Genesis and Catastrophe of the Chaotic Double-Bell Attractor

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Abstract: - In this paper a 4th order, nonlinear and non-autonomous electric circuit has been studied. We have investigated the low frequency response of the circuit, both theoretically and experimentally. The circuit is based on the 3rd order Chua's canonical circuit and is driven by a sinusoidal voltage source. We have studied the dynamics of the circuit for different amplitude values of the input signal $v_s(t)$ in the low frequency area, which will lead us in future investigation of circuit's application in secure communication systems.

Key-Words: - Non linear circuit, Chaos, Strange attractors, Secure communication systems.

1 Introduction

Chaos is a phenomenon that occurs widely in dynamical systems. It is a noise like phenomenon that occurs due to the inevitable nonlinearity of physical systems. Many scientists have observed chaotic behaviors in different areas of science, such as in meteorology, physics, mathematics, et. al. Theoretical study was much easier than the experimental one, because of the difficulty in searching of chaotic behaviours.

In recent papers [1, 2] we have studied the dynamics of a 4th order autonomous electronic circuit with a nonlinear resistor and a negative conductance. The antimonotonicity, the formation of "bubbles" in the bifurcation region as well as the chaotic behaviour of this circuit were reported in two recent publications [3, 4].

The low frequency response of a 4th order non autonomous, nonlinear electronic circuit has been studied here. The electronic circuit consists of two active elements, one linear negative conductance and one nonlinear resistor exhibiting a symmetrical piecewise linear v-i characteristic of N-type. The circuit contains also two capacitances C_1 and C_2, two inductances L_1 and L_2 and a sinusoidal input source $v_s(t)$.

Due to the advantages which electric and electronic circuits offer to experimental chaos studies, such as robustness and convenient implementation, most chaotic and bifurcation effects cited in the literature, have been observed in such circuits. Such circuits exhibit among others the period-doubling route to chaos [5, 6], the intermittency route to chaos [7, 8], the quasiperiodicity route to chaos [9, 10] and of course the crisis [11, 12].

2 The 4th Order Implemented Circuit

The circuit, we have studied, is shown in Fig.1, while in Figs.2 and 3 we can see the v-i characteristic of the nonlinear resistor and negative conductance, respectively.

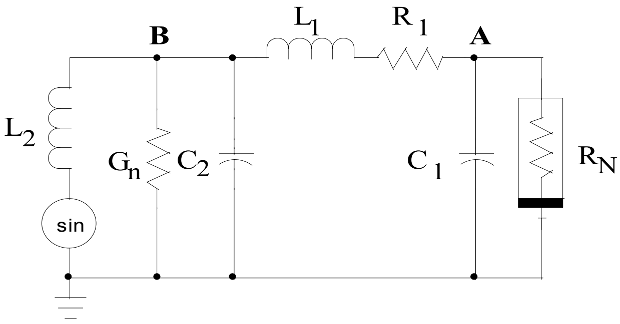

Fig.1. The 4th order non autonomous electronic circuit.

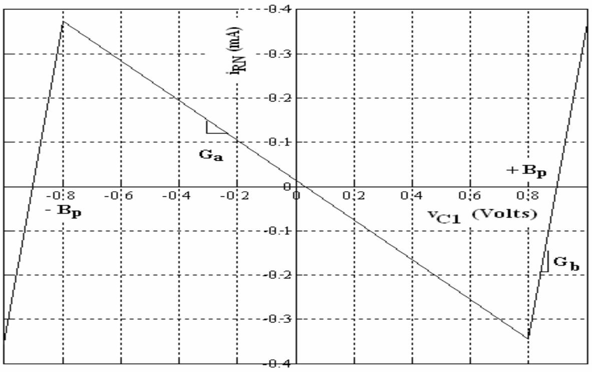

Fig.2. v-i characteristic of the N-type nonlinear resistor R_N.

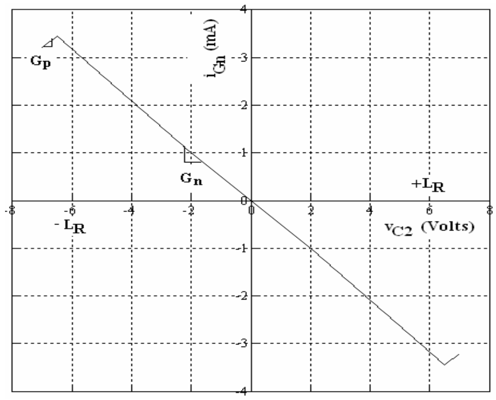

Fig.3. v-i characteristic of the negative conductance.

The state equations of the circuit are:

$$\frac{dv_{C1}}{dt}=\frac{1}{C_1}\left(i_{L1}-i\right) \tag{1}$$

$$\frac{dv_{C2}}{dt}=-\frac{1}{C_2}\left(G_n\cdot v_{C2}+i_{L1}+i_{L2}\right) \tag{2}$$

$$\frac{di_{L1}}{dt}=\frac{1}{L_1}\left(v_{C2}-v_{C1}-R_1 i_{L1}\right) \tag{3}$$

$$\frac{di_{L2}}{dt}=\frac{1}{L_2}\left(v_{C2}-R_2 i_{L2}-v_s(t)\right) \tag{4}$$

where the nonlinear function i is described by

$$i=g(v_{C1})=G_b v_{C1}+0.5(G_a-G_b)(|v_{C1}+B_p|-|v_{C1}-B_p|) \tag{5}$$

and $v_s(t)$ is the input sinusoidal signal.

The parameters of the circuit are: L_1 =100mH, L_2 =100mH, R_{L1} =101.0Ω, R_{L2} =101.2Ω, C_1 =33nF, C_2 =75nF, R_1 =1KΩ, G_n =-0.50mS =-G_p, L_R =7.5V, G_a=-0.35mS, G_b=5.0mS and B_p=0.8V. All the above parameters are consider unchangeable during the study that follows.

3 Computer Simulation Results

The theoretical-simulation phase portraits v_{C2} vs. v_{C1} for f =30Hz and f =35Hz and various values of the amplitude of input sinusoidal signal are shown in paragraphs 3.1 and 3.2, respectively.

3.1 Simulation Results for f = 30Hz

In Figs.4-10 we can see the theoretical phase portraits v_{C2} vs. v_{C1} for f =30Hz and various values of the amplitude V_o of the input sinusoidal signal $v_s(t)$.

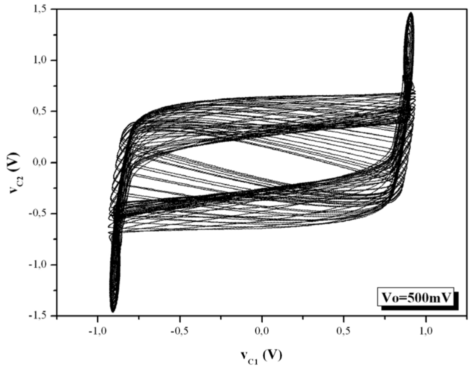

Fig.4. Theoretical v_{C2} vs. v_{C1} for V_o=500mV.

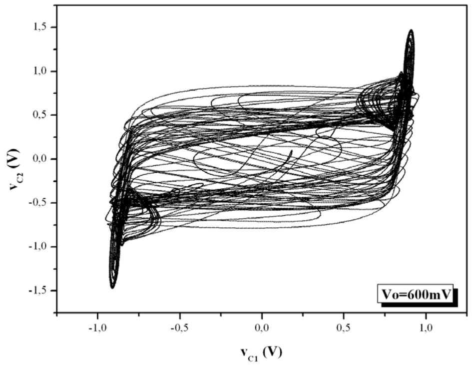

Fig.5. Theoretical v_{C2} vs. v_{C1} for V_o=600mV.

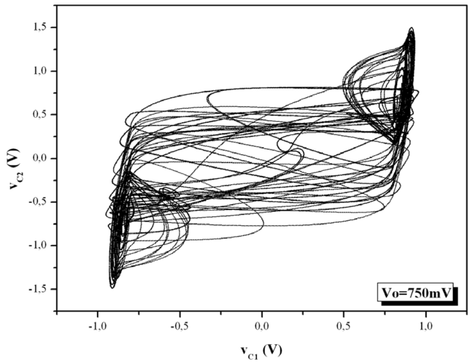

Fig.6. Theoretical v_{C2} vs. v_{C1} for V_o=750mV.

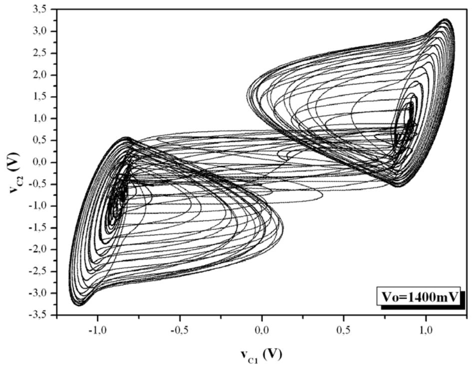

Fig.7 Theoretical v_{C2} vs. v_{C1} for V_o=1400mV.

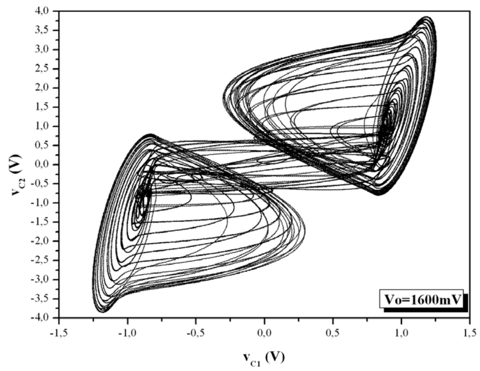

Fig.8. Theoretical v_{C2} vs. v_{C1} for V_o=1600mV.

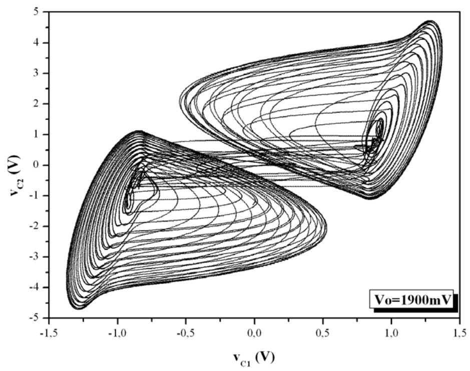

Fig.9. Theoretical v_{C2} vs. v_{C1} for V_o=1900mV.

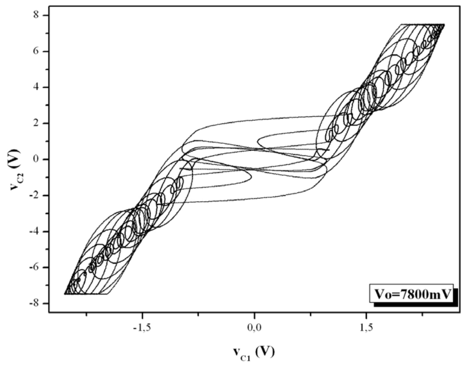

Fig.10. Theoretical v_{C2} vs. v_{C1} for V_o=7800mV.

3.2 Simulation Results for f = 35Hz

In Figs.11-16 we observe the theoretical phase portraits v_{C2} vs. v_{C1} for f = 35Hz and various values of the amplitude V_o of the sinusoidal signal $v_s(t)$.

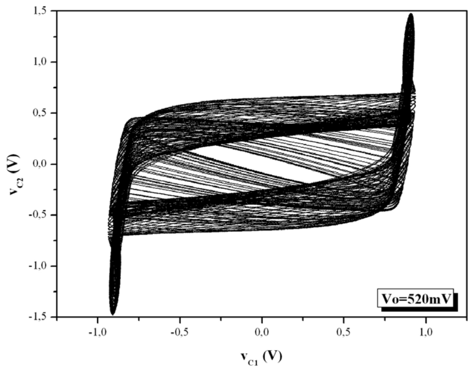

Fig.11. Theoretical v_{C2} vs. v_{C1} for V_o=520mV.

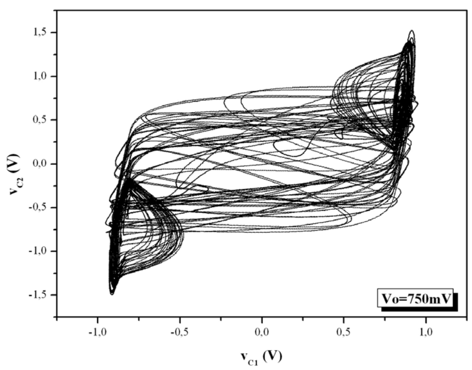

Fig.12. Theoretical v_{C2} vs. v_{C1} for V_o=750mV.

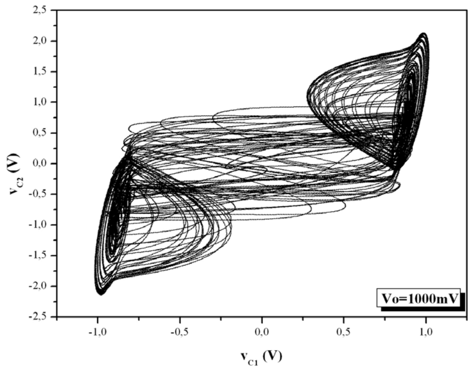

Fig.13. Theoretical v_{C2} vs. v_{C1} for V_o=1000mV.

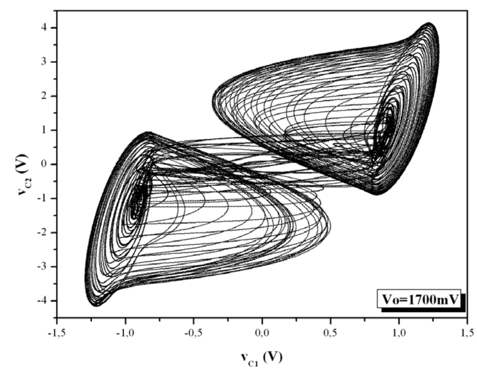

Fig.14. Theoretical v_{C2} vs. v_{C1} for V_o=1700mV.

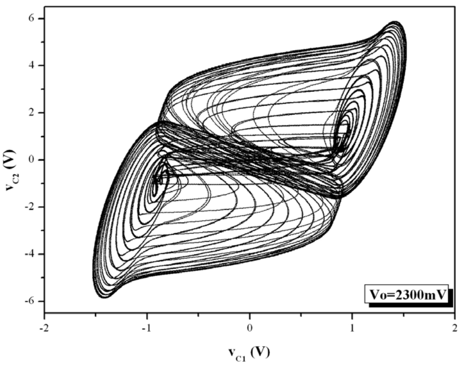

Fig.15. Theoretical v_{C2} vs. v_{C1} for V_o=2300mV.

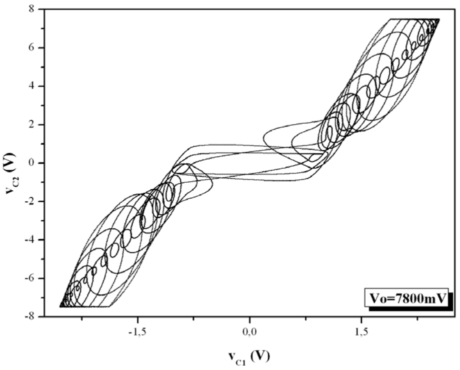

Fig.16. Theoretical v_{C2} vs. v_{C1} for V_o=7800mV.

4 Experimental Results

Using the implemented circuit of Fig.1, we appose the experimental phase portraits v_{C2} vs. v_{C1} for f=30Hz and f=35Hz and various values of the amplitude V_o of the input sinusoidal signal.

4.1 Experimental Results for f = 30Hz

In Figs.17-23 we observe the experimental phase portraits v_{C2} vs. v_{C1} for f =30Hz.

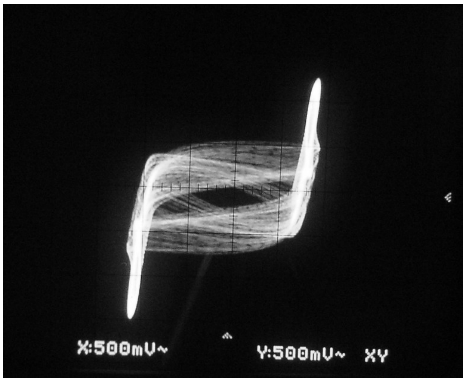

Fig.17. Experimental v_{C2} vs. v_{C1} for V_o=500mV.

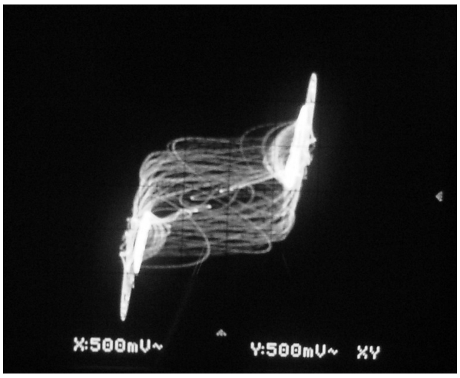

Fig.18. Experimental v_{C2} vs. v_{C1} for V_o=600mV.

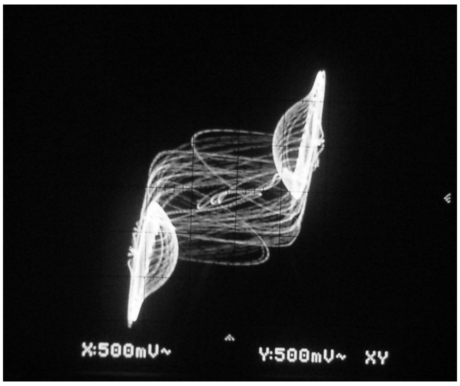

Fig.19. Experimental v_{C2} vs. v_{C1} for V_o=750mV.

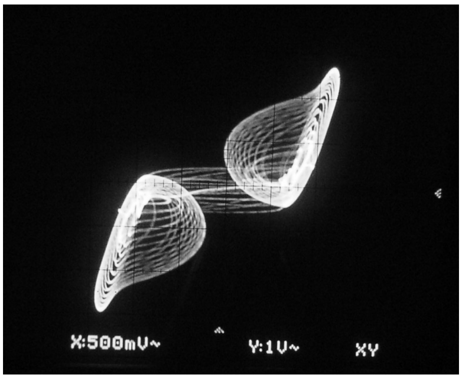

Fig.20. Experimental v_{C2} vs. v_{C1} for V_o=1400mV.

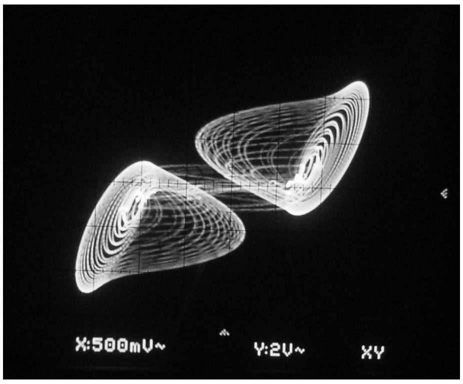

Fig.21. Experimental v_{C2} vs. v_{C1} for V_o=2000mV.

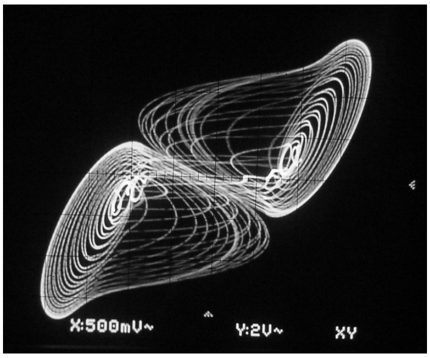

Fig.22. Experimental v_{C2} vs. v_{C1} for V_o=2800mV.

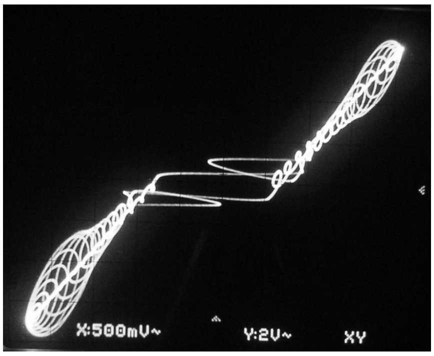

Fig.23. Experimental v_{C2} vs. v_{C1} for V_o=7800mV.

4.2 Experimental Results for f = 35Hz

In Figs.24-29 we can see the experimental phase portraits v_{C2} vs. v_{C1} for f =35Hz.

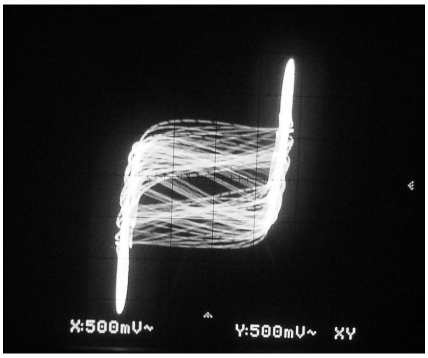

Fig.24. Experimental v_{C2} vs. v_{C1} for V_o=500mV.

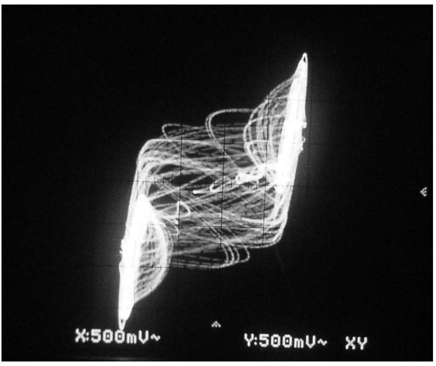

Fig.25. Experimental v_{C2} vs. v_{C1} for V_o=750mV.

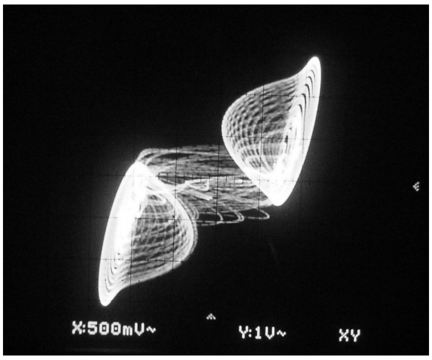

Fig.26. Experimental v_{C2} vs. v_{C1} for V_o=1300mV.

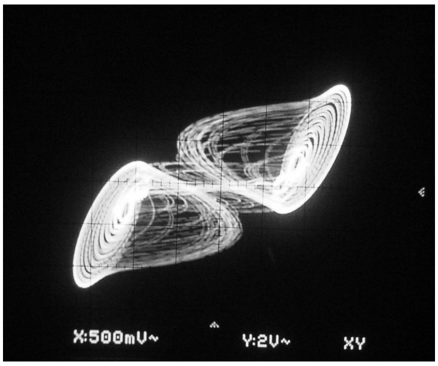

Fig.27. Experimental v_{C2} vs. v_{C1} for V_o=2000mV.

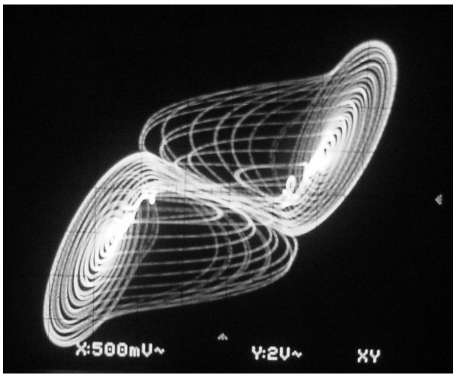

Fig.28. Experimental v_{C2} vs. v_{C1} for V_o=2500mV.

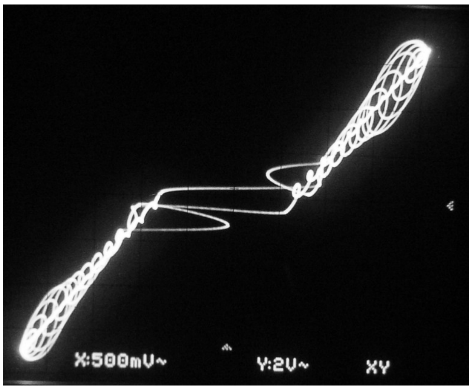

Fig.29. Experimental v_{C2} vs. v_{C1} for V_o=7800mV.

5 Conclusion

In this paper we have studied a nonlinear and non autonomous 4th order electric circuit, which contains a non linear resistor R_N with a N-type v-i characteristic. We have studied the dynamics of the circuit for various values of the input sinusoidal signal $v_s(t)$ in the low frequency area.

In particularly, we have observed the genesis and catastrophe of the "Double-Bell" strange attractor for low frequencies of the input signal, both theoretically and experimentally. We have demonstrated the formation of "Double-Bell" attractor for f=30Hz (see Figs.5 & 18) and the process in which the two independent attractors merge (see Figs.9-10 & 22-23). Moreover, the circuit exhibits the same behaviour for f=35Hz. So, we have seen genesis (see Figs.5 & 18) and catastrophe (see Figs.9-10 & 22-23) of "Double-Bell" attractor for f=35Hz, both theoretically and experimentally, too.

This work is precursor of future investigation of circuit's application in secure communication systems.

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