Improved Blind Equalization Scheme Using Variable Step Size Constant Modulus Algorithm

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Abstract - The blind equalizer relies on the knowledge of signal's constellation and its statistics to perform equalization. The major drawback is that the blind equalizer will typically take a longer time to converge as compared to a trained equalizer. Variable Step-Size LMS (VSLMS) algorithms have been introduced to optimize the speed and steady-state error. The relationships between the a priori and a posteriori error signals have been used to quickly and easily characterize the stability and robustness of the given adaptive algorithms. Our proposed algorithm uses both variable step size and a posteriori updates to increase the speed of convergence while reducing the trade off between the convergence speed and steady state error.

Keywords – Variable step size, blind equalizer, convergence speed, steady state error

1 Introduction

Intersymbol interference (ISI) has been recognized as the major obstacle to high speed data transmission over wireless channels. ISI caused by multipath in bandlimited time dispersive channels distorts the transmitted signal. Since the channel is mostly random and time varying, the equalizer used to combat ISI must be able to track and adapt to these time varying characteristics [1]. In some adaptive filter applications, it may be undesirable to employ a training signal. For example, in wireless communication from a basestation to multiple receivers, having the basestation transmit a training signal to each receiver reduces the effective data rate at which information can be broadcast and complicates the transmission protocol. For this reason blind adaptive algorithms have been developed that can, acquire a desirable solution without a training signal [2]. In case of blind adaptive equalizers, convergence is slow as compared to trained equalizers and hence blind equalizers are limited in use for situations with slowly changing channel condition. There exists a strong complementary dependence between the convergence speed and the steady state error of an adaptive equalizer. A major role in this scenario is played by the stepsize used. In case of a large step size value, initially when the equalizer is far from its optimal values, the convergence rate is satisfactorily high but as the output of the equalizer converges, the steady state error is large. But in case of a small stepsize value, although the steady state error is fairly low, the initial convergence is very slow and hence not helpful in most of the cases when the channel conditions are random and time varying. The *a priori* and *a posteriori* error signals can be used to characterize the robustness of the given algorithm and putting some necessary constraints on the step size value ensuring the system's stability. One of the widely used blind adaptation algorithm is the Constant Modulus Algorithm (CMA). Different schemes for varying the step-size of CMA have been proposed in [3, 4]. We propose a variable stepsize scheme for CMA driven by the output error, resulting in high convergence speed and low output error.

2 System Model

A system model of a digital communication system is shown in Fig. 1.

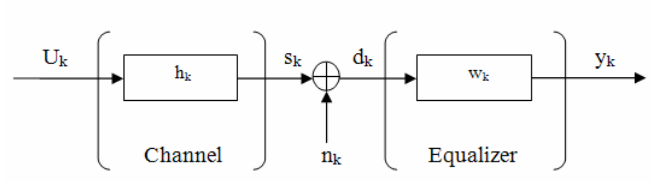

Fig. 1. Digital communication system model

Here U_k represents the source symbols for an independent and identically distributed (i.i.d.) source transmitting BPSK symbols. The vector $h_k = [h_0,...,h_{Nh-1}]^t$ is an N_h x 1 vector representing the channel impulse response and $w_k = [w_0,\ldots,w_{Nw-1}]^t$ is the equalizer coefficient vector of size Nw x 1. s_k is the output data from the channel. The number of channel and equalizer coefficients is denoted by N_h and N_w respectively. The received signal corrupted by intersymbol interference and noise is denoted by d_k and is given by

$$d(n) = \sum_k h(k)u(n-k) + n_k \quad (1)$$

Where n_k is the additive white noise, and is assumed to be Gaussian. The purpose here is to perform equalization on the received signal hence minimizing the effects of ISI and get an approximation of U_k that is denoted by y_k i.e.

$$y_k = U_k \quad (2)$$

Where y_k denotes the equalizer's output and is given by

$$y(n) = \sum_k w(k)d(n-k) \quad (3)$$

3 CMA Algorithm

The constant modulus algorithm [5] is the most widely used blind equalization algorithm. The reason behind that is its robustness and simplicity in implementation. The algorithm [6] is given by

$$y(k) = w(k)d(k) \quad (4)$$

$$w(k+1) = w(k) + \mu y(k)(A - |y(k)|^2)d(k) \quad (5)$$

Where (3) gives the coefficient update rule. y(k)is the adaptive equalizer's output while d(k) is the input of the equalizer. μ is the step size used. Here A is the statistical constant (also called the dispersion constant) giving knowledge about the transmitted signal constellation and is given by

$$A = \frac{E\{a^4(k)\}}{E\{a^2(k)\}} \quad (6)$$

Where $a(k)$ denotes the symbol ensemble that the source is transmitting. The error signal is given by

$$e(k) = y(k)(A - |y(k)|^2) \quad (7)$$

CMA suffers a serious problem in terms of slow convergence when it is implemented with a constant step size [7]. Clearly, the idea is to use large steps when the equalizer is far away from the optimum value to get a fast convergence and a small step size when the equalizer is near its optimum values, thus to have a minimum steady state error. This situation essentially calls for the need of a variable step size.

4 Stability Analysis of the Algorithm

Regarding the nature of the performance and stability analysis of an algorithm, the most important is the choice of error criterion from which the algorithm's behavior can be evaluated. The error function is given by (6). If the value of the error function tends to reduce over time, we can say that the algorithm is properly converging. But such a condition does not necessarily guarantee the stability for the given algorithm. The relationship between the *a priori* and *a posteriori* error signals have already been used to quickly and easily characterize the stability and robustness of the given adaptive algorithm [2].

The *a posteriori* error signal is given by

$$e_p(k) = y_p(k)(A - |y_p(k)|^2) \quad (8)$$

Where $y_p(k)$ is the *a posteriori* output and is given by the relationship

$$y_p(k) = d^T(k)w(k+1) \quad (9)$$

The relationships shown by (7, 8) have been used to analyze and put a certain upper limit constraint on the step size [2]. The range of the step size is given by

$$0 < \mu(k) < \frac{\mu_{\max}(y(k), A)}{\|x(k)\|^2} \tag{10}$$

Where

$$\mu_{\max}(y(k),A) = \begin{cases} \frac{1}{A^2 - |y(k)|^2}\left(1 - \sqrt{\frac{2A^2}{|y(k)|^2} - 1}\right) & if\, |y(k)| \le A\sqrt{2} \\ \frac{2}{|y(k)|^2 - A^2} & if\, |y(k)| > A\sqrt{2} \end{cases} \tag{11}$$

and y(k) and A refer to equalizer's output and dispersion constant respectively. The relation in (9) can be used to test and adjust the value of μ(k) at each iteration to ensure the system's stability.

5 Proposed Variable Step Size Algorithm

A variable step size scheme has been developed to cater with the problems of slow convergence speed and relationship between the convergence speed and steady state error. The proposed algorithm uses two adaptive equalizers that work in parallel. Structure of the proposed algorithm is shown in Fig. 2.

The equalizer $w_2(k)$ uses a constant and large step size $\mu_2=\mu_{max}$ because it works for the speed mode and helps for fast initial convergence, while the equalizer $w_1(k)$ uses a variable step size $\mu_1(k)$ and caters for the accuracy mode and reduction of steady state error [8]. The large step size μ_{max} has to be chosen on each iteration within the bounds given by (9) thus to ensure system's stability. The output is always taken from the accuracy equalizer The essence of CMA lies in the variation of the equalizer output from a constant modulus [4].

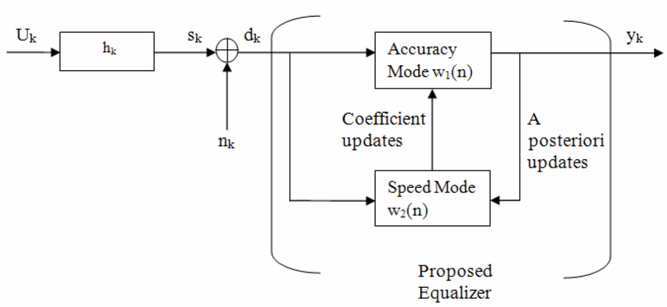

Fig. 2. Proposed equalizer structure for variable step size algorithm.

The working of the algorithm is described in the following equations.

$$y_1(k) = w_1(k)d(k) \tag{12}$$

$$y_2(k) = w_2(k)d(k) \tag{13}$$

Here w(k) is the coefficient of the equalizer and d(k) is the tap-input vector while y(k) represents the equalizer output. The subscripts 1 and 2 refer to speed mode and accuracy mode equalizers respectively. The coefficients of the speed mode equalizer are updated by

$$w_2(k+1) = w_2(k) + \mu_{\max} e_2(k)d(k) \tag{14}$$

Correspondingly, the coefficients of the accuracy mode equalizer are updated as

$$w_1(k+1) = \begin{cases} w_2(k+1) & if \sum_{i=k-T+1}^{k} e_2^2(i) < \sum_{i=k-T+1}^{k} e_1^2(i) \\ w_1(k) + \mu_1(k)e_2(k)d(k) & else \end{cases} \tag{15}$$

Where the error signal e(k) is given by (6). The step size of the accuracy mode equalizer is updated by

$$\mu_2(k+1) = \begin{cases} \frac{\mu_x(k) + \mu_{\max}}{2} & if \begin{cases} \sum_{i=k-T+1}^{k} e_2^2(i) < \sum_{i=k-T+1}^{k} e_1^2(i) \\ and \quad k = T, 2T, \ldots \end{cases} \\ \max(\alpha\mu_2(k), \mu_{\min}), & if \begin{cases} \sum_{i=k-T+1}^{k} e_2^2(i) < \sum_{i=k-T+1}^{k} e_1^2(i) \\ and \quad k = T, 2T, \ldots \end{cases} \\ \mu_2(k), & otherwise \end{cases} \tag{16}$$

Where T and α € [0, 1] are constant parameters.

6 Behaviour of the Proposed Algorithm

The behavior of the proposed algorithm can be described by considering the starting phase. In this phase, the speed mode equalizer acts as an adaptive equalizer with a fixed and large step size. The accuracy equalizer also acts similar to an adaptive equalizer with a fixed step size for a number of T iterations. At the end of this interval, the sum of the squared errors of both the equalizers is compared. If the sum for the accuracy mode equalizer is larger than the sum for the speed mode equalizer, it means that the speed mode equalizer performed better during the last T iterations and the coefficients of the accuracy mode equalizer are updated to the coefficients of the speed mode equalizer and also the step size is increased. But this is for the case of initial convergence when the coefficients of the equalizer are far away from the optimal values. When they are close to the optimal values, the sum of squared errors for the speed mode equalizer will be larger than that of the accuracy mode equalizer and hence the step size of the accuracy mode equalizer will be decreased. The step size of the accuracy mode equalizer will be equal to μ_{min} (or nearly equal) at the steady state.

The parameters α and T also play an important role in the convergence of the proposed algorithm [8]. Since the overall equalizer output is taken from the accuracy equalizer, if the value of α is taken to be too small, it reduces the stepsize very quickly and hence the convergence process gets slow. The length of the interval T also controls the convergence speed of the algorithm. If T is set too large, then the adaptation of the step-size of the accuracy mode filter is lost and if T is too small, the step-size $\mu_1(n)$ will have large variations at the steady state.

7 Simulations and Results

Performance of the proposed variable step-size CMA (VSSCMA) was observed by simulating a BPSK source under noisy channel conditions using an SNR of 30dB. In the simulation, the variables of the algorithms were set so as to get the minimum possible mean square error (MSE) so that the convergence rate can be easily compared. For the proposed VSSCMA μ_{min} = 0.00001 and μ_{max} = 0.13. The upper limit for the value of step size given by (9) was tested at each iteration for the stability of the algorithm. The equalizers were taken as finite impulse response (FIR) transversal filter of length N=7. The channel is a well-behaved channel with an impulse response given by h = [-0.005 0.009 -0.024 0.854 -0.218 0.049 -0.016].

For the variable step size algorithm the constant α has been set to 0.9 while the length of the interval under which the errors are compared is taken as T=50 which gives satisfactory performance. The performance of proposed VSSCMA has been compared to that of the fixed step-size CMA and to a single block variable step size CMA algorithm [7]. It can be seen in Fig. 3 that the variable step-size algorithm, which is based on a double block structure, in noisy channel conditions outperforms both of the algorithms in achieving its convergence point. Also evident from Fig. 3 is that MSE at steady state for all the algorithms is comparable, but proposed VSSCMA converges to its steady state error very quickly as compared to the remaining algorithms.

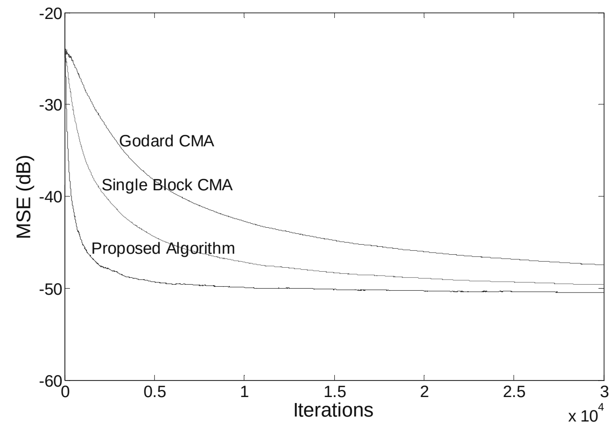

Fig. 3. MSE trajectories for fixed step-size CMA, single block VSSCMA and double block VSSCMA for noisy channel for SNR=30dB. Source signal is modulated with BPSK.

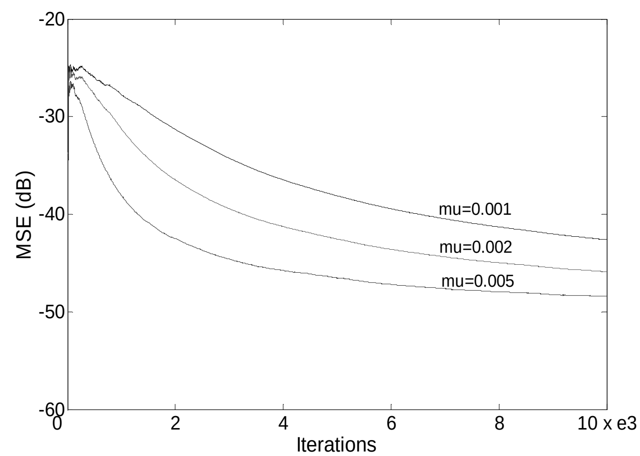

Fig. 4. MSE trajectories for fixed step-size CMA with different values of the step-size.

Secondly, we take a BPSK modulated source signal in noisy channel conditions at SNR=30 dB. The step-size was changed to 3 different values (0.001, 0.002 and 0.005) and we can clearly see the change in MSE trajectories. Fig. 4 shows the effect of changing step size on the performance of the algorithm but due to the non-varying value of the step-size, the convergence rate and the achieved MSE is lower as compared to the case of varying parameter. The operating step-size changes from a large value to a small value as the equalizer goes near the optimal values. This effect is shown in Fig. 5 where the values of step-sizes are shown against the process iterations. It clearly shows that towards steady state conditions, the equalizer attains its minimum allowed value of step-size.

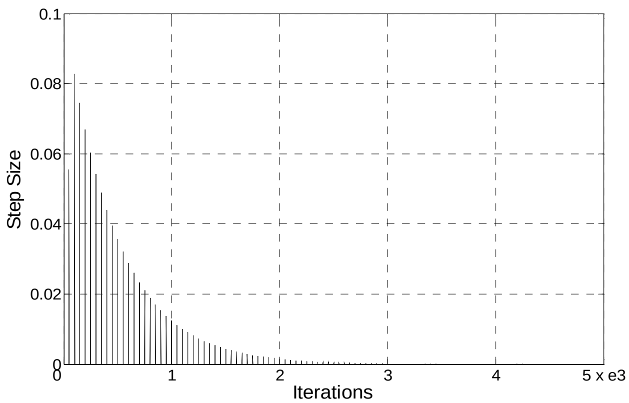

Fig. 5. Step-size behavior for increasing iterations. Clearly the step size is seen to reduce as equalizer gets close to optimal values.

8 Conclusion

In this paper, we proposed a variable step-size constant modulus algorithm. It is shown that the proposed scheme eliminates the existing trade-off between fast convergence and steady state error of the algorithm. Hence giving fast convergence towards the optimal solution and at the same time, reducing the steady state error to a certain limit. Simulation results have shown that the proposed algorithm performs quite well under noisy conditions thus yielding a higher performance for blind adaptive channel equalization than the fixed step size CMA algorithm.

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