A State-Space Robust Feedforward Compensator for Industrial Robot Manipulators Needless to Computed Torque Control

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Abstract: This paper presents a unified robust motion control scheme for robot manipulators in state space. The controller design task is carried out based on feedforward control sketch in state space framework with no need to the Computed Torque Control (CTC) which is usually used in many other control approaches. In the proposed approach, the load torque and some other uncertainties in the linear model of the actuators as external disturbances are successfully mitigated by the used control law. Six degrees of freedom PUMA560 arm, including completed models of actuators is simulated and the results satisfy the common technical specifications including good tracking and robustness. The proposed control approach can guarantee the stability and a satisfactory tracking performance via a simple design method.

Key-Words: Manipulator, computed torque control, feedforward control, state space, Trajectory planning.

1 Introduction

The problem of robust tracking control of a robot has been a topic of considerable interest over the past decade in industrial electronics and so many robust control schemes have been reported with the aim of coping with the internal model uncertainties and counteracting the external disturbances [1]-[3]. In the meantime, the best tracking controller is ideally the perfect tracking controller (PTC) which controls the object with zero tracking error and it can be achieved using a feed forward control scheme [4].

However, this methodology frequently suffers from several inherent weaknesses [5]:

1. They require to measure the disturbance, and need a very good model for the process;
2. The changes in the process parameters cannot be compensated unless a reliable system identification procedure is incorporated;
3. They may lead to improper transfer functions, so some important simplifications must be done to reach realizable results.

For minimum phase linear systems, the perfect tracking is relatively easy to achieve, but the control problem for non-minimum phase systems becomes hypersensitive due to the fundamental limitations on the transient tracking performance characterized by the number and location of the zeros which are non-minimum phase [6].

For this reason, many researches have been performed by researchers. For instance, for linear continuous-time systems, [7] show that the asymptotic tracking problem is solvable if and only if, a set of linear matrix equations is solvable. This was later generalized to nonlinear systems by replacing the linear matrix equations by a set of first order partial differential equations [8]. These approaches asymptotically track any member in a given family of signals generated by an exosystem.

[9] proposed a stable inversion approach to avoid the use of exosystems, and, in the case of non-minimum phase systems, improve the transient performance by using pre-actuation. But, due to parametric variations and unmodeled dynamics in the industrial process, this approach seems rather ill-posed.

Also, for Linear systems, an optimal feedforward control design is addressed in [10] with smoothing constraints on input and output, whereby the output trajectory is constructed via transition polynomials. This concept has also been used with the anti-causal stable inversion for uncertain non-minimum phase systems [11], and has been applied to a motor-position servo control system [12]. However, all of the aforementioned methods are based on the solving some complicated algebra equations.

It most be noted that, the feedforward signal leads to asymptotic tracking of any trajectory only in absence of disturbance. To mitigate the disturbance's effect it is usually needed to CTC approach.

The computed torque or inverse dynamics technique is a special application of feedback linearization of nonlinear systems, which is utilized to linearize the nonlinear equation of robot motion by cancellation of some, or all nonlinear terms [13]. However, there are many reasons that cause, the computed torque strategy to present poor efficiency [14]-[17]:

1. In industrial applications, there are many uncertainties such as system parameter variations, external disturbance, friction , and unmodeled dynamics that influence the tracking performance of computed torque control especially in high speed operations [14]-[16]. The situation is more severe for direct-drive robots without transmissions, which significantly reduce unmodelled dynamic effects.
2. A recursive equation is needed to compute the CTC term with a powerful processor to handle the required calculations.

Therefore, several computed torque controllers have tried to overcome the problem of uncertainties by using adaptive techniques [18]–[21]. For instance, in [18], a computed torque control approach using the sliding mode technique is introduced, and the uncertainty bound is estimated by an adaptive scheme. Ref. [19] describes an adaptive robust computed torque control where a time-varying gain in the controller is estimated by an adaptation law.

This work attempts to partially address a unified robust motion control scheme for a six degree of freedom robotic manipulator using linear state feedback and needless to computed torque control. An analytical consideration in state space for the tracking problem is presented including complete models of the actuators.

2 Problem Formulation

Suppose that a linear system in controllable form in the state space is given by

$$\dot{\mathbf{x}}_i = \mathbf{A}_i \mathbf{x}_i + B_i \mathbf{u}_i + \mathbf{d}_i \tag{1}$$

$$\mathbf{y}_i = \mathbf{c}_i \mathbf{x}_i \tag{2}$$

where $\mathbf{u}_i$ is defined as:

$$\mathbf{u}_i = -\mathbf{k}_i \mathbf{x}_i + k_{0,i} \mathbf{v}_i \tag{3}$$

$\mathbf{x}_i$ is the state vector, y_i is the output vector of the i-th coordinate, that only consist of joints positions, K_i and $K_{0,i}$ are the design parameters for pole placement, $\mathbf{d}_i$ is the vector of external disturbances and V_i is the robustifying control input. Substituting equation (3) into (1) leads to:

$$\dot{\mathbf{x}}_i = \left(\mathbf{A}_i - \mathbf{B}_i \mathbf{k}_i\right)\mathbf{x}_i + B_i k_{0,i} \mathbf{v}_i + \mathbf{d}_i \tag{4}$$

Next, we are defining an algorithm to adjust the control input V_i so that, the tracking error minimized. On the other hand, suppose that the desired closed loop state equations of i-th coordinate are given by:

$$\dot{\mathbf{x}}^d_i = \left(\mathbf{A}_i - \mathbf{B}_i \mathbf{k}_i\right)\mathbf{x}^d_i + B_i k_{0,i} \mathbf{v}^d_i \tag{5}$$

$$\mathbf{y}^d_i = \mathbf{c}_i \mathbf{x}^d_i \tag{6}$$

where $\mathbf{v}^d_i$ is the reference linear output for the i-th coordinate and $\mathbf{y}^d_i$ is the i-th reference output. Therefore, the coefficient vector $\mathbf{k_i}$ is determined such that the $\mathbf{y}^d_i$ will closely follow $\mathbf{v}^d_i$. So we can use $\mathbf{v}^d_i$ to specify the desired trajectory in joint space. Subtracting (5) from (4) and (6) from (2), we obtain the following sets of tracking error equations:

$$\dot{\mathbf{e}}'_i = \left(\mathbf{A}_i - \mathbf{B}_i \mathbf{k}_i\right)\mathbf{e}'_i + \mathbf{B}_i k_{0,i} \mathbf{v}'_i + \mathbf{d}_i \qquad i=1,...,n \tag{7}$$

$$\mathbf{y}'_i = \mathbf{c}_i \mathbf{e}'_i \tag{8}$$

where

$$\mathbf{y}'_i = \mathbf{y}_i - \mathbf{y}^d_i \tag{9}$$

$$\mathbf{e}'_i = \mathbf{x}_i - \mathbf{x}^d_i \tag{10}$$

$$\mathbf{c}_i = [1 \quad 0....... \quad 0] \tag{11}$$

To achieve high tracking accuracy, we need to generate an auxiliary linear control input $\mathbf{v}'_i$ to drive the tracking error $\mathbf{e}'_i\left(\mathbf{y}'_i\right)$ to zero asymptotically. For this reason, the following quantities are defined [22]:

$$z_i = \mathbf{e}'^{(p)}_i - \sum_{j=1}^{p} b_j \mathbf{e}'^{(p-j)}_i \tag{12}$$

$$s_i = \mathbf{v}_i'^{(p)} - \sum_{j=1}^{p} b_j \mathbf{v}_i'^{(p-j)} \tag{13}$$

Differentiating (12) with respect to time and using (7) and (13) and also, supposing that $\mathbf{d}_i$ can be modeled by a pth-order ordinary differential equation as fourteenth equation [22], we will have:

$$\mathbf{d}_i^{(p)} = \sum_{j=1}^{p} b_j \mathbf{d}_i^{(p-j)} \tag{14}$$

$$\dot{\mathbf{z}}_i = \left(\mathbf{A}_i - \mathbf{B}_i \mathbf{k}_i\right)\mathbf{z}_i + \mathbf{B}_i k_{0,i} \mathbf{s}_i \tag{15}$$

$$\mathbf{y}_i'^{(p)} = \mathbf{c}_i \mathbf{e}_i'^{(p)} = \sum_{j=1}^{p} b_j \mathbf{y}_i'^{(p-j)} + \mathbf{c}_i z_i \tag{16}$$

It must be noted that, the order p of this differential equation reflects the dynamic structure of $\mathbf{d}_i$, which in most cases is regarded one or two for simplicity. If we define $\mathbf{Z}_i$ as follows:

$$\mathbf{Z}_i = \begin{bmatrix} \mathbf{y}_i' & \dot{\mathbf{y}}_i' & \cdots & \mathbf{y}_i'^{(p-1)} & z_i \end{bmatrix}^{\mathrm{T}} \tag{17}$$

The following state equation can be obtained:

$$\dot{\mathbf{Z}}_i = \Lambda_i \mathbf{Z}_i + \psi_i \mathbf{s}_i \tag{18}$$

$$\mathbf{y}_i = H_i \mathbf{Z}_i \tag{19}$$

Where

$$\Lambda_i = \begin{bmatrix} 0 & 1 & 0 \cdots 0 & 0 \\ \vdots & 0 & 1 \cdots 0 & 0 \\ \vdots & \vdots & \vdots\ \vdots & \\ b_p & b_{p-1} & b_1 & c_i \\ 0 & 0 & \cdots 0 & \mathbf{A}_i - \mathbf{B}_i \mathbf{k}_i \end{bmatrix} \tag{20}$$

$$\psi_i = [0 \ \dots \ 0 \ \ \mathbf{B}_i k_{0,i} \ ]^{\mathrm{T}} \tag{21}$$

$$H_i = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{22}$$

It is noted that if $((\mathbf{A}_i - \mathbf{B}_i \mathbf{k}_i), \mathbf{B}_i k_{0,i})$ is controllable, then (Λ_i, ψ_i) is also controllable [22]. With this in mind we define a control law as follows.

$$\mathbf{s}_i = -\mu_i \mathbf{Z}_i \tag{23}$$

Or equivalently:

$$\mathbf{s}_i = -\mu_{p,i} y_i' - \mu_{p-1,i} \dot{y}_i' - \dots - \mu_{p,i} y_i'^{(p-1)} - \mu_{0,i} z_i \tag{24}$$

Substituting (12) and (13) into (24), we obtain

$$\left(v_i' + \mu_{0,i} \mathbf{e}_i'\right)^{(p)} = -\sum_{j=1}^{p} \mu_{j,i} y_i'^{(p-j)} + \sum_{j=1}^{p} b_j \left(v_i' + \mu_{0,i} \mathbf{e}_i'\right)^{(p-j)} \tag{25}$$

Whereas $\mathbf{y}_i'$ is a function of tracking error $\mathbf{e}_i'$, therefore $\mathbf{v}_i'$ depends on the tracking error only and finally it ensure that the tracking error $\mathbf{e}_i'$ converging to zero asymptotically. The final step is to adjust the linear control input in (4) to account for the effects of disturbance as follows:

$$v_i = v_i^{d} + v_i' \tag{26}$$

A suitable way of calculating μ_i is to select proper closed-loop poles of (Λ_i, Ψ_i) first and then to determine the the value of μ_i correspondingly [23]. Therefore two sets of closed-loop system poles must be decided:

1. Inner linear control part: to place closed-loop poles in desired and so control the output y_i to follow the reference input v_i.
2. Outer linear control part: to place closed-loop poles of equation (18) in desired places to suppress effects of uncertainties.

The Laplace transform of equation (4) is given by

$$X_i(s) = \left(SI - \mathbf{A}_i + \mathbf{B}_i \mathbf{k}_i\right)^{-1} \left(B_i k_{0,i} \mathbf{V}_i(s) + \mathbf{d}_i(s)\right) \tag{27}$$

Where $X_i(s)$ is Laplace transform of $\mathbf{x}_i$. Further more the laplace transform of equation (5) defined as

$$B_i k_{0,i} \mathbf{V}_i^{d}(s) = \left(SI - \mathbf{A}_i + \mathbf{B}_i \mathbf{k}_i\right) X_i^{d}(s) \tag{28}$$

With multiplication extremes of equation (26) in $B_i K_{0,i}$ will have:

$$B_i K_{0,i} v_i(s) = B_i K_{0,i} v_i^{d}(s) + B_i K_{0,i} v_i'(s) \tag{29}$$

In this scheme the set point is fed through a signal generator and then compared with the process output. The feedforward branch is inversion of controlled process by state feedback theory ($SI-A_i+B_ik_i$). Ideally the feedforward branch generates a signal when applied to controlled process, $(SI-A_i+B_ik_i)^{-1}$, produces the desired output in response to set point changes. The linear control law which acts on the error will make corrections if there is uncertainty in process that is under control. The block diagram of the proposed approach is represented in Figure 1.

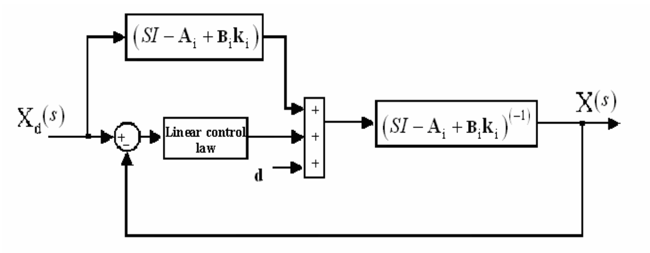

Fig.1 Robust Feedforward Compensator

According to [24], good tracking can be achieved with relatively low uncertainty model error (p=1 or 2). To this end we can select the poles of equation (18) further to the left of the imaginary axis in the complex S-plane than the inner control loop closed-loop poles.

3 Dynamic model of the actuators

We first consider the familiar differential equations of motion which describe DC motors driving an *n* degree of freedom robot. These equations for *n* actuators are given as

$$J_{m_i}L_i\dddot{\theta} + (J_{m_i}R_i + L_iB_{m_i})\ddot{\theta}_i + (R_iB_{m_i} + K_{m_i}K_{b_i})\dot{\theta} = r_iK_{m_i}v_i - r_i^2(R_i\tau_{l_i} + L_i\frac{d\tau_{l_i}}{dt}) \quad (30)$$

Where, all of parameter defines as [25]. It must be noted that, the load torque τ_{li} is placed on motor shaft by the manipulator. The load torque vector for simulation is calculated by dynamic equation of robot that defines as equation (18) in [26].

Selecting the position, velocity and acceleration as state variables, $X = \begin{bmatrix}\theta & \dot{\theta} & \ddot{\theta}\end{bmatrix}^T$, lead to a state-space equation such as equation (1), where:

$$A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & \frac{-(R_iB_{m_i} + K_{m_i}K_{b_i})}{J_{m_i}L_i} & \frac{-(J_{mi}R_i + L_iB_{m_i})}{J_{m_i}L_i} \end{bmatrix}$$

$$B_i = \begin{bmatrix} 0 & 0 & \frac{r_iK_{m_i}}{J_{m_i}L_i} \end{bmatrix}^T \quad (31)$$

$$\mathbf{d}_i = -\begin{bmatrix}0\\0\\1\end{bmatrix}\frac{r_i^2(R_i\tau_{li} + L_i\dot{\tau}_i)}{J_{m_i}L_i}$$

With the last representation, the torques' dynamics are thoroughly modeled as disturbances.

4 Simulation result

Reference [26] gives the details of the six DOF robot manipulator system used in this paper. The major steps of the proposed algorithm can be summarized as follow:

A. The desired trajectory is chosen as:

$$\theta = -a\cos(\frac{\pi}{T}t) + a, \ t \geq 0 \quad (32)$$

Where we set $a = 0.5rad$, and $T = 1\sec$.

B. By the procedure described in section 2 and choosing the closed loop poles of each actuator, the resulting state feedback vector K is calculated as presented in Table1.

Table1. Gains of the Controllers

Joint	K		
1	[87.111	-6.5389	-0.0597]
2	[159.781	-6.0157	-0.0376]
3	[81.505	-7.3243	-0.0557]
4	[2.3166	-3.8364	-0.1399]
5	[2.6325	-3.456	-0.159]
6	[2.6325	-3.223	-0.1588]

C. Modeling of uncertainty by a p'th-order differential equation, Set the uncertainty equation to zero and finally getting b_j. In this step, if we choose p=1 for the uncertainty, we have.

$$d_i^{(1)} = b_1d_i \quad (33)$$
$$= 0$$

That is b_1 set to zero and consequently, d_i would be an arbitrary constant (a step function).

D. With the procedures described in section 2, the μ vector is calculated and given by Table2.

On the basis of mentioned to them in above, Figure 2 shows the tracking errors of 3 first joints in absence of external disturbance. To check the robustness of the system, we apply the load torques on the motors shaft. Figure 3 show the applied load torques. The tracking error is limited within about 0.0014 rad, which is acceptable due to the adverse effect of the disturbances with a good tracking performance. The results are shown in Figure 4.

Table2. Gains of the Controllers

Joint	μ			
1	[2991.3	66.2595	0.522	0.0015]
2	[2808.4	52.5182	0.346	0.0008]
3	[2941.7	65.7412	0.5205	0.0015]
4	[3300	135.33	2	0.0111]
5	[2719.4	117.7	1.8	10.5]
6	[1722.7	82.6074	1.4196	0.0089]

Because the characteristics of the motors' currents and voltages are of importance, as an example, Figure 5 indicates the voltages waveforms for the motors of joints 1-3. As seen they are bounded with acceptable time-variation. The response of the manipulator is shown in the Figure 6.

5 Conclusions

A two degree of freedom robust control scheme has been used to control a robot manipulator system. The controller design approach presented uses the linear model of the motors while the robot' torques and some model uncertainties are simply modeled as external disturbances and then rejected by an appropriate robust controller. The simulation results prove the robustness and excellent tracking performance of the linear control system proposed for the system under study. Considering some major severities appear in adaptive and nonlinear control methods due to a need for calculating the torque control, the proposed approach can be considered as a successful, simple, practical, and with low computation burden approach to control robotic manipulator systems.

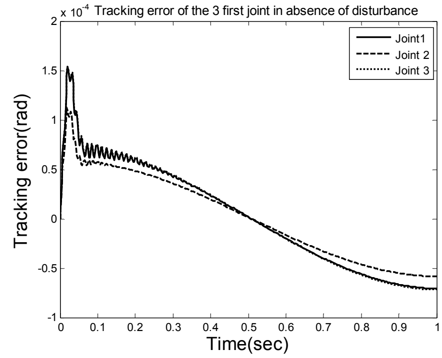

Fig.2 tracking errors for joints 1-3

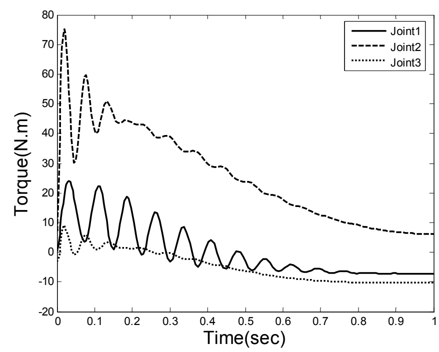

Fig 3: the torques applied on joints 1-3

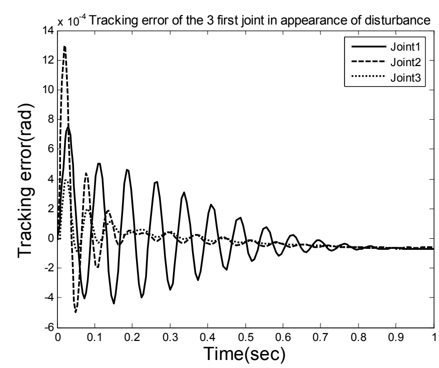

Fig.4 tracking errors for joints 1-3

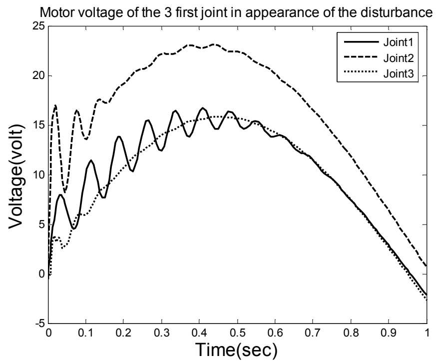

Fig.5 Motors voltages for joints 1-3

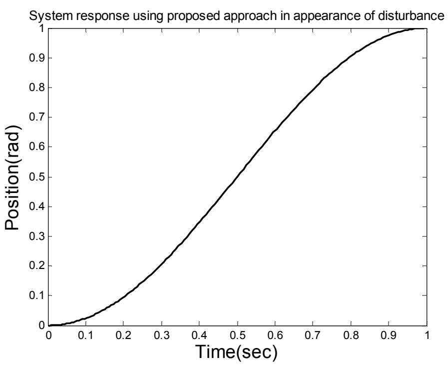

Fig.6 Output trajectory with the effect of disturbance

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