Structural Stability of Polynomial Matrices Related to Linear Time-Invariant Singular Systems

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Abstract:- We consider the set of quadruples of matrices defining singular linear time-invariant dynamical systems and show that there is a one-to-one correspondence between this set and a subset of the set of polynomial matrices of degree two. This correspondence preserves the equivalence relations introduced in both sets (feedback-similarity and strict equivalence): two quadruples of matrices are feedback-equivalent if, and only if, the polynomial matrices associated to them are also strictly equivalent. We characterize structurally stable polynomial matrices (stable elements under small perturbations) describing singular systems and derive a lower bound on the distance to the orbits of polynomial matrices with strictly lower dimension.

Key-Words:- singular systems, polynomial matrices, structural stability.

1 Introduction

Singular systems arise in a number of scientific and engineering applications. For example, when modelling multibody systems in robotics, in electrical circuit simulation, in models of chemical processes, in fluid dynamics, in optimal control theory, in the study of Hamiltonian systems with constraints, in the analysis of stiff differential equations, etc.

The study of singular systems began at the end of the 1970s and attracted the interest of many scientists, who studied not only the generalization of classical system theory results but also specific ones. See, for example, [5] for a complete survey on singular systems, or [2], where a behavioral approach can be found.

Linear time-invariant singular systems may be described by

$$\left\{ \begin{array}{rl} E\dot{x}(t) & = Ax(t) + Bu(t) \\ y(t) & = Cx(t) \end{array} \right. \qquad (1)$$

where $E, A \in \mathcal{M}_n(\mathbb{C})$, $B \in \mathcal{M}_{n\times m}(\mathbb{C})$, $C \in \mathcal{M}_{p\times n}(\mathbb{C})$ and $\operatorname{rk} E < n$.

It is well known that the behavior of the system (1) depends on the properties of the matrix pencil $\alpha E - \beta A$ obtained as a natural generalization of the matrix pencil $\lambda I - A$ considered for standard systems. System (1) and the corresponding matrix pencil are called regular if $\det(\alpha E - \beta A) \neq 0$ for some $(\alpha, \beta) \in \mathbb{C}^2$.

The necessary and sufficient condition for the existence and uniqueness of the solution to the system (1) is regularity. Many of the results on the behavior of singular systems depend on the assumption of regularity. When a system is not regular, conditions may be studied to know whether the system can be *regularized*; that is to say, it can be transformed into a uniquely solvable closed loop system (for any given control and consistent initial values).

The matrix pencil $\alpha E - \beta A$ does not allow to know whether the system is regularizable. Let us consider, for example, the system

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t)$$

This system is regularizable, since the feedback $u(t) = y(t) + w(t)$ yields the close loop system

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} w(t)$$

which is uniquely solvable for every consistent initial condition and any control $w(t)$. But the matrix pencil $\det(\alpha E - \beta A)$ is not regular, since $\det(\alpha E - \beta A) = 0$, for all $(\alpha, \beta) \in \mathbb{C}^2$. We conclude that this matrix pencil is not the most suitable to obtain information about regularizable conditions. In [3], sufficient regularizability conditions are presented using state and derivative feedbacks (regularity of the matrix pencil $\det(\alpha E - \beta A)$ is not required).

In this work, we will consider an alternative approach, associating to the quadruples which define singular systems a special type of polynomial matrices of degree two and introducing in the space of polynomial matrices the concept of "strict equivalence" which generalizes, in the natural way, the notion of strict equivalence of matrix pencils.

The notion of structural stability has become a central one in the study of systems because of its practical importance. When a dynamical system is structurally stable, small perturbations (like those introduced by a numerical approximation when simulating, or noise in an experiment) will not qualitatively alter the observed dynamics.

We will consider the concept of structural stability as appears in [9] and characterize structurally stable polynomial matrices related to singular systems.

In this paper we will make use of geometrical techniques, similar to those which are used in [1]: strict equivalence will be viewed as the equivalence relation defined on the differentiable manifold of polynomial matrices of degree two related to singular systems by the action of a Lie group acting on it, giving rise to orbits which are in turn also differentiable manifolds and the tangent spaces to orbits will be described.

2 Polynomial matrices related to singular linear systems

It is known that equivalent triples of matrices defining regular systems, under the equivalence relation derived from: basis changes in the state, input and output spaces, state feedback and output injection, are those having strictly equivalent associated matrix pencils (see [8], for example). In this Section we show a similar result in the case of quadruples of matrices defining singular systems.

Let us now consider a linear time-invariant singular dynamical system described by

$$\begin{cases} E\dot{x}(t) &= Ax(t) + Bu(t) \\ \quad y(t) &= Cx(t) \end{cases}$$

where $E, A \in M_n(\mathbb{C})$, $B \in M_{n\times m}(\mathbb{C})$, $C \in M_{p\times n}(\mathbb{C})$ and $\operatorname{rk} E < n$.

We will consider the following elementary transformations: basis similarity for the state space, basis changes for the control space, basis changes for the output space, output injection, state feedback, derivative feedback.

These transformations lead to the definition of the following equivalence relation on the space of quadruples of matrices defining singular linear time-invariant dynamical systems.

Definition 1 *The quadruples* (E_1, A_1, B_1, C_1), (E_2, A_2, B_2, C_2) *are feedback-similar if, and only if, there exist matrices* $P \in Gl_n(\mathbb{C})$, $R \in Gl_m(\mathbb{C})$, $S \in Gl_p(\mathbb{C})$, $U, V \in M_{m\times n}(\mathbb{C})$ *and* $W \in M_{n\times p}(\mathbb{C})$ *such that*

$$\begin{pmatrix} P & W \\ 0 & Q \end{pmatrix}\begin{pmatrix} E_1 & A_1 & B_1 \\ 0 & C_1 & 0 \end{pmatrix}\begin{pmatrix} P^{-1} & 0 & 0 \\ 0 & P^{-1} & 0 \\ V & U & S \end{pmatrix} = \begin{pmatrix} E_2 & A_2 & B_2 \\ 0 & C_2 & 0 \end{pmatrix}$$

That is to say, two quadruples of matrices are said to be feedback-similar when one can be obtained from the other by means of one, or more, of the elementary transformations (1) - (6) above.

This equivalence relation generalizes block-similarity of pairs of matrices and equivalence of triples. In particular, if (E_1, A_1, B_1, C_1), (E_2, A_2, B_2, C_2) are feedback- similar, the pairs (E_1, B_1) and (E_2, B_2) are block-similar and the triples (A_1, B_1, C_1) and (A_2, B_2, C_2) are equivalent (we referred to these equivalence relations at the Introduction).

We associate to each quadruple describing a singular system a polynomial matrix of degree two and introduce in the set of polynomial matrices an equivalence relation, *strict equivalence*, which generalizes strict equivalence for matrix pencils.

Definition 2 *We will say that two polynomial matrices $M_1(\lambda)$ and $M_2(\lambda)$ are strictly equivalent when there exist constant regular matrices L and R such that $LM_1(\lambda)R = M_2(\lambda)$.*

Let us associate to the quadruple (E, A, B, C) the polynomial matrix

$$M(\lambda) = \begin{pmatrix} E & A & B \\ 0 & C & 0 \end{pmatrix} + \lambda \begin{pmatrix} I & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \lambda^2 \begin{pmatrix} 0 & I & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

We will denote by $\mathcal{P}$ the set of polynomial matrices of degree two and by $\mathcal{M} \subset \mathcal{P}$ the subset of polynomial matrices of the type above.

We can state now the main result in this Section.

Theorem 1 *(E_1, A_1, B_1, C_1) and (E_2, A_2, B_2, C_2) are feedback-similar if, and only if, $M_1(\lambda)$ and $M_2(\lambda)$ are strictly equivalent.*

Proof Let us assume (E_1, A_1, B_1, C_1) and (E_2, A_2, B_2, C_2) feedback-similar. Then there exist $P \in Gl_n(\mathbb{C})$, $R \in Gl_m(\mathbb{C})$, $S \in Gl_p(\mathbb{C})$, $U, V \in M_{m\times n}(\mathbb{C})$ and $W \in M_{n\times p}(\mathbb{C})$ such that

$$\begin{pmatrix} P & W \\ 0 & Q \end{pmatrix} \begin{pmatrix} E_1 & A_1 & B_1 \\ 0 & C_1 & 0 \end{pmatrix} \begin{pmatrix} P^{-1} & 0 & 0 \\ 0 & P^{-1} & 0 \\ V & U & S \end{pmatrix} = \begin{pmatrix} E_2 & A_2 & B_2 \\ 0 & C_2 & 0 \end{pmatrix}$$

It suffices to take

$$L = \begin{pmatrix} P & W \\ 0 & Q \end{pmatrix}, \; R = \begin{pmatrix} P^{-1} & 0 & 0 \\ 0 & P^{-1} & 0 \\ V & U & S \end{pmatrix}$$

Conversely. We assume that

$$L\left[\begin{pmatrix} E_1 & A_1 & B_1 \\ 0 & C_1 & 0 \end{pmatrix} + \lambda \begin{pmatrix} I & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \lambda^2 \begin{pmatrix} 0 & I & 0 \\ 0 & 0 & 0 \end{pmatrix}\right] R =$$

$$\begin{pmatrix} E_2 & A_2 & B_2 \\ 0 & C_2 & 0 \end{pmatrix} + \lambda \begin{pmatrix} I & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \lambda^2 \begin{pmatrix} 0 & I & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where

$$L = \begin{pmatrix} L_1 & L_2 \\ L_3 & L_4 \end{pmatrix}, \; R = \begin{pmatrix} R_1 & R_2 & R_3 \\ R_4 & R_5 & R_6 \\ R_7 & R_8 & R_9 \end{pmatrix}$$

From this equality we conclude that L_1 is regular, $R_1 = {L_1}^{-1}$, $L_3 = 0$, $R_2 = 0$, $R_3 = 0$, $R_4 = 0$ and $R_5 = 0$.

Therefore (E_1, A_1, B_1, C_1) and (E_2, A_2, B_2, C_2) are feedback-similar.

3 Characterization of structurally stable polynomial matrices in $\mathcal{M}$

One of the goals of this work is to characterize structurally stable polynomial matrices related to singular systems. The effect of small perturbations in the parameters, structural stability and sensibility analysis are some of the most studied problems in Control Theory. The relevance of this concept is easy to understand: when a system is not structurally stable, small perturbations may lead to great changes in the behavior of the system. This is specially important when considering families of systems depending on parameters.

First we will recall the notion of structural stability as introduced by Willems.

Definition 2 *If $\mathcal{M}$ is a topological space and $\sim$ is an equivalence relation defined on $\mathcal{M}$, $x \in \mathcal{M}$ is structurally stable when there exists a neighborhood $\mathcal{U}$ of x such that $x \sim y$ for all $y \in \mathcal{U}$.*

Remark 1 In the case where $\mathcal{M}$ is a differentiable manifold and $\sim$ is the equivalence relation defined by the action of a Lie group acting on $\mathcal{M}$, giving rise to orbits which are also differentiable manifolds, if we denote by $\mathcal{O}(x)$ the orbit of an element $x \in X$, we have that the following statements are equivalent:

(a) x is structurally stable.

(b) $\mathcal{O}(x)$ is an open submanifold.

(c) $\dim \mathcal{O}(x) = \dim \mathcal{M}$.

(d) $\dim T_x\mathcal{O}(x)^\perp = 0$ (for any Hermitian product considered).

Note that the set $\mathcal{P}$ of polynomial matrices of degree two and order $(n+p)\times(2n+m)$, and the subset $\mathcal{M}$ of $\mathcal{P}$ consisting of polynomial matrices of degree two associated to singular linear time-invariant dynamical systems are differentiable manifolds.

It is easy to check that the orbits under the action of the Lie group

$$\mathcal{G} = Gl_{n+p}(\mathbb{C}) \times Gl_{2n+m}(\mathbb{C})$$

acting on $\mathcal{P}$ via

$$\alpha : \mathcal{G} \times \mathcal{P} \longrightarrow \mathcal{P}$$

$$((L,R), M(\lambda)) \longrightarrow LM(\lambda)R$$

coincide with equivalence classes under strict equivalence.

Using elementary theory of Lie groups, it is straightforward to see that $T_{M(\lambda)}\mathcal{O}(M(\lambda))$ is the set $\{LM(\lambda)+M(\lambda)R$ with $L \in M_{n+p}(\mathbb{C})$, $R \in M_{2n+m}(\mathbb{C})\}$. The orthogonal spaces to the tangent spaces to the orbits would be easier to calculate, identifying them with the set of solutions of suitable linear equations systems.

We consider in $\mathcal{P}$ the natural Hermitian inner product deduced after to identify $\mathcal{P}$ with $\mathbb{C}^{(2n+m)(n+p)}$.

$$< M(\lambda), N(\lambda) >= \operatorname{tr}(M_0\bar{N}_0^t)+\operatorname{tr}(M_1\bar{N}_1^t)+\operatorname{tr}(M_2\bar{N}_2^t)$$

if $M(\lambda) = M_0 + \lambda M_1 + \lambda^2 M_2$, $N(\lambda) = N_0 + \lambda N_1 + \lambda^2 N_2$.

Theorem 2 *Let $M(\lambda) = M_0 + \lambda M_1 + \lambda^2 M_2 \in \mathcal{P}$. Then $N(\lambda) = X + \lambda Y + \lambda^2 Z$ is an element of the orthogonal to the tangent space to the orbit, with respect to the Hermitian product considered above, if and only if,*

$$\left.\begin{array}{rl} M_0\bar{X}^t + M_1\bar{Y}^t + M_2\bar{Z}^t & = 0 \\ \bar{X}^t M_0 + \bar{Y}^t M_1 + \bar{Z}^t M_2 & = 0 \end{array}\right\}$$

Proof It follows from direct calculation taking into account the description of the tangent space to the orbit given above and the Hermitian product defined on $\mathcal{M}$.

If $M(\lambda) \in \mathcal{M}$ we can restrict the orthogonal to $\mathcal{M}$. For that, we restrict the action to the subgroup $\mathcal{G}_0 = \{(L,R) \in \mathcal{P}\}$ with

$$L = \begin{pmatrix} P & W \\ 0 & Q \end{pmatrix}, \; R = \begin{pmatrix} P^{-1} & 0 & 0 \\ 0 & P^{-1} & 0 \\ V & U & S \end{pmatrix}$$

it acts over $\mathcal{M}$ via

$$\alpha_0 : \mathcal{G}_0 \times \mathcal{M} \longrightarrow \mathcal{M}$$

$$((L,R), M(\lambda)) \longrightarrow LM(\lambda)R$$

we can state the following result.

Theorem 3 *If*

$$M(\lambda) = \begin{pmatrix} E & A & B \\ 0 & C & 0 \end{pmatrix} + \lambda \begin{pmatrix} I & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \lambda^2 \begin{pmatrix} 0 & I & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

is a matrix polynomial of degree two associated to a singular system, then

$$\begin{pmatrix} X & Y & Z \\ 0 & T & 0 \end{pmatrix} + \lambda \begin{pmatrix} I & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \lambda^2 \begin{pmatrix} 0 & I & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

is an element of the orthogonal to the tangent space to the orbit, with respect to the Hermitian product considered above, if, and only if,

$$\left.\begin{array}{rl} E\bar{X}^t - \bar{X}^t E + A\bar{Y}^t - \bar{Y}^t A + B\bar{Z}^t - \bar{T}^t C & =0 \\ \bar{X}^t B & =0 \\ \bar{Y}^t B & =0 \\ C\bar{Y}^t & =0 \\ \bar{Z}^t B & =0 \\ C\bar{T}^t & =0 \end{array}\right\}.$$

As consequences, we obtain the following characterizations of structurally stable polynomial matrices in $\mathcal{P}$ as well as in $\mathcal{M}$.

Corollary 1 *A polynomial matrix in $\mathcal{P}$, $M(\lambda) = M_0 + \lambda M_1 + \lambda^2 M_2$ is structurally stable under strict equivalence if, and only if, the system*

$$\left.\begin{array}{rl} M_0 X_1 + M_1\ Y_1 + M_2 Z_1 & = 0 \\ X_1 M_0 + Y_1 M_1 + Z_1 M_2 & = 0 \end{array}\right\}$$

has no non-trivial solutions.

Proof One only needs to take into account that $M(\lambda)$ is structurally stable under strict equivalence if, and only if, $\dim \mathcal{O}(M(\lambda))^\perp = 0$.

Corollary 2 *A polynomial matrix $M(\lambda) = M_0 + \lambda M_1 + \lambda^2 M_2$ in $\mathcal{P}$ is structurally stable under strict equivalence if, and only if,*

$$\operatorname{rank} \mathbf{m}_{\mathcal{P}}(M(\lambda)) = 2n^2 + mn + 2np + mp$$

where $\mathbf{m}_{\mathcal{P}}(M(\lambda))$ is the following matrix

$$\mathbf{m}_{\mathcal{P}}(M(\lambda)) = \begin{pmatrix} I_{n+p} \otimes M_0 & I_{n+p} \otimes M_1 & I_{n+p} \otimes M_2 \\ M_0^t \otimes I_{2n+m} & M_1^t \otimes I_{2n+m} & M_2^t \otimes I_{2n+m} \end{pmatrix}$$

Proof It suffices to apply the vec-operator and the Kronecker product to the equations of $T_{M(\lambda)}\mathcal{O}(M(\lambda))^{\perp}$.

Corollary 3 *A polynomial matrix of degree two $M(\lambda)$ associated to a singular system (E, A, B, C) is structurally stable under strict equivalence if, and only if, the system*

$$\begin{cases} EX_1 - X_1E + AY_1 - Y_1A + BZ_1 - T_1C & = 0 \\ X_1B & = 0 \\ Y_1B & = 0 \\ CY_1 & = 0 \\ Z_1B & = 0 \\ CT_1 & = 0 \end{cases}$$

has no non-trivial solutions.

Corollary 4 *A polynomial matrix of degree two $M(\lambda)$ associated to a singular system (E, A, B, C) is structurally stable under strict equivalence if, and only if,*

$$\operatorname{rank} \mathbf{m}_{\mathcal{M}}(M(\lambda)) = 2n^2 + mn + np$$

where $\mathbf{m}_{\mathcal{M}}(M(\lambda))$ is the following matrix

$$\begin{pmatrix} E^t \otimes I - I \otimes E & 0 & 0 & -I \otimes B & 0 & 0 \\ A^t \otimes I - I \otimes A & 0 & 0 & 0 & -I \otimes B & C^t \otimes I \\ B^t \otimes I & -I \otimes B & 0 & 0 & 0 & 0 \\ -I \otimes C & 0 & C^t \otimes I & 0 & 0 & 0 \end{pmatrix}^t$$

Remark The structurally stable character of a two degree polynomial matrix $M(\lambda) \in \mathcal{M}$ associated to a singular system (E, A, B, C) ensures that in a neighborhood of this system all systems in it, are normalizable, stabilizable and detectable.

4 A lower bound on the distance to a polynomial matrix with orbit of strictly lower dimension

In this last Section, we will obtain a bound for the distance between a structurally stable polynomial matrix associated to a singular system and the nearest one not being structurally stable. That is to say, a bound for the value of the radius of a ball which is a neighborhood of a structurally stable polynomial matrix containing only elements which are also structurally stable.

More concretely, we will obtain a bound for the distance between a polynomial matrix in $\mathcal{M}$ and the nearest polynomial matrix in $\mathcal{M}$ having an orbit of strictly lower codimension, with respect to strict equivalence. To do this, we will proceed in a similar way to that in [6], in the case of matrix pencils related to pairs of matrices under block-similarity, or that in [4].

The starting point is the relationship between the polynomial matrix $M(\lambda)$ and the Frobenius norm of the matrices $\mathbf{m}_{\mathcal{P}}(M(\lambda))$ or $\mathbf{m}_{\mathcal{M}}(M(\lambda))$ (for the definitions on norms of matrices see, for example, [7]), given in terms of a constant depending only upon the order of the matrices defining the system.

Theorem 4 *For a given polynomial matrix $M(\lambda) \in \mathcal{P}$ (or $\in \mathcal{M}$ with orbit of codimension d, a lower bound on the distance to the nearest polynomial matrix $N(\lambda)$ having an orbit of strictly greater codimension is given by*

$$\frac{1}{\sqrt{3n + m + p}} \sigma_{2n^2+mn+2np+mp-d}(\mathbf{m}_{\mathcal{P}}(M(\lambda))),$$

$$(\textit{or } \frac{1}{\sqrt{3n + m + p}} \sigma_{2n^2+mn+np-d}(\mathbf{m}_{\mathcal{M}}(M(\lambda))))$$

where $\sigma_1 \geq \cdots \geq \sigma_{2n^2+mn+2np+mp}$, (or $\sigma_1 \geq \cdots \geq \sigma_{2n^2+mn+np}$) denote the non-zero singular values of $\mathbf{m}_{\mathcal{P}}(M(\lambda))$, ($\mathbf{m}_{\mathcal{M}}(M(\lambda))$).

Proof The smallest perturbation in the Frobenius norm that reduces the rank of $\mathbf{m}_{\mathcal{P}}(M(\lambda))$ from $2n^2 + mn + 2np + mp - d$ to $2n^2 + mn + 2np + mp - d - \alpha$ (or that reduces the rank of $\mathbf{m}_{\mathcal{M}}(M(\lambda))$ from $2n^2+mn+np-d$ to $2n^2+mn+np-d-\alpha$)is:

$$\sqrt{\sum_{i=2n^2+mn+2np+mp-d-\alpha+1}^{2n^2+mn+2np+mp-d} \sigma_i^2(\mathbf{m}_{\mathcal{P}}(M(\lambda)))} \geq \sigma_{2n^2+mn+2np+mp-d}(\mathbf{m}_{\mathcal{P}}(M(\lambda)))$$

(or

$$\sqrt{\sum_{i=2n^2+mn+np-d-\alpha+1}^{2n^2+mn+np-d} \sigma_i^2(\mathbf{m}_{\mathcal{M}}(M(\lambda)))} \geq \sigma_{2n^2+mn+np-d}(\mathbf{m}_{\mathcal{M}}(M(\lambda)))$$

) (Eckart-Young-Mirsky Theorem). Then we only need to apply the following (not difficult to prove) relationship: $\|\mathbf{m}_{\mathcal{P}}(M(\lambda))\|_F^2 = (3n+m+p)\|M(\lambda)\|_F^2$, (or $\|\mathbf{m}_{\mathcal{M}}(M(\lambda))\|_F^2 \leq (3n+m+p)\|M(\lambda)\|_F^2$).

As a consequence, we obtain a bound for the distance between a structurally stable polynomial matrix in $\mathcal{M}$ under strict equivalence and the nearest non-structurally stable one in $\mathcal{M}$.

Corollary 5 *For a given structurally stable polynomial matrix $M(\lambda) \in \mathcal{P}$ (or $\in \mathcal{M}$ a lower bound on the distance to the nearest non-structurally stable polynomial matrix $N(\lambda)$ is given by*

$$\frac{1}{\sqrt{3n+m+p}}\sigma_{2n^2+mn+np}(\mathbf{m}_{\mathcal{P}}(M(\lambda)))$$

(or

$$\frac{1}{\sqrt{3n+m+p}}\sigma_{2n^2+mn+np}(\mathbf{m}_{\mathcal{M}}(M(\lambda)))$$

) where $\sigma_{2n^2+mn+np}$ denotes the smallest non-zero singular value of $\mathbf{m}_{\mathcal{P}}(M(\lambda))$, (or $\mathbf{m}_{\mathcal{M}}(M(\lambda))$).

5 Conclusion

In this work a controllability matrix for second order generalized linear systems in the form $E\ddot{x} = A_1\dot{x}+A_2x+Bu$, is presented. This matrix depends only on the matrices E, A_1, A_2 and B defining the system and gives us a simple method to analyze the controllability of the system.

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