fluid Rayleigh number

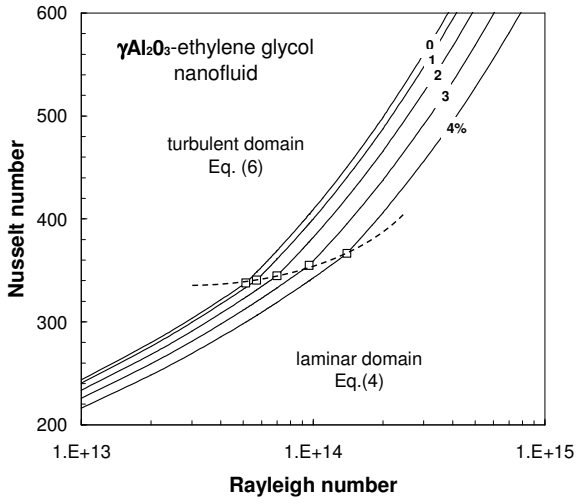

Fig. 3 Nusselt number for Ethylene glycol-γAl_2O_3 versus the base-fluid Rayleigh number

At a given particle loading and in the base-fluid Rayleigh number range wherein the two nanofluids present a laminar behaviour in flowing, for $Ra^*_{bf} < 3.10^{11}$, comparing the two ethylene glycol-γAl_2O_3 and water-γAl_2O_3 nanofluids gives a enhancement about 6% in the heat transfer in favour of the ethylene glycol-Al_2O_3. On the other hand, what is singular is that the opposite behaviour is observed for $Ra^*_{bf} > 5.10^{13}$ where the two nanofluids evolve in turbulent regime. Using Ethylene Glycol-γAl_2O_3 nanofluid would induce a heat transfer degradation as important as 31% in the turbulent regime, for a given volume fraction.

To have a precise idea on how the laminar-to-turbulent threshold varies with the use of nanofluids, Fig. 4 presents its evolution versus the particle loading. It can be seen that a similar trend is observed for the two nanofluids under study. Thus, increasing the particle volume fraction induces an augmentation of the critical Rayleigh number, which results in a delay of the occurrence of the turbulent regime. In other words, the belief of increasing the heat transfer by provoking an earlier start of the laminar-turbulent transition does not seem to be valid, according to the results of the present study.

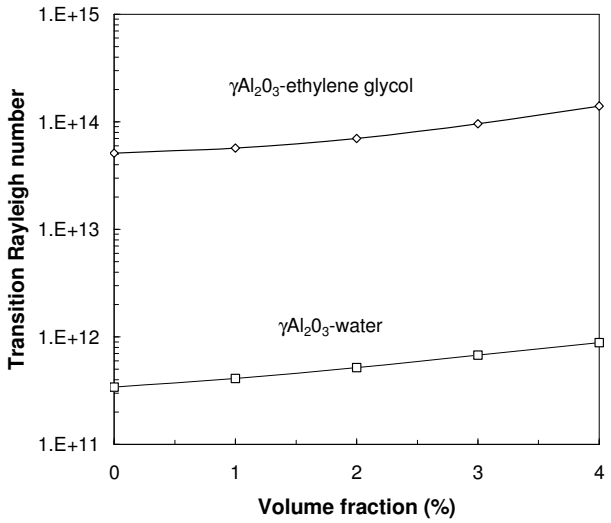

Fig. 4 Critical (transition) Rayleigh number versus the nanofluid volume fraction

The variation of the nanofluid critical Rayleigh number $Ra^*_{nf}\big|_c$ (Eq. 8) is reported in Figure 5, on which are also reported the numerical results deduced from Fig. 2 and Fig. 3 by paying attention to transpose these results using Eq. (3). It can be seen that the evolution versus the nanofluid Prandtl number is easily and suitably fitted by the correlation proposed by Varga et al. (2004) for single-phase fluid, namely:

$$Ra^*_{nf}\big|_c = 1.32\ 10^{10}\, Pr_{nf}^{1.58} \qquad (16)$$

This correlation gives a very interesting estimate of the order of magnitude of the transition threshold and should be extended to other kinds of Newtonian nanofluids.

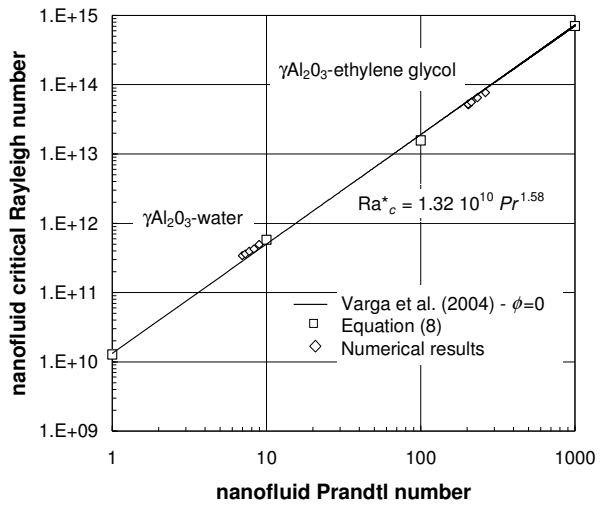

Fig. 5 Critical (transition) Rayleigh number versus the nanofluid Prandtl number

5 Conclusion

The laminar and turbulent natural convection boundary layer along a vertical wall subjected to a uniform heat flux density has been theoretically investigated for ethylene glycol-γAl_2O_3 and water-γAl_2O_3 nanofluids for a wide range of the Prandtl numbers (Pr = 6.96 to 262.42), and particle volume fractions up to 4%. The modelling is based on the integral formalism taking into account the consideration of distinct thicknesses of both the thermal and dynamical boundary layers, and by assuming that laminar effects can be extended to the turbulent region. The approach is based on both a macroscopic modelling and the use of an empirical viscosity model.

Despite the lack of data for comparison, the results obtained have been found to be consistent with the few earlier results in the literature regarding the heat transfer enhancement capability of nanofluids in natural convection flows.

The following conclusions can be drawn from this theoretical analysis:

1/ Regardless the nanofluid used (ethylene glycol-γAl_2O_3 or water-γAl_2O_3), increasing the nanoparticle volume fraction leads to the degradation of the external free convection heat transfer while compared to that of the base-fluid. Such a result, which confirms previous findings from other studies on similar cases, seems to indicate that the use of nanofluids for heat transfer enhancement purposes in an external free convection is not possible.

2/ Flow regime is a parameter that has a notable influence on the heat transfer. Indeed, at a given volume fraction, comparing the two ethylene glycol-γAl_2O_3 or water-γAl_2O_3 nanofluids gives an enhancement about 6% in the heat transfer in the laminar regime in favour of the ethylene glycol-γAl_2O_3. *A contrario*, using this same nanofluid induces important heat transfer degradation as important as 31% in the fully turbulent regime.

3/ For the two nanofluids used (ethylene glycol-γAl_2O_3 or water-γAl_2O_3), increasing the particle volume fraction induces an augmentation of the critical Rayleigh number, which results in a delay of the turbulence occurrence.

4/ Knowledge of the nanofluid Prandtl number allows to use the correlation $Ra^*_{nf}\big|_c = 1.32\ 10^{10}\, Pr_{nf}^{1.58}$ to estimate the critical Rayleigh number.

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