Control by Feedback Linearization of the Torque and the Flux of the Induction Motor

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Abstract:- The theory of input-output feedback linearization is extensively used for the control of nonlinear systems. In this paper, this theory is developed of the mathematical view point, and then, applied for the regulation of the torque and the flux of the induction machine. The results of simulations testify the hardiness of the method developed.

Key-Words: - feedback linearization, nonlinear system, induction machine, robustness

1 Introduction

The development of different methods of induction motor's control is justified by the need to take into account its nonlinear state structure. It is for this reason that, in spite of its weakest cost and its least clutter, the industry had preferred until the 80 to use de motors, and after, the synchronous motor for their linear structure that facilitated the control. A method of control of the induction motor, called rotor-flux-oriented that reduces the nonlinear dynamics of the induction motor to a linear structure has been proposed in 1972, [1], [2]. If this method remained little exploited until the beginning of the years 80, the progress in the power semiconductors technique's and in the micro-electronics permitted its use in the present industrial variators. However, the experience showed the weakness of this method facing the parameter's uncertainties, that they are measured, such as the motor's speed or that they vary under working, as the resistances of the stator and the rotor. Otherwise, the electric motor control was revealed to be a field of the methodologies of the nonlinear control, developed since the years 70. Indeed, the modeling of the ac motors is well mastered, to be nonlinear models characterized by a limited number of state variables. So, several techniques has been developed for the induction machine's control [3]. Our contribution is focused in this context. In the first section, a modelisation of the induction machine is presented; the second section is dedicated, after a short exposition of linearization techniques to the application to the induction motor. Simulations perfectly illustrate the relevance of the control developed.

An industrial applications of feedback application can be seen at [9] and [10].

2 Modelisation of the induction machine [4], [5], [6]

In this section we present briefly, and in non exhaustive way a mathematical model of the induction machine. This model, fluently used to synthesis a control law is defined in a referential rotating frame (indication (d,q)). This referential frame is defined from the natual referential three phase frame of the induction machine with the help of adapted mathematical transformation.

Principle:

The dynamics of the stator current and rotor defined in a referential rotating flux defined in a referential rotating frame (d,q), are given by:

$$\dot{x} = f(x) + g(x).u = \begin{bmatrix} f1 \\ f2 \\ f3 \\ f4 \end{bmatrix} + [g_1 \ g_2].u \quad (1)$$

Where the vector x regroup the stator currents and the rotor fluxes and u represent the applied voltages to the motor expressed in the reference frame (d,q).

$$x^T = [x_1 \ x_2 \ x_3 \ x_4] = [i_{ds} \ i_{qs} \ \varphi_{dr} \ \varphi_{qr}]$$

$$u^T = [u_{ds} \ u_{qs}] \quad (2)$$

with

$$g_1^T = [b1 \ 0 \ 0 \ 0] \ , \ g_2^T = [0 \ b2 \ 0 \ 0] \quad (3)$$

$$\begin{cases} f1 = a11.x_1 + a12.x_2 + a13.x_3 + a14.x_4.\omega \\ f2 = a21.x_1 + a22.x_2 + a23.x_3.\omega + a24.x_4 \\ f3 = a31.x_1 + a32.x_2 + a33.x_3 + a34.x_4.\omega \\ f4 = a41.x_1 + a42.x_2 + a43.x_3.\omega + a44.x_4 \end{cases} \quad (4)$$

the elements a_{ij} and b_i are given by:

$$\begin{cases} a11 = a22 = -\frac{R_t}{\sigma.Ls}, a12 = a21 = 0 \\ a13 = a24 = \frac{Lm}{\sigma.Ls.Lr.Tr}, a14 = -a23 = \frac{Lm}{\sigma.Ls.Lr} \\ b1 = b2 = \frac{1}{\sigma.Ls}, a31 = a42 = \frac{Lm}{Tr}, \\ a32 = a41 = 0 \\ a33 = a44 = -\frac{1}{Tr}, a34 = -a41 = -1 \\ R_t = Rs + \frac{Lm^2}{Lr^2}.Rr \\ \sigma = 1 - \frac{Lm^2}{Ls.Lr}, Tr = \frac{Lr}{Rr} \end{cases} \quad (5)$$

where

R_s, L_s : resistance et inductance stator,

R_r, L_r : resistance et inductance rotor,

L_m : mutual inductance between stator and rotor,

R_t : total resistance brought back to the rotor,

σ : total leakage coefficient,

T_r : rotor time constant,

ω : rotor angular frequency,

The associated mechanical equations are given by :

$$C_{em} = p.\frac{Lm}{Lr}.(i_{qs}.\varphi_{dr} - i_{ds}.\varphi_{qr}) \quad (6.1)$$

$$\frac{d\Omega}{dt} + \frac{f}{J}.\Omega = \frac{1}{J}.(C_{em} - C_r) \quad (6.2)$$

C_{em} : electromagnetic torque of the machine,

C_r : load torque,

Ω : mechanical speed of the rotor $\Omega = \omega / p$

J : moment of inertia

f : damping constant,

p : number of motors's pole

The parameters to be regulated are the rotor flux Φ and the electromagnetic torque C_{em} that are given by:

$$y_1(x) = h_1(x) = \phi = 0.5.(\varphi_{dr}^2 + \varphi_{qr}^2) = 0.5.(x_3^2 + x_4^2) \quad (7)$$

$$y_2(x) = h_2(x) = C_{em} = p\frac{Lm}{Lr}(x_2.x_3 - x_1.x_4) \quad (8)$$

3 Feedback linearization principle

[7], [8]

Let a SISO system of order n, described by the nonlinear state representation:

$$\dot{x} = f(x) + g(x).u \quad (9)$$

$$y = h(x) \quad (10)$$

where the functions f, g and h are analytic

Definition

Let $L_f h(.)$ the directional derivation of h according to the vector field f.

$$L_f h(x) = \sum_{i=1}^{n} \frac{\partial h(x)}{\partial x_i} f_i(x) \quad (11)$$

The system is said of relative degree r at x_0 for all x in a neighborhood of x_0 if:

$$L_g h(x) = ... = L_g L_f^{r-2} h(x) = 0 \quad (12)$$

$$L_g L_f^{r-1} h(x_0) \neq 0 \quad (13)$$

The computation of the output derivative drives to:

$$\forall\, 0 \le k \le r-1 \quad y^{(k)}(t) = L_f^k h(x) \qquad (14)$$

$$y^{(r)}(t) = L_f^r h(x) + L_g L_f^{r-1} h(x).u \qquad (15)$$

and while putting:

$$u = \frac{1}{L_g L_f^{r-1} h(x)}.(-L_f^r h(x) + v) \qquad (16)$$

where the variable represent an external excitation, one gets $y^{(r)}(t) = v(t)$.

Let $y_d(t)$ a reference to the trajectory, the ulterior choice,

$$v(t) = y_d^r(t) + \sum_{i=1}^{r} \alpha_i.(y_d^{(r-i)}(t) - L_f^{r-i} h(x)) \qquad (17)$$

Leads to the dynamics

$$e^{(r)}(t) + \sum_{i=1}^{r} \alpha_i.e^{(r-i)}(t) = 0 \qquad (18)$$

on the tracking error $e = y - y_d$. If the α_i are the coefficients of a Hurwitz polynomial, the convergence to zero of the tracking error is guaranteed. Knowing that for r<n, the feedback conceived makes unobservable a part of the dynamics, it is necessary to verify the stability of the intern evolution corresponding to a reference trajectory [7]. Which is a difficult problem for which it doesn't exist a general solution, the study must be made case by case.

These very simple reviews can be generalized to MIMO systems, [7]. In conclusion, this strategy of command make the system to look like a chains of a decoupled integrators, followed by a pole placement.

4 Application to the induction motor

While applying the analogous procedure, it is easy to verify that, the controls appear for the first time in $\ddot{y}_1$ and $\dot{y}_2$, one gets thus :

$$\begin{bmatrix} \ddot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} L_f^2 h_1(x) \\ L_f h_2(x) \end{bmatrix} + \begin{bmatrix} Lg_1 L_f h_1(x) & Lg_2 L_f h_1(x) \\ Lg_1 h_2(x) & Lg_2 h_2(x) \end{bmatrix}.u \qquad (19)$$

That is to say an expression of the forme :

$$\begin{bmatrix} \ddot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \alpha(x) + \beta(x).u \qquad (20)$$

where the vector $\alpha(x)$ is given by:

$$\alpha(x) = [\alpha_1(x) \;\; \alpha_2(x)]^T \qquad (21)$$

with

$$\alpha_1(x) = \begin{cases} f_3^2 + f_4^2 + x_3.(a33.f_3 + a34.\dot{\omega}.x_4 \\ + a34.\omega.f_4) + x_4.(a43.\dot{\omega}\, x_3 + \\ a43.\omega.f3 + a44.f4) + f_1.(x_3.a31 + \\ x_4.a41) + f_2.(x_3.a32.x_4.a42) \end{cases} \qquad (22)$$

$$\alpha_2(x) = \frac{Lm}{Lr}.(x_3.f_2 + x_2.f_3 - x_1.f_4 - x_4.f_1) \qquad (23)$$

In the same manner, the matrix $\beta(x)$ is given by:

$$\beta = \begin{bmatrix} b1.(x3.a31 + x4.a41) & b2.(x3.a32 + x4.a44) \\ -x4.b1.\frac{Lm}{Lr} & x3.b2.\frac{Lm}{Lr} \end{bmatrix} \qquad (24)$$

One immediately verifies that in steady state, the matrix $\beta(x)$ called matrix of decouplage is invertible, $\det(\beta) = (x_3^2 + x_4^2)$, and the vectorial reltive degree of the system is equal to (2,1), second derivative of y_1, first derivative of y_2. It is therefore possible to linearize the system by the introduction of the control

$$u = \beta^{-1}(x).(v - \alpha(x)) \qquad (25)$$

Therefore, the system looks like a double integrator between y_1 et v_1, and et un simple intégrateur between y_2 and v_2.

Poles placement

The computation of the control v according to the reference trajectory $y_d = [y_{1d} \;\; y_{2d}]^T$ .is done by drawing the relation (17), in the previous expression, one gets:

$$v = \begin{bmatrix} \ddot{y}_{1d} + \alpha 11.(\dot{y}_{1d} - \dot{y}_1) + \alpha 12.(y_{1d} - y_1) \\ \dot{y}_{2d} + \alpha 2.(y_{2d} - y_2) \end{bmatrix} \quad (26)$$

let

$$e_1 = y_1 - y_{1d} \quad (27)$$

The dynamics of the error is given by:

$$\ddot{e}_1 + \alpha 11.\dot{e}_1 + \alpha_{12} = 0 \quad (28)$$

The error is asymptotically stable if the polynomial

$$p^2 + \alpha 11.p + \alpha 12 \quad (29)$$

is a Hurwitz polynomial. Besides, the coefficients αij are chosen according to specifications of the desired output.

From the equation (27) and (28), one drawn the expressions of the second derivative of y_1

$$\ddot{y}_1 = \ddot{y}_{1d} - \alpha 11.\dot{e}_1 - \alpha 12 \quad (30)$$

In the same way, let

$$e_2 = y_2 - y_{2d} \quad (31)$$

The dynamics of the error is given by:

$$\dot{e}_2 + \alpha_2.e_2 = 0 \quad (32)$$

The error is asymptotically stable if $\alpha 2 \succ 0$.

From the equation (31) and (32), one computes the first derivative of y_2.

$$\dot{y}_2 = \dot{y}_{2d} - \alpha 2.e_2 \quad (33)$$

For a second order system, the characteristic polynomial is given by:

$$p^2 + 2.\xi.\omega_n.p + \omega_n^2 \quad (34)$$

The settling time at 2 % and the damping factor are given by:

$$tr1 = \frac{4}{\xi.\omega_n}, \quad D = \exp\left(-\frac{\xi.\pi}{\sqrt{1-\xi^2}}\right) \quad (35)$$

Therefore, if one fixes in advance the settling time and the damping factor, one can determine the parameters ξ and ω_n which are given by:

$$\xi = -\frac{Ln(D)}{\sqrt{(Ln^2 D + \pi^2)}} \quad \text{et} \quad \omega_n = \frac{4}{\xi.tr1} \quad (36)$$

While equalizing polynomials (29) and (34), one gets

$$\alpha 11 = 2.\xi.\omega_n, \quad \alpha 12 = \omega_n^2 \quad (37)$$

In the same way, for a first order system, the settling time at 2 % is given by $tr2 = 3.912 \times T$, where T is the time constant. From the equation (32), the time constant is given by :

$$T = \frac{1}{\alpha 2} = \frac{tr2}{3.912} \quad (38)$$

therefore

$$\alpha 2 = \frac{3.912}{tr2} \quad (39)$$

Finally, knowing the three wanted parameters which are the settling time $tr1$, $tr2$ and the damping factor D, one can determine the coefficients $\alpha 11, \alpha 12$ and $\alpha 2$ by using formulas (36), (37) et (39).

5 Simulations

The simulation has been made o a time interval of 10 secondes bwhile using the MATLAB software.

(N.B) a value without unit refers to the international system.

The real parameters of the machine are given by:

$R_s = 52m\Omega$, $R_r = 70m\Omega$, $L_m = 31m\Omega$

$L_s = 31.7mH$, $L_r = 32.3mH$, $p = 2$.

$J = 0.41.Kg.m^2$, $f = 0.5.N$, $C_r = 0.1.N.m$.

The initial conditions are given by:

$i_{ds}(0) = 0.1$, $i_{qs}(0) = -0.2$, $\varphi_{dr}(0) = 0.3$, $\varphi_{qr}(0) = -0.4$, $\Omega(0) = 0.2$.

The steady state desired parameters are:

$$\phi_{ref} = 1Wb, \quad C_{emref} = 10.N.m$$

And the settling time imposed are $t_{r1} = t_{r2} = 4s$, the damping factor D=20 %.

After calculation, one finds that

$\omega_n = 2.1932$, $\xi = 0.4559$, $\alpha 11 = 4.8102$, $\alpha 12 = 2$, $\alpha 2 = 0.75$.

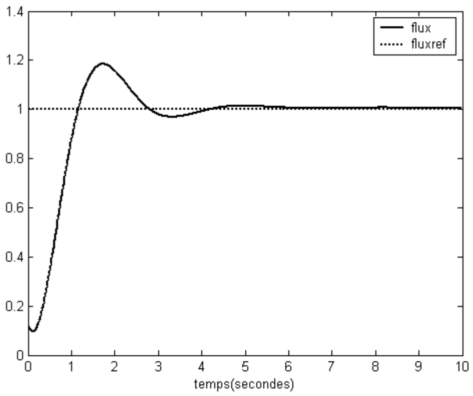

Fig. 1: flux (__), reference flux (...)

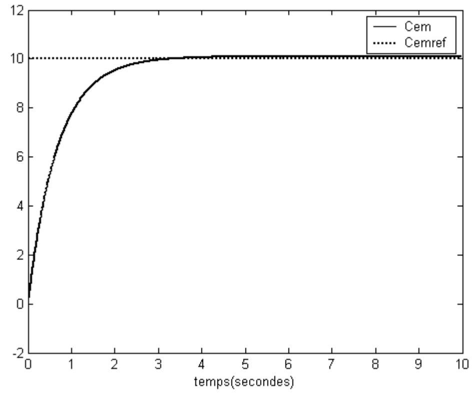

Fig. 2: torque (__), reference torque (...)

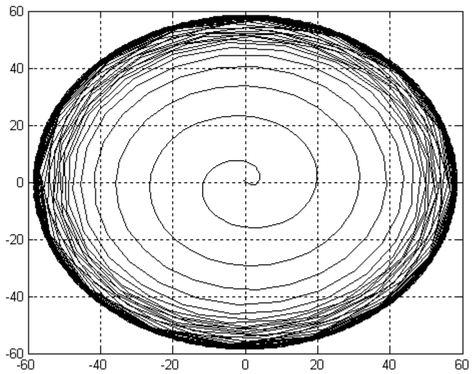

Fig. 3: voltage u_{qs} vs u_{ds}

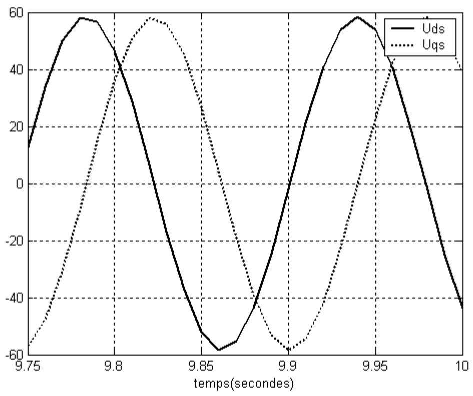

Fig. 4: voltage u_{ds} (__), voltage u_{qs} (..)

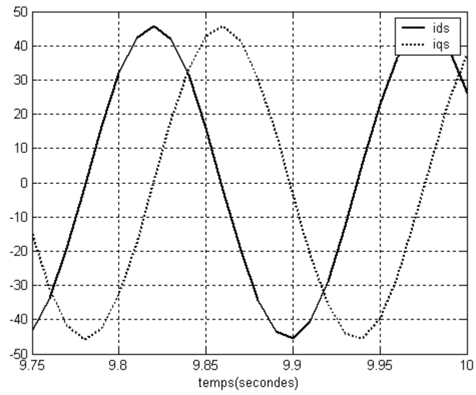

Fig. 5: current i_{ds} (__), current i_{qs} (..)

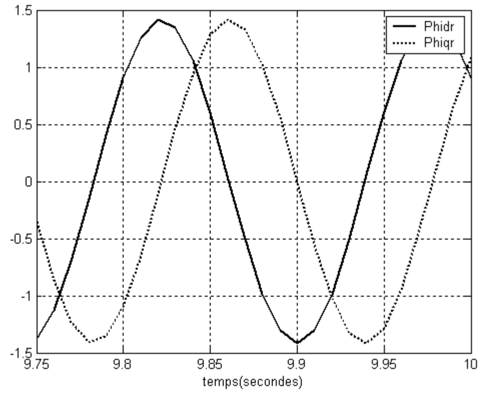

Fig. 6: flux φ_{dr} (__), flux φ_{qr} (..)

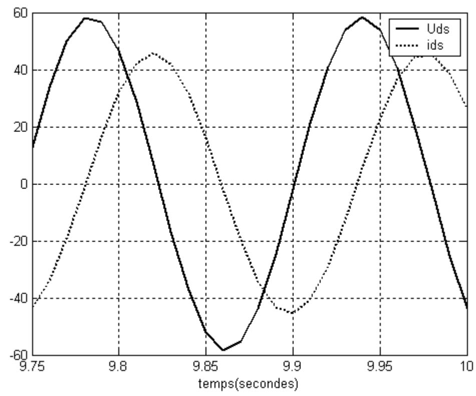

Fig. 7: voltage u_{ds} (__), current i_{ds} (..)

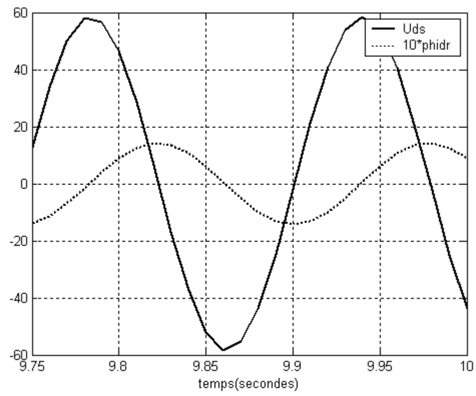

Fig. 8: voltage u_{ds} (__), 10*flux φ_{dr} (..)

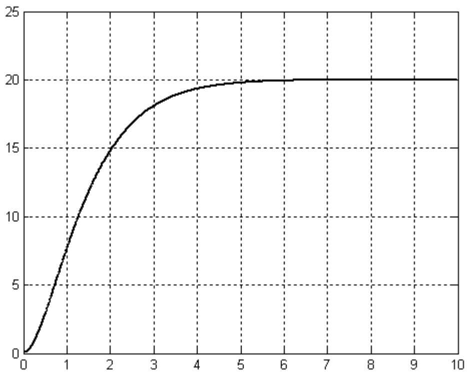

Fig. 9: speed Ω

The voltages u_{ds} and u_{qs} are sinusoidal and in quadrature.

$$u_{ds} = a_m . \sin(\omega_s .t)$$
$$u_{qs} = a_m . \sin(\omega_s .t - \pi / 2)$$

with $a_m = \sqrt{u_{ds}^2 + u_{qs}^2}$, ω_s is the frequency of the signal, one measures $\omega_s = 39.76$ and $a_m = 57.74$. The voltages applied to our motor are given by:

$$u_{ds} = 57.74. \sin(39.76.t)$$
$$u_{qs} = 57.74. \sin(39.76.t - \pi / 2)$$

The stator currents are also sinusoidal and in quadrature with the same frequency that the voltages

$$i_{ds} = I_m . \sin(\omega_s .t + \varphi)$$
$$i_{qs} = I_m . \sin(\omega_s .t + \varphi - \pi / 2)$$

with $I_m = \sqrt{I_{ds}^2 + I_{qs}^2}$, φ is the phase angle between the voltage u_{ds} and the current i_{ds}, therefore

$$i_{ds} = 45.76. \sin(39.76.t - 1.43)$$
$$i_{qs} = 45.76. \sin(39.76.t - 1.43 - \pi / 2)$$

The rotor fluxes are sinusoidals, and in quadrature with the same frequency that the voltages

$$\varphi_{dr} = \phi. \sin(\omega_s .t + \psi)$$
$$\varphi_{qr} = \phi. \sin(\omega_s .t + \psi - \pi / 2)$$

with $\phi = \sqrt{\varphi_{dr}^2 + \varphi_{qr}^2}$ and ψ is the phase angle between voltages u_{ds} and the φ_{dr}, therefore

$$\varphi_{dr} = 1.41. \sin(39.76.t - 1.51)$$
$$\varphi_{qr} = 1.41. \sin(39.76.t - 1.51 - \pi / 2)$$

In steady state, the speed Ω is given by the relation

$$\Omega = \frac{C_{em} - C_r}{f} = \frac{10 - 0.}{0.5} = 19.8$$

6 Interpretation

Figures 1 and 2 shows the well perfect follow-up of the flux and the torque toward their wanted values that means $\Phi = 0.5.(\varphi_{dr}^2 + \varphi_{qr}^2) = 1, C_{em} = 10$. the figure 3 represents the u_{qs} voltage versus u_{ds} voltage, after the transient response, the two voltages describe a circle that proves that they are in quadrature of π/2 and of amplitude 57.74, the following figures represent the different signals of our machine and the phase angle between them, the figure 9 represents the mechanical speed which is equal in steady state to 19.8.

Conclusion

In this steady, one developed a method of nonlinear control known an decoupled control by feedback linearization. This method has been applied with success to control the torque and the flux of an induction motor that is extensively used in industry. Results of the simulation testify the hardiness of the method. An extension of this work is considered in order to apply this method of decouplage to other type of motors.

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