A Method for Determining Stabilizeability of a Class of Switched Systems

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Abstract: In this paper, the problem of finding a stabilizing feedback controller for single input single output switched systems of order two is addressed. To ensure stability under arbitrary switching, the existence of a common Lyapunov function (CLF) needs to be established. A method for establishing the existence of CLFs, given suitable feedback controllers, is presented. The method presented here is computationally less demanding compared to those that depend on linear matrix inequalities solvers.

Key–Words: Hybrid systems, Common Lyapunov functions, Stability under arbitrary switching, Brunovsky canonical form, State transformation

1 Introduction

Switched systems are a class of hybrid systems that is made up of a collection of linear subsystems with rules that govern switching between these subsystems [1]. Stability of switched systems is an area of important study as switching sequences and dwell time affect system behaviours and performances. Several methods for determining stability of switched systems have been recognised. Among these are the existence of common or multiple Lyapunov functions, modifying theorems, Poincaré mappings and Lagrange-based methods [2]. In this paper, our attention is focussed on finding the existence of common Lyapunov functions, as this ensures stability for arbitrary switching sequences. Some progress in this area have been made, for example, in [3, 4, 5, 6]. Here, we present a method for determining stabilizeability of single input single output (SISO) switching systems where each subsystem is a second order linear time invariant (LTI) system.

2 Problem Definition

In this paper, we consider the class of switching system denoted by

$$\dot{x} = A_i x + B_i u \tag{1}$$

where $x \in \mathbb{R}^2$, $u \in \mathbb{R}$, $A \in \mathbb{R}^{2\times 2}$, $B \in \mathbb{R}^{2\times 1}$ and $i = 1, \ldots, N$.

The problem is to ensure quadratic stabilization of (1). Formally, it is defined as follows.

Definition 1 *System (1) is said to be quadratically stabilizeable if there exists a set of state feedback control laws $u_i = A_i + B_i K_i$ such that $A_i + B_i K_i,\ i = 1, 2, \ldots, N$, share a common quadratic Lyapunov function $x^T M x$.*

3 The LMI Approach

One method of obtaining solutions to the problem posed by Definition 1 is by solving the following linear matrix inequalities (LMI) [6]:

$$\begin{array}{rcl} MA_i + A_i^T M + B_i Z_i + Z_i^T B_i^T & < & 0, \\ M & > & 0 \end{array} \tag{2}$$

Then,

$$K_i = Z_i M^{-1} \tag{3}$$

ensures stability of System (1).

While this is a powerful method which has been shown to be capable of providing stable pole placement feedback control of switched systems [6], the computational burden could prove to be a limiting factor. As an example, consider the case of four switching models given by

$$A_1 = \begin{bmatrix} -4 & -2 \\ 9 & 5 \end{bmatrix}, \quad b_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 2 & -\frac{1}{3} \\ -3 & 0 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 5 \\ -6 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -7 & -6 \\ \frac{19}{2} & 8 \end{bmatrix}, \quad b_3 = \begin{bmatrix} 6 \\ -7 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} -12 & -11 \\ 13 & 12 \end{bmatrix}, \quad b_4 = \begin{bmatrix} -7 \\ 8 \end{bmatrix}. \qquad (4)$$

It was found that feasibility of (2) could not be determined using Matlab's LMI Lab.

4 The Proposed Method

We now present our methodology which is founded on the well known fact that a single input system can be uniquely transformed to its Brunovsky controllable canonical form [7].

Lemma 2 *Let (A, b) be a single input linear system. Then, there exists a unique state transformation matrix C which converts the system into the Brunovsky canonical form*

$$CAC^{-1} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & & \cdots & \\ & & & \cdots & 1 \\ a_1 & a_2 & a_3 & \cdots & a_n \end{bmatrix}, \ Cb = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 1 \end{bmatrix}$$

where

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} b & Ab & A^2b & \ldots & A^{n-1}b \end{bmatrix}^{-1} A^n b$$

with

$$D_n = b,$$

$$D_{i-1} = AD_i - a_i b; \quad i = n, n-1, \ldots, 2,$$

$$D = \begin{bmatrix} D_1 & D_2 & D_3 & \ldots & D_n \end{bmatrix}$$

and

$$C = D^{-1}$$

Lemma 3 *Given*

$$M = \begin{bmatrix} m_1 & m_2 \\ m_2 & m_3 \end{bmatrix} > 0,$$

there exists a feedback control law $u = Kx = \begin{bmatrix} k_1 & k_2 \end{bmatrix} x$ *such that*

$$\widetilde{A} = A + bK = \begin{bmatrix} 0 & 1 \\ \alpha & \beta \end{bmatrix}$$

(i.e. the resulting closed loop system) has M defining its quadratic Lyapunov function if, and only if, $m_2 > 0$.

Proof: No loss of generality arises from assuming $m_1 = 1$. Then, for $M > 0$ to have the required property, it is required that $M\widetilde{A} + \widetilde{A}^T M < 0$, or equivalently,

$$Q := \begin{bmatrix} 2\alpha m_2 & 1 + \alpha m_3 + \beta m_2 \\ 1 + \alpha m_3 + \beta m_2 & 2(m_2 + \beta m_3) \end{bmatrix} < 0.$$

Given that the system is in the Brunovsky form, β is the trace of $\widetilde{A}$ and hence, $\beta < 0$ is a necessary condition for closed loop stability. Also, $m_2 = 0$ is not allowed, and $\alpha m_2 < 0$ is necessary.

Suppose now that $m_2 < 0$ and $\alpha > 0$. Then, for $Q < 0$ to hold, $\det(Q)$ is required to be positive. It is also easy to see that the maximum value of this determinant occurs when $\beta m_2 = \alpha m_3 - 1$, and that it is negative. Hence, $m_2 > 0$ is a necessary condition for $Q < 0$.

If $m_2 > 0$ and $\alpha < 0$, any $\alpha < 0$ can be chosen and then, let $\beta = \frac{\alpha m_3 - 1}{m_2}$. Then, $\det(Q) = -4\alpha(m_3 - m_2^2) > 0$ and sufficiency is established. □

Definition 4 *A matrix*

$$M = \begin{bmatrix} m_1 & m_2 \\ m_2 & m_3 \end{bmatrix}$$

is said to be 'an equivalence' to the controllable canonical form if $m_2 > 0$.

Lemma 5 *Given a nonsingular matrix*

$$C = \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix},$$

there exists 'an equivalence' matrix $M > 0$ such that $C^T MC$ also has this property if, and only if, the quadratic equation

$$c_3c_4x^2 + (c_2c_3 + c_1c_4)x + c_1c_2 > 0 \qquad (5)$$

has positive solution $x > 0$.

Proof: To prove sufficiency, proceed as follows. Given $M > 0$ with $m_2 > 0$, and calculating $\widetilde{M} = C^T M C$ gives

$$\begin{aligned} H(m_1, m_2, m_3) &:= \widetilde{m}_{12} \\ &= c_3c_4m_3 + (c_2c_3 + c_1c_4)m_2 + c_1c_2m_1, \end{aligned} \quad (6)$$

and from (5), $H(1, x, x^2) > 0$. Hence, by continuity, there exists a small enough $\epsilon > 0$ such that $H(1, x, x^2 + \epsilon) > 0$. Setting $m_1 = 1$, $m_2 = x$ and $m_3 = x^2 + \epsilon$ now gives an M with the required property.

To prove necessity, assume, without loss of generality, that there exists

$$M = \begin{bmatrix} 1 & m_2 \\ m_2 & m_3 \end{bmatrix} > 0$$

such that both M and $C^T M C$ have the desired property, i.e. $m_2 > 0$ and $H(1, x, x^2) > 0$. Now, if $c_3c_4 > 0$, clearly (5) has a positive solution. If $c_3c_4 < 0$, then, since $m_3 > m_2^2$,

$$H(m_1, m_2, m_2^2) \geq H(m_1, m_2, m_3) > 0, \quad (7)$$

and m_2 is a positive solution of (5). □

For a given state transformation matrix C, the solution of (5) consists of one or two open intervals, or it could be empty. Use I to denote the set of solutions.

Still with the single input assumption, let $\Lambda = \{1, 2, \ldots, N-1\}$ be a finite set. Then, for each switched model, denote the state transformation matrix which converts it to the Brunovsky canonical form by C_i, $i = 1, 2, \ldots, N$. By Lemma 2, each transformation matrix is uniquely defined. Let $z_i = C_i x$ and set $T_j = C_1 C_{j+1}^{-1}$, $j = 1, 2, \ldots, N-1$. In this notation, T_j is the state transformation matrix from z_{j+1} to z_1, i.e.

$$z_1 = C_1 x = C_1 C_{j+1}^{-1} z_{j+1} = T_j z_{j+1}.$$

Next, classify T_j according to $t_3^j t_4^j$ into the following three categories:

$$\begin{aligned} S_p &= \{j \in \Lambda \mid t_3^j t_4^j > 0\}, \\ S_n &= \{j \in \Lambda \mid t_3^j t_4^j < 0\}, \\ S_z &= \{j \in \Lambda \mid t_3^j t_4^j = 0\}. \end{aligned}$$

Then, $\Lambda = S_p \cup S_n \cup S_z$. Also, for $j \in S_z$, define

$$\begin{aligned} r_j &= t_2^j t_3^j + t_1^j t_4^j \\ s_j &= t_1^j t_2^j \end{aligned}, \quad j \in S_z, \quad (8)$$

and, for the other cases, define

$$p_j = \frac{t_1^j}{t_3^j}, \quad q_j = \frac{t_2^j}{t_4^j}, \quad j \in S_p \cup S_n. \quad (9)$$

For each $j \in S_z$, define a linear form as

$$L_j = r_j x + s_j, \quad j \in S_z$$

and, for each $j \in S_p$ or $j \in S_n$, define a quadratic form as

$$Q_j = x^2 + (p_j + q_j)x + p_j q_j, \quad j \in S_p \cup S_n. \quad (10)$$

Then, by Lemma 5, obtain (or solve) x from

$$\begin{aligned} Q_j(x) &< 0, \quad j \in S_n, \\ Q_j(x) &> 0, \quad j \in S_p, \\ L_j(x) &> 0, \quad j \in S_z. \end{aligned}$$

Note also that the polynomial $Q_j(x)$ of (10) has roots $\{-p_j, -q_j\}$ and hence, the solution set can be defined as follows:
If $j \in S_p$, an open set is defined as

$$I_j = (-\infty, \min(-p_j, -q_j) \cup (\max(-p_j, -q_j), \infty).$$

If $j \in S_n$, an open set is defined as

$$I_j = (\min(-p_j, -q_j) \cup (\max(-p_j, -q_j)).$$

If $j \in S_z$, and since T_j is nonsingular, it follows that $r_j \neq 0$. Hence, an open set can be defined as

$$I_j = \begin{cases} (-\frac{s_j}{r_j}, \infty), & r_j > 0, \\ (-\infty, -\frac{s_j}{r_j}), & r_j < 0. \end{cases}$$

The following result now follows immediately from Lemma 5.

Theorem 6 *1. A sufficient condition for the switched system (1) to be quadratically stabilizeable is*

$$I = \bigcap_{j=1}^{N-1} I_j \neq \emptyset. \quad (11)$$

2. *If all $j \in S_p$, $j = 1, \cdots, N-1$, then the switched system (1) is always stabilizeable.*

3. *If all $i \in S_n$, $i = 1, \cdots, N-1$, (11) is also necessary.*

Proof: To prove 1, choose $m_2 \in I$ and $m_3 = m_2^2 + \epsilon$. Then, it follows immediately that, for small enough ϵ, the corresponding matrix (in z_1 co-ordinates)

$$M = \begin{bmatrix} 1 & m_2 \\ m_2 & m_3 + \epsilon \end{bmatrix}$$

becomes (for suitably chosen controls) the common quadratic Lyapunov function.

In the case of 2, choose m_2 large enough and then, $m_2 \in I$.

In the case of 3, the inequality (7) from the proof of Lemma 5 shows that $m_2 \in I$, and therefore, $I \neq \emptyset$. □

In the case of $N = 2$, the above result is, from Lemma 5, necessary and sufficient. In general however, this property does not always hold. The following result gives conditions for the general case.

Theorem 7 *Let $\Lambda = \{1, 2, \ldots, N-1\}$. Then, the system considered here is quadratically stabilizeable if, and only if, there exists a positive x such that*

$$\begin{aligned} \min_{j \in S_n} Q_j(x) &< 0, \\ \min_{j \in S_p} Q_j(x) &> \min_{j \in S_n} Q_j(x), \\ L_j(x) &> 0, \quad I \in S_z. \end{aligned} \tag{12}$$

Proof: Assume that there is a quadratic Lyapunov function in z_1 coordinates which is expressed as

$$M_1 = \begin{bmatrix} 1 & m_2 \\ m_2 & m_3 \end{bmatrix}$$

and, by Lemma 3, $m_2 > 0$. It is easy to see from the proof of Lemma 5 that M_1 is a common quadratic Lyapunov function for the other models if, and only if, $H_j(1, m_2, m_3) > 0$, $j = 1, 2, \ldots, N-1$, or

$$\begin{aligned} m_3 + (p_j + q_j)m_2 + p_j q_j &> 0, \quad j \in S_p, \\ m_3 + (p_j + q_j)m_2 + p_j q_j &< 0, \quad j \in S_n, \\ r_j m_2 + s_j &> 0, \quad j \in S_z. \end{aligned} \tag{13}$$

Also, since $m_3 > m_2^2$, the first two equations in (13) can be rewritten as

$$\begin{aligned} e + m_2^2 + (p_j + q_j)m_2 + p_j q_j &> 0, \quad j \in S_p, \\ e + m_2^2 + (p_j + q_j)m_2 + p_j q_j &< 0, \quad j \in S_n \end{aligned} \tag{14}$$

where $e > 0$, and the necessity of (12) is obvious.

To prove sufficiency, assume that there is a solution x such that $\min\limits_{j \in S_p} Q_j(x) > 0$. Then, m_2 and m_3 can be chosen such that $m_2 = x$ and $m_3 = x^2 + \epsilon$, with $\epsilon > 0$ small enough to ensure that (13) holds. Otherwise, set $w = \min\limits_{j \in S_p} Q_j(x) \leq 0$. Then choose $m_2 = x$ and

$$m_3 = x^2 + \frac{1}{2}\left(\min_{j \in S_p} Q_j(x) - \max_{j \in S_p} Q_j(x)\right) - w,$$

and it is easy to see that (13) holds. Hence, the matrix

$$M = \begin{bmatrix} 1 & m_2 \\ m_2 & m_3 \end{bmatrix}$$

in this case meets the requirement. □

Return now to Theorem 6. Then, in fact, it has been proven that the system is stabilizeable if all $j \in S_p$ or, if all $j \in S_n$, $j = 1, \ldots, N-1$, then, (11) is also necessary. When $N \leq 3$, however, the following is true:

Corollary 8 *If $N \leq 3$, then (11) is also necessary.*

Proof: For the case when $N = 2$, it was proved in Lemma 5. To prove this result for the case when $N = 3$, first, construct T_1 and T_2. If both $j = 1$ and $j = 2$ are in S_p (or S_n), the result has been proven in Theorem 6. Without loss of generality, assume that $j = 1 \in S_p$ and $j = 2 \in S_n$ and to establish necessity, assume $I_1 \cap I_2 = \emptyset$. Then

$$p_1 \geq p_2 \geq q_2 \geq q_1.$$

Again, without loss of generality, it can be assumed that $p_1 \geq q_1$ and $p_2 \geq q_2$. Now, it is obvious that

$$Q_2(x) \geq Q_1(x), \quad x \in (q_2, p_2) \tag{15}$$

and the result follows immediately. □

Now, note that the first inequality in (12) is equivalent to

$$\min\{p_k, q_k\} < x < \max\{p_k, q_k\}, \quad k \in S_n$$

and the second inequality in this set is equivalent to

$$(p_j + q_j - p_k - q_k)x + p_j q_j - p_k q_k > 0, \quad j \in S_p, \ k \in S_n$$

(or each Q_j, $j \in S_p$ is greater than each Q_k, $k \in S_n$). Also, since a positive solution is required, the following result ensues.

Corollary 9 *The switched system considered here is quadratically stabilizeable if, and only if, the following set of linear inequalities have a solution*

$$\begin{aligned} \min\{p_k, q_k\} < x < \max\{p_k, q_k\}, &\quad k \in S_n, \\ (p_j + q_j - p_k - q_k)x + p_j q_j - a_k q_k > 0, &\quad j \in S_p, \\ &\quad k \in S_n, \\ r_j x + s_j > 0, &\quad j \in S_z, \\ x > 0. & \end{aligned} \tag{16}$$

Recalling the proof of Lemma 3, a stabilizing control law is easily constructed.

Theorem 10 *Let (A, b) be a canonical system of the form*

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ a_{21} & a_{22} \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad (17)$$

and

$$M = \begin{bmatrix} m_1 & m_2 \\ m_2 & m_3 \end{bmatrix}$$

with $m_2 > 0$*. Then* $x^T Mx$ *is a quadratic Lyapunov function for the closed loop system under the action of the control law*

$$u = kx = \begin{pmatrix} (\alpha - a_{21}) & \left(\frac{\alpha m_3 - m_1}{m_2} - a_{22}\right) \end{pmatrix} x \qquad (18)$$

where $\alpha < 0$ *is an arbitrary real number.*

Note that (18) is not unique.

We now perform this method, step by step, on the example system (4) shown previously.

- **Step 1** Use Lemma 2 to obtain the state transition matrices C_i such that in the coordinate frame z_i, the i-th switching model is in the Brunovsky canonical form.

$$C_1 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -2 & -\frac{5}{3} \\ 1 & \frac{2}{3} \end{bmatrix},$$

$$C_3 = \begin{bmatrix} \frac{7}{6} & 1 \\ \frac{4}{3} & 1 \end{bmatrix}, \quad C_4 = \begin{bmatrix} \frac{8}{3} & \frac{7}{3} \\ -\frac{5}{3} & -\frac{4}{3} \end{bmatrix}.$$

- **Step 2** Define another set of state transformation matrices $T_j = C_1 C_{j+1}^{-1}$, $j = 1, \ldots, N-1$, such that $z_1 = T_j z_{j+1}$.

$$T_1 = \begin{bmatrix} 1 & 4 \\ -1 & -1 \end{bmatrix}, \quad T_2 = \begin{bmatrix} -4 & 5 \\ 2 & -1 \end{bmatrix},$$

$$T_3 = \begin{bmatrix} -3 & -6 \\ 1 & 1 \end{bmatrix}.$$

- **Step 3** Calculate p_j and q_j by (9) if $j \in S_p \cup S_n$, and r_j and s_j by (8) if $j \in S_z$.

  For T_1,

$$t_3^1 t_4^1 = 1, \quad j = 1 \in S_p,$$
$$p_1 = -1, \qquad q_1 = -4.$$

  For T_2,

$$t_3^2 t_4^2 = -2, \quad j = 2 \in S_n,$$
$$p_1 = -2, \qquad q_1 = -5.$$

  For T_3,

$$t_3^3 t_4^3 = 1, \quad j = 3 \in S_p,$$
$$p_1 = -3, \qquad q_1 = -6.$$

- **Step 4** Construct the system of inequalities (16) and find a solution $x = x_0$. *Note: If there is no solution, the problem considered is not mathematically stabilizeable.*

  $j = 1 \in S_p$ and $k = 2 \in S_n$:

$$(p_1 + q_1 - p_2 - q_2)x + p_1 q_1 - p_2 q_2 = 2x - 6 > 0.$$

  $j = 3 \in S_p$ and $k = 2 \in S_n$:

$$(p_3 + q_3 - p_2 - q_2)x + p_3 q_3 - p_2 q_2 = -2x + 8 > 0.$$

  The complete set of inequalities (16) is then

$$\begin{array}{rcl} 2 < & x & < 5, \\ 2x - 6 & > & 0, \\ -2x + 8 & > & 0, \\ x & > & 0 \end{array}$$

  with solution $3 < x < 4$. Choose for example, $x_0 = 3.5$.

- **Step 5** Using the inequalities (14), set $m_2 = x_0$ to find a positive solution $e > 0$. Set $m_3 = m_2^2 + e$. Construct a positive definite matrix

$$M_1 = \begin{bmatrix} 1 & m_2 \\ m_2 & m_3 \end{bmatrix} > 0$$

  which is a common quadratic Lyapunov function for all switching models with certain feedback control laws. *Note: If Step 4 has a solution, then, mathematically, there exist solutions for the inequalities (14).*

  With $x_0 = 3.5$,

$$\begin{array}{rcl} e + 12.5 + (-1-4) \times 3.5 + 4 & > & 0, \\ e + 12.5 + (-3-6) \times 3.5 + 18 & > & 0, \\ e + 12.5 + (-2-5) \times 3.5 + 10 & > & 0, \\ e & > & 0 \end{array}$$

  giving $1 < e < 2$. Choose, for example, $e = 1.5$. Then,

$$M_1 = \begin{bmatrix} 1 & 3.5 \\ 3.5 & 14 \end{bmatrix}.$$

- **Step 6** Convert M_1 to each canonical coordinate system using

$$M_{j+1} = T_j^T M_1 T_j, \quad j = 1, \ldots, N-1.$$

Then, convert model (A_i, b_i) to its canonical representation

$$\widetilde{A}_i = C_i A_i C_i^{-1}, \quad \widetilde{b}_i = \begin{bmatrix} 0 & 1 \end{bmatrix}^T, \\ i = 1, \ldots, N$$

and use (18) to construct the feedback gains k_i.

$$M_2 = \begin{bmatrix} 8 & 0.5 \\ 0.5 & 2 \end{bmatrix}, \quad M_3 = \begin{bmatrix} 16 & 1 \\ 1 & 4 \end{bmatrix},$$

$$M_4 = \begin{bmatrix} 2 & 0.5 \\ 0.5 & 8 \end{bmatrix}.$$

$$\widetilde{A}_1 = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}, \quad \widetilde{A}_2 = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix},$$

$$\widetilde{A}_3 = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}, \quad \widetilde{A}_4 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Set, for example, $\alpha = -1 < 0$, and hence,

$$k_1 = \begin{bmatrix} -3 & -\frac{37}{7} \end{bmatrix}, \qquad k_2 = \begin{bmatrix} -2 & -22 \end{bmatrix}, \\ k_3 = \begin{bmatrix} 0 & -21 \end{bmatrix}, \qquad k_4 = \begin{bmatrix} -2 & -20 \end{bmatrix}.$$

- **Step 7** Revert to the original coordinate x, where the controls are

$$K_i = k_i C_i, \quad i = 1, \ldots, N.$$

The common quadratic Lyapunov function for all closed loop switching models is then

$$M = C_1^T M_1 C_1.$$

Hence,

$$K_1 = \begin{bmatrix} -11.29 & -8.29 \end{bmatrix}, \\ K_2 = \begin{bmatrix} -18 & -11.33 \end{bmatrix}, \\ K_3 = \begin{bmatrix} -28 & -21 \end{bmatrix}, \\ K_4 = \begin{bmatrix} 28 & 22 \end{bmatrix}$$

and

$$M_0 = \begin{bmatrix} 32 & 26.5 \\ 26.5 & 22 \end{bmatrix}.$$

Clearly,

$$M_0(A_i + b_i K_i) + (A_i + b_i K_i)^T M_0 < 0, \\ i = 1, 2, 3, 4.$$

Hence, this system is quadratically stabilizeable.

5 Conclusion

The problem of finding stabilizing feedback controllers for single input single output switching systems comprising of subsystems that are second order linear time-invariant has been addressed. The problem relies on finding a common Lyapunov function (CLF) for all subsystems. A method has been presented by which the existence of a CLF that is shared by all subsystems, given appropriate control parameters, could be determined. Using appropriate transformations to and from the Brunovsky form, the required control parameters can then be identified.

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