# Influences of Induction Motor Parameters on Stability in Case of Operation at Variable Frequency

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*Abstract:* - This paper analyzes the way in which the induction motors parameters influence the stability when operating at variable frequency. In this purpose the used mathematical model is presented. A new analysis method is also detailed, the method being conceived by the members of the staff. Finally the results and the conclusions of the study are presented.

*Key-Words:* - induction motor, stability, variable frequency, parameters, MATLAB, simulation.

## 1 Introduction

The stability is a qualitative feature of the systems associated with the dynamic behaviour.

This problem is a very present one, especially in the field of the driving systems [1], [3], [8] etc.. The systems having induction motors operating at variable frequency does not make an exception, too.

This paper aims to analyze this problem, first of all from mathematical point of view. In the second part a new analysis method will be detailed, method conceived by the members of the staff.

## 2 Problem Formulation

Further on we aim that, by using known representations of the transfer locus and of the amplitude-phase characteristics, to analyze the influences of the machine windings resistances on the stability. In this purpose it is imposed, at the beginning, to establish an adequate mathematical model of the system which is referred to.

Further on the induction motor is supposed to be supplied by means of a sinusoidal voltage source having variable frequency. In order to obtain the relations necessary to carry out the proposed study the following matrix equation, written in accordance with [6], will be used:

$$s\begin{vmatrix} I_{ds}(s) \\ I_{qs}(s) \\ I_{df}(s) \\ I_{qf}(s) \end{vmatrix} = \begin{vmatrix} -\left(\frac{1}{T_s}-\frac{1}{T_r}\right)\cdot\frac{1}{\sigma} & \omega_r & \frac{1}{T_r\sigma} & \frac{\omega}{\sigma} \\ -\omega_r & -\left(\frac{1}{T_s}-\frac{1}{T_r}\right)\cdot\frac{1}{\sigma} & -\frac{\omega}{\sigma} & \frac{1}{T_r\sigma} \\ -\frac{1}{T_s} & 0 & 0 & \omega_s \\ 0 & -\frac{1}{T_s} & -\omega_s & 0 \end{vmatrix} \cdot \begin{vmatrix} I_{ds}(s) \\ I_{qs}(s) \\ I_{df}(s) \\ I_{qf}(s) \end{vmatrix} + \begin{vmatrix} \frac{1}{L_s\sigma} \\ 0 \\ \frac{1}{L_s} \\ 0 \end{vmatrix} \cdot U_{ds}(s), \tag{1}$$

where the following notations have been used:

$T_s = \dfrac{L_s}{R_s}$ - time constant of the stator winding;

$T_r' = \dfrac{L_r'}{R_r'}$ - time constant of the rotor winding;

$\sigma = 1 - \dfrac{L_{sh}^2}{L_s L_r'}$ - leakage coefficient of the machine windings;

$$I_{df}(s) = I_{ds}(s) + \frac{L_{sh}}{L_s} I_{dr}'(s),$$
$$I_{qf}(s) = I_{qs}(s) + \frac{L_{sh}}{L_s} I_{qr}'(s). \tag{2}$$

The motion equation is added to the relation:

$$\frac{J}{p}\frac{d\omega}{dt} = \frac{3}{2} pL_{sh}(i_{qs}i^{/}_{dr} - i_{ds}i^{/}_{qr}) - m_r . \quad (3)$$

If the electrical transient process is considered to be much faster than the mechanical one (J is very great), it results that the mathematical model which will be used further on, is limited to the matrix equation.
This one may be also written as:

$$s[Y(s)] = [A]\cdot[Y(s)] + [B]\cdot[U(s)], \quad (4)$$

or, equivalently:

$$[Y(s)] = (s[I]-[A])^{-1}\cdot[B]\cdot[U(s)], \quad (5)$$

which is the input-output operational equation for the analyzed multi-variable system.
The transfer matrix is obtained from this equation:

$$[H(s)] = (s[I]-[A])^{-1}\cdot[B] = \frac{adj(s[I]-[A])}{\det(s[I]-[A])}\cdot[B], \quad (6)$$

where I is the unit matrix.
The transfer function denominator will have the form:

$$n(s) = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4, \quad (7)$$

with

$$a_0 = \frac{1}{T_s^2 T_r^2 \sigma^2} + \frac{\omega_s^2}{T_r^2\sigma^2} + \frac{\omega_r^2}{T_s^2\sigma^2} + 2\frac{\omega_s\omega_r}{T_s T_r\sigma^2}(1-\sigma)$$

$$a_1 = \frac{2}{T_s T_r \sigma^2}\cdot\left(\frac{1}{T_s}+\frac{1}{T_r}\right) + \frac{2\omega_r}{T_s\sigma} + \frac{2\omega_s}{T_r\sigma}, \quad (8)$$

$$a_2 = \omega_s^2 + \omega_r^2 + 2\frac{\sigma+1}{T_s T_r \sigma^2} + \frac{1}{T_s^2\sigma^2} + \frac{1}{T_r^2\sigma^2}$$

$$a_3 = \frac{2}{\sigma}\left(\frac{1}{T_s}+\frac{1}{T_r}\right),$$

$$a_4 = 1.$$

Further on the following approximation will be used:

$$\mu = \frac{1}{2\sigma}\left(\frac{1}{T_s}+\frac{1}{T_r}\right) \cong \frac{1}{T_s\sigma} \cong \frac{1}{T_r\sigma}. \quad (9)$$

For a concrete case of an induction motor ($R_s$ =7,5 $\Omega$, $R^{/}_r$=5,5 $\Omega$, $L_s = 0{,}529$ H, $L^{/}_r = 0{,}528$ H, $L_{sh} = 0{,}498$ H) it is obtained:

$$\sigma = 0{,}112, \qquad \mu = 110. \quad (10)$$

In order to determine the transfer functions poles for the studied system, their denominator will be made zero:

$$n(s) = 0, \quad (11)$$

with n(s) given by the relations (7) and (8).
By solving this equation the following poles have been obtained:

a)If $\mu \geq \frac{\omega}{2\sqrt{1-\sigma}}$,

$$s_{1,2,3,4} = -\mu \pm \sqrt{\mu^2(1-\sigma) - \frac{\omega^2}{4}} \pm j(\omega + 2\omega_r)/2; \quad (12)$$

b) If $\mu < \frac{\omega}{2\sqrt{1-\sigma}}$,

$$s_{1,2,3,4} = -\mu \pm j\left[\sqrt{\frac{\omega^2}{4} - \mu^2(1-\sigma)} \pm (\omega + 2\omega_r)/2\right]. \quad (13)$$

As one can observe, in the relation (12), $\mu > \sqrt{\mu^2(1-\sigma) - \frac{\omega^2}{4}}$, irrespective of the machine parameters or the operation point. So, the real part of the transfer functions poles is always negative. In conclusion, the studied system is always stable.
Since the inductances $L_s$ and $L^{/}_r$ are inverse proportional with μ, these ones have a non-stabilizing effect on the induction motor operating at variable frequency.

## 3 A New Method for Stability Analysis

This method, detailed in [2], has as starting point the following relations, written in per unit values:

$$hs\cdot\Delta\omega^* = -k\Delta i^{/*}_{dr}, \quad (14)$$

$$\Delta i_{dr}^{/} = \frac{s + j\omega_s^* + \varepsilon}{s^2 + (s_{ks} + s_{kr} + j\omega_s^*)s + s_{kr}(\varepsilon + j\omega_s^*)} \cdot$$

$$\cdot k(\Delta\omega^* - \Delta\omega_s^*), \qquad (15)$$

where

$$\varepsilon = (1 - k^2)s_{ks} = \frac{r_s^*}{x_s^*} = \frac{r_s^*}{x_r^{*/}}. \qquad (16)$$

At the same time it must be mentioned that the per unit quantities used in the previous equations have been noted with " * ", the quantities which are present here having the significance from [2].

The equation (15) may also be written in the following form:

$$\Delta\omega^* = -\frac{k}{hs} \cdot \Delta i_{dr}^{/*}, \qquad (17)$$

or, equivalently:

$$\Delta\omega^* = G_1(s) \cdot \Delta i_{dr}^{/*}, \text{ with } G_1(s) = -\frac{k}{hs}. \qquad (18)$$

Similarly, the equation (16) becomes:

$$\Delta i_{dr}^{/*} = G_2(s) \cdot (\Delta\omega_s^* - \Delta\omega^*), \qquad (19)$$

where

$$G_2(s) = -\frac{s + j\omega_s^* + \varepsilon}{s^2 + (s_{ks} + s_{kr} + j\omega_s^*)s + s_{kr}(\varepsilon + j\omega_s^*)} \cdot k \qquad (20)$$

The previous relations lead to the equivalent scheme of the induction machine operating at variable frequency.

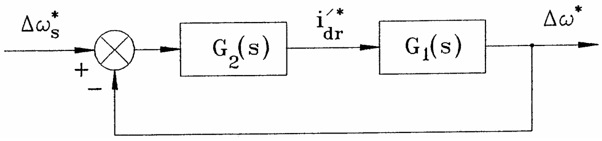
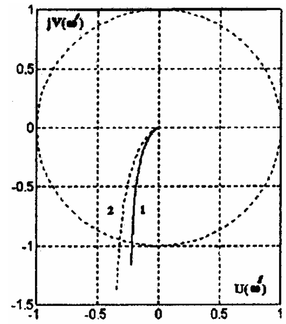



Fig.1. Block scheme of the machine in the mentioned situation.

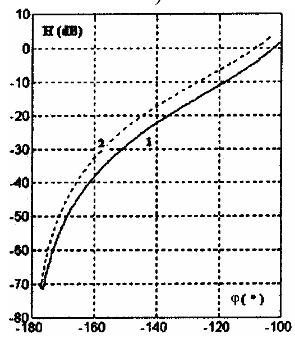
# 4 Results

A MATLAB program for the stability analysis has been conceived, by using the previous scheme. The representations from the figures 2, 3 and 4 have been obtained by running this program, for the concrete case of a motor rated at 1,1 kW.

There must be also mentioned the importance of the introduced method resulting from the possibility to emphasize the machine parameters influence and especially the inertia moment influence, on the stability when operating at variable frequency, fact that provides originality to this method.

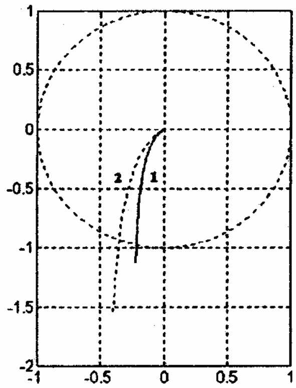


a)



b)

Fig.2. Transfer locus (a) and amplitude-phase characteristics (b) obtained in the case of the inductances modification: $L_s = 0{,}529$ H (1) and $L_s = 0{,}549$ H (2);



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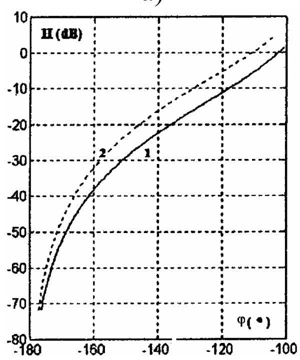
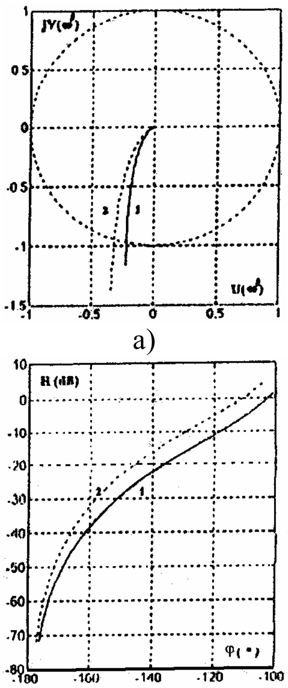


b)

Fig.3. Transfer locus (a) and amplitude-phase characteristics (b) obtained in the case of the inductances modification: $L_r^{/} = 0{,}528$ H (1) and $L_r^{/} = 0{,}548$ H (2);




a)

b)

Fig.4. Transfer locus (a) and amplitude-phase characteristics (b) obtained in the case of the inductances modification: $L_{sh} = 0{,}498$ H (1) and $L_{sh} = 0{,}558$ H (2).

In order to catch quantitatively these interdependences the following tables have been filled.

Table

| Param. | Absolute value | Per unit param. | Per unit value | Phase Margin [$^0$] |
|---|---|---|---|---|
| $L_s$ | 0,529 | $x_s^*$ | 2,1907 | 75,54 |
| | 0,549 | | 2,2735 | 69,13 |
| $L_r^/$ | 0,528 | $x_r^{/*}$ | 2,1865 | 75,54 |
| | 0,548 | | 2,2694 | 75,31 |
| $L_{sh}$ | 0,498 | $x_{1m}^*$ | 2,0623 | 75,54 |
| | 0,558 | | 2,3108 | 71,32 |

## 4 Conclusion

These results help us to emphasize a few important conclusions regarding the resistances influence on the studied system stability:

- the increase of the inductance $L_s$ leads to the stability decrease;
- at the same time with the rotor inductance increase the system stability decreases;
- the increase of the main inductance has a stabilizing effect.

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