Bidirectional Coupling of two Duffing-type Circuits

Ch. K. VOLOS, I. M. KYPRIANIDIS, and I. N. STOUBOULOS
Physics Department
Aristotle University of Thessaloniki
Thessaloniki, 54124
GREECE

Abstract: - In this paper we have studied experimentally the case of chaotic synchronization of two identical nonlinear electric circuits. This is a very interesting research area because of its applications to the field of secure communications. The circuit we have used is a second order, Duffing-type, nonlinear electric circuit driven by a sinusoidal voltage source. The nonlinear element has a cubic i-v characteristic of the form, $i(v) = p \cdot v + q \cdot v^3$. We have studied the dynamic behavior of the system in the case of the bidirectional coupling via a linear resistor. Both experimental and simulation results have shown that chaotic synchronization is possible.

Key-Words: - Chaos, Duffing equation, Chaotic synchronization, Bidirectional coupling.

1 Introduction

Synchronization, among dynamical variables in coupled chaotic systems would appear to be almost an oxymoron as the definition of chaos. Since the beginning of the '90s, many researchers have discussed the synchronization of two coupled chaotic systems [1-4]. Synchronization of chaotic systems plays an important role in several research areas. For example, neural signals in the brain are observed to be chaotic and it is worth to consider further their possible synchronization [5]. Other interesting examples may be seen from the working artificial neural networks [6], biological networks [7], coupled chaotic neurons [8], multimode lasers [9], coupled map lattices [10, 11], and coupled electric oscillators [12]. Also, the topic of synchronization has risen great interest as a potential mean in communication [13, 14]. The last few years, a considerable effort has been devoted to extend the chaotic communication applications to the field of secure communications.

Generally, there are two methods of chaos synchronization available in the literature. In the first method, a stable subsystem of a chaotic system could be synchronized with a separate chaotic system, under certain suitable conditions. The second method to achieve chaos synchronization between two identical nonlinear systems is due to the effect of resistive coupling without requiring to construct any stable subsystem [15-17]. As we know from the bibliography, periodically forced synchronized chaotic circuits are much more noise-resistant than autonomous synchronized chaotic circuits.

In this paper we have studied the case of bidirectional coupling of two identical, second order Duffing-type electrical oscillators.

2 The Duffing-Type Circuit

Duffing's equation,

$$\frac{d^2x_1}{dt^2} + \varepsilon \cdot \frac{dx_1}{dt} + a \cdot x_1 + b \cdot x_1^3 = B \cdot \cos(\omega \cdot t) \quad (1)$$

is one of the most famous and well studied nonlinear non-autonomous equations, exhibiting various dynamic behaviors, including chaos and bifurcations. One of the simplest implementations of the Duffing equation has been presented by Kyprianidis et al. [18]. It is a second order nonlinear circuit, which is excited by a sinusoidal voltage source and contains two op-amps (LF411) operating in the linear region Fig.1. This circuit has also a very simple nonlinear element, implementing a cubic function of the form

$$i(v) = p \cdot v + q \cdot v^3 \quad (2)$$

which is shown in Fig.2.

Denoting by x_1 and x_2 the voltages across capacitors C_2 and C_4 respectively, we have the following state equations.

$$\frac{dx_1}{dt} = -\frac{1}{C_2 \cdot R_2} \cdot x_1 + \frac{1}{C_2 \cdot R_3} \cdot x_2 \quad (3)$$

$$\frac{dx_2}{dt} = -\frac{R_0}{C_4 \cdot R_5} \cdot f(x_1) + \frac{V_0}{C_4 \cdot R_5} \cdot \cos(\omega \cdot t) \quad (4)$$

where, $f(x_1) = p \cdot x_1 + q \cdot x_1^3$, is a cubic function.

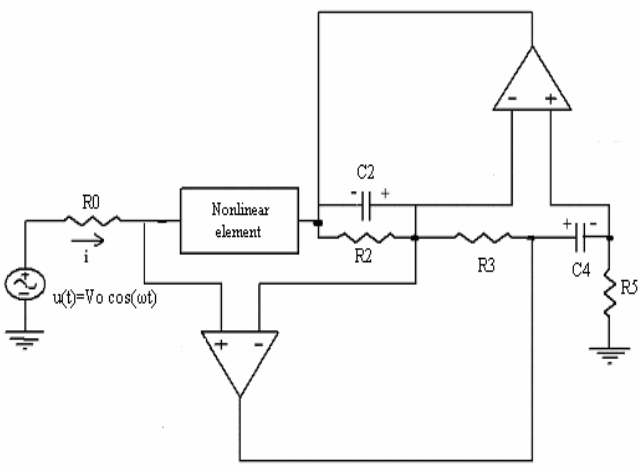


Fig.1. The electric circuit obeying Duffing's equation.

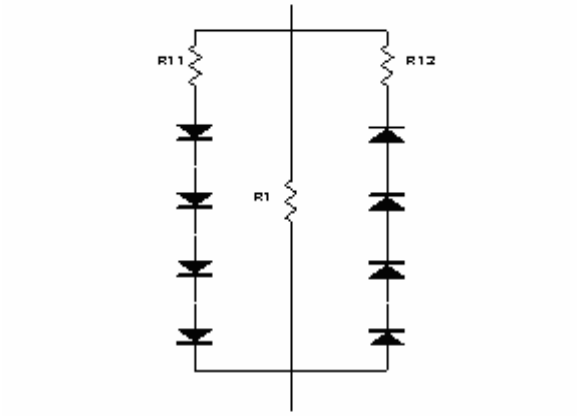


Fig.2. The nonlinear element implementing the cubic function of the form $i(v) = p \cdot v + q \cdot v^3$

Finally, from equations (3) and (4), we take the Duffing equation (1), where,

$$\varepsilon = \frac{1}{C_2 \cdot R_2}, \qquad a = \frac{p \cdot R_0}{C_2 \cdot C_4 \cdot R_3 \cdot R_5}$$

$$b = \frac{r \cdot R_0}{C_2 \cdot C_4 \cdot R_3 \cdot R_5}, \quad B = \frac{V_0}{C_2 \cdot C_4 \cdot R_3 \cdot R_5} \qquad (5)$$

The values of circuit parameters are R_0=2.05kΩ, R_2=5.248kΩ, R_3=R_5=1kΩ, R_{11}=R_{12}=0.557kΩ, R_1=8.11kΩ, C_2=105.9nF, C_4=9.79nF, V_0=2V and f=1.273kHz, so the normalized parameters take the following values a=0.25, b=1, ε=0.18, ω= 0.8 and B=20. The phase portrait of x_2 vs. x_1 is shown in Fig.3, where we can see that the circuit has a chaotic behavior.

3 The Coupled System

The system of two identical Duffing circuits bidirectionally or two-way coupled via a linear resistor R_C is shown in Fig.4.

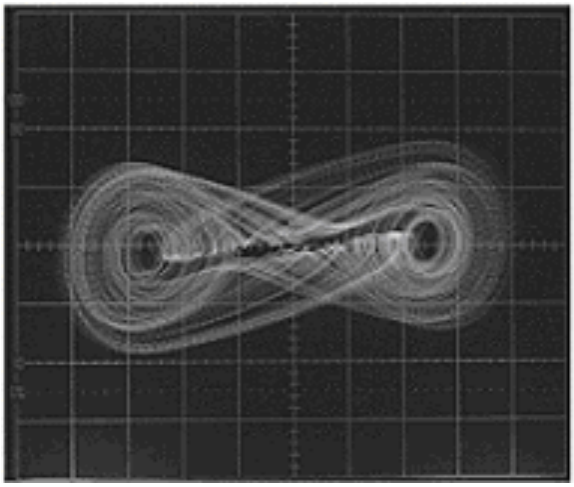


Fig.3. Experimental phase portrait of x_2 vs. x_1 for a=0.25, b=1, ε=0.18, ω= 0.8 and B=20 (Horiz.: 1V/div., Vert.: 5V/div.)

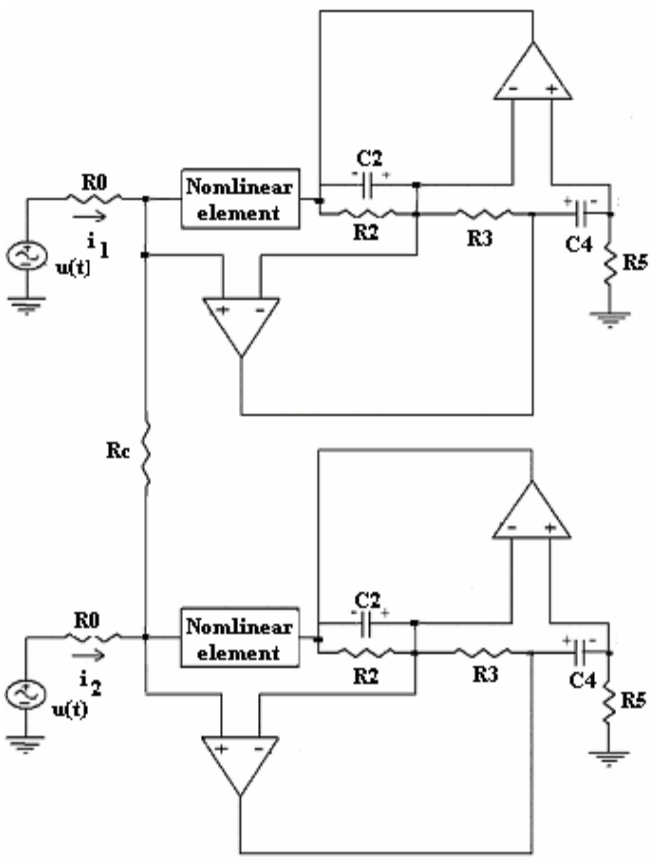


Fig.4. Two Duffing circuits bidirectionally coupled via a linear resistor.

The state equations of the system of Fig.4 has the form of equations (6-9), or the form of equations (10, 11), where, $x_1 = \upsilon_{C2}$, $x_2 = \upsilon_{C4}$, $x_1' = \upsilon_{C2}'$, $x_2' = \upsilon_{C4}'$, and $\xi = \frac{R_0}{R_C + 2R_0}$, is the coupling factor.

We have chosen the following values of the normalized parameters, a=0.25, b=1, ε=0.18, ω=0.8

and B=20, so the single circuit is in a chaotic steady state, as we saw before.

$$\frac{dx_1}{dt} = -\frac{1}{C_2R_2}x_1 + \frac{1}{C_2R_3}x_2 \quad (6)$$

$$\frac{dx_2}{dt} = -\frac{R_0}{C_4R_5}f(x_1) + \frac{q}{C_4R_5}\cos(\omega t) - \frac{R_0^{\,2}}{C_4R_5(R_C + 2R_0)}\left(f(x_1')-f(x_1)\right) \quad (7)$$

$$\frac{dx_1'}{dt} = -\frac{1}{C_2R_2}x_1' + \frac{1}{C_2R_3}x_2' \quad (8)$$

$$\frac{dx_2'}{dt} = -\frac{R_0}{C_4R_5}f(x_1') + \frac{q}{C_4R_5}\cos(\omega t) - \frac{R_0^{\,2}}{C_4R_5(R_C + 2\cdot R_0)}\left(f(x_1)-f(x_1')\right) \quad (9)$$

$$\frac{d^2x_1}{dt^2} + \varepsilon\frac{dx_1}{dt} + \alpha(1-\xi)x_1 + b(1-\xi)x_1^3 + \alpha\xi x_1' + b\xi\left(x_1'\right)^3 = B\cdot\cos(\omega t) \quad (10)$$

$$\frac{d^2x_1'}{dt^2} + \varepsilon\frac{dx_1'}{dt} + \alpha(1-\xi)x_1' + b(1-\xi)\left(x_1'\right)^3 + \alpha\xi x_1 + b\xi x_1^{\,3} = B\cdot\cos(\omega t) \quad (11)$$

The two single circuits have different initial conditions and we study the dynamics of the system, as the coupling coefficient ξ is increased from zero (uncoupled circuits).

4 Chaotic Synchronization

Considering the case, that the two coupled circuits are identical and are driven by signals of the same amplitude, we have studied chaotic synchronization as the coupling factor ξ is increased. The bifurcation diagram x_1 - x_1' versus ξ is shown in Fig.5. When the difference x_1 - x_1' becomes equal to zero, this means that the two circuits are in chaotic synchronization.

In Figs.6-14 we can see the experimental results from the coupled system for various values of the coupling resistor R_C. The system has a variety of dynamical behavior as we saw at the bifurcation diagram (Fig.5).We observe that the system appears phase-locked states of period-1, period-2, e.t.c. in different ranges of values of the coupling factor ξ (Figs.6, 8, 9, 12). Also, the system passes from chaotic states (Figs.7, 10, 11, 13) to a chaotic synchronization (Fig.16) as we expect form the bifurcation diagram. The phenomenon of chaotic synchronization appears for $\xi > 0.48$. So, the coupled circuits confirmed the theoretical results we took from the simulation of the dynamical system, as we saw in Figs.6, 11, 12, 14.

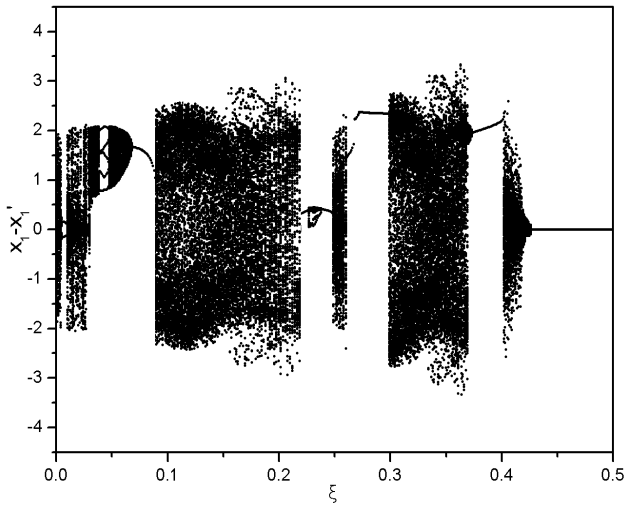
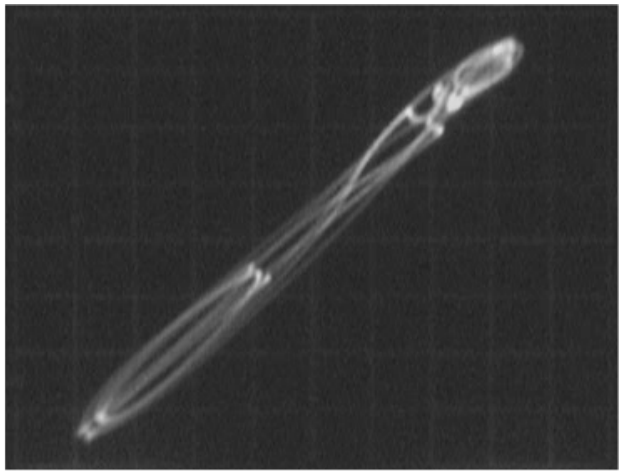
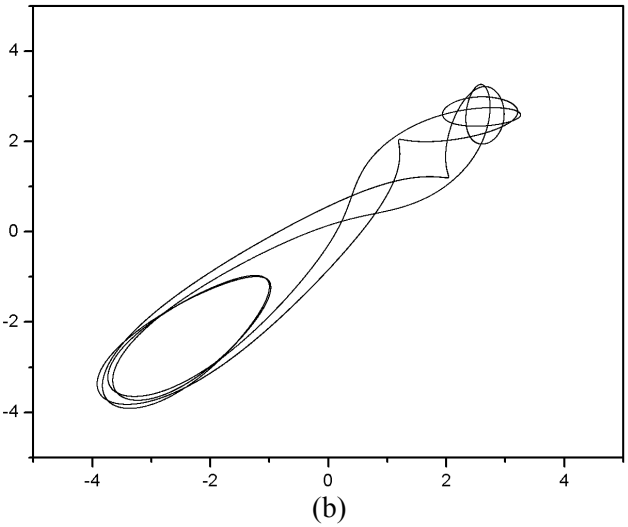


Fig.5. The bifurcation diagram x_1 - x_1' versus ξ for a=0.25, b=1, ε=0.18, ω=0.8 and B=20.



(a)



(b)

Fig.6. (a) Experimental phase portrait x_1' versus x_1 (Horiz.V_{C2}: 1V/div., Vert. V_{C2}': 1V/div.),
(b) Theoretical phase portrait x_1' versus x_1, for R_C=182kΩ (ξ=0.011). The system is in period-2.

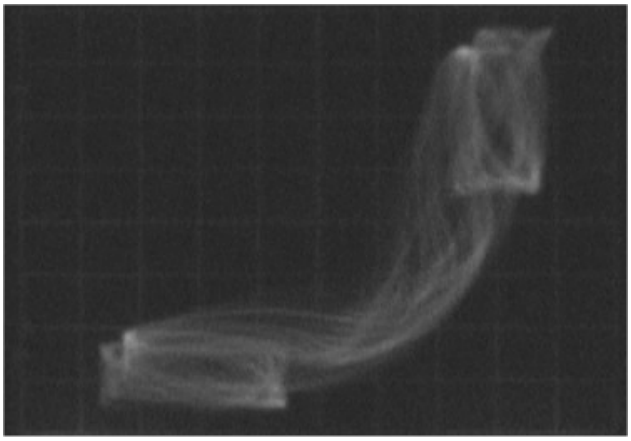


Fig.7. Experimental phase portrait x_1' versus x_1 for R_C=54kΩ (ξ=0.035).
(Horiz.V_{C2}: 1V/div., Vert. V_{C2}': 1V/div.).
The system is in a chaotic state.

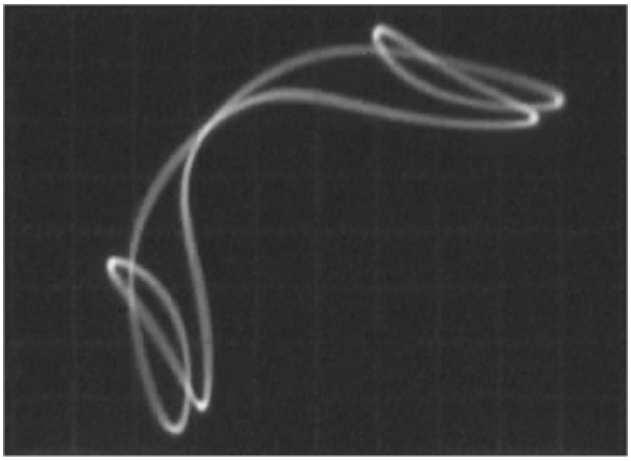


Fig.8. Experimental phase portrait x_1' versus x_1 for R_C=41kΩ (ξ=0.08).
(Horiz.V_{C2}: 1V/div., Vert. V_{C2}': 1V/div.).
The system is in period-1 state.

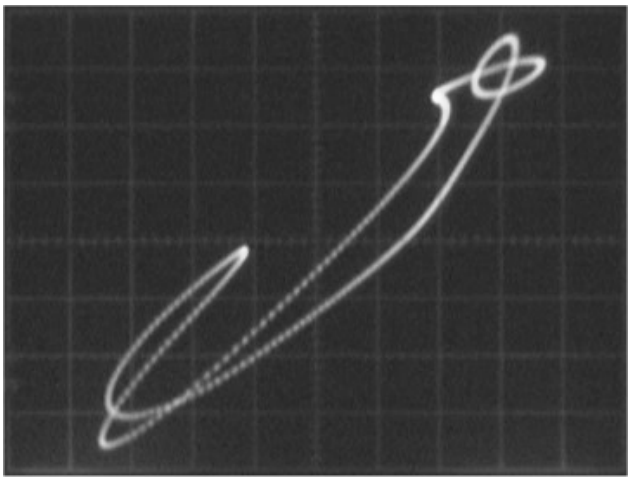


Fig.9. (a) Experimental phase portrait x_1' versus x_1 for R_C=5.2kΩ (ξ=0.22). (Horiz.V_{C2}: 1V/div., Vert. V_{C2}': 1V/div.). The system is in a period-1 state.

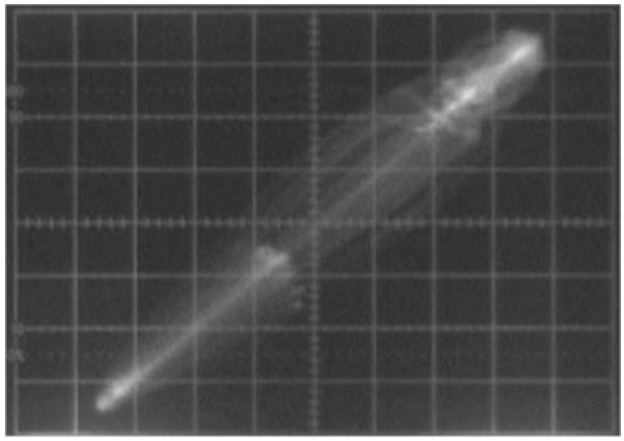
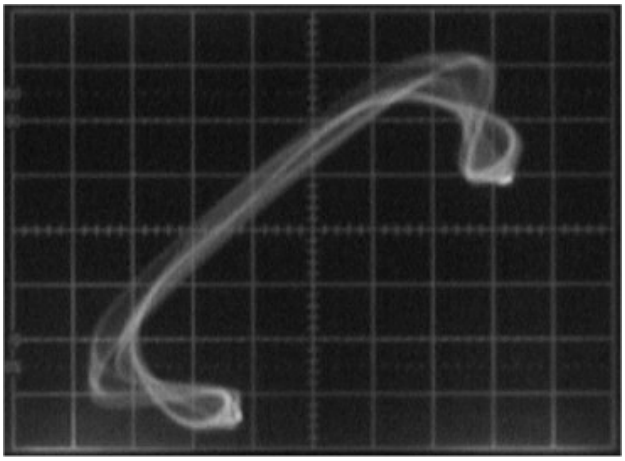
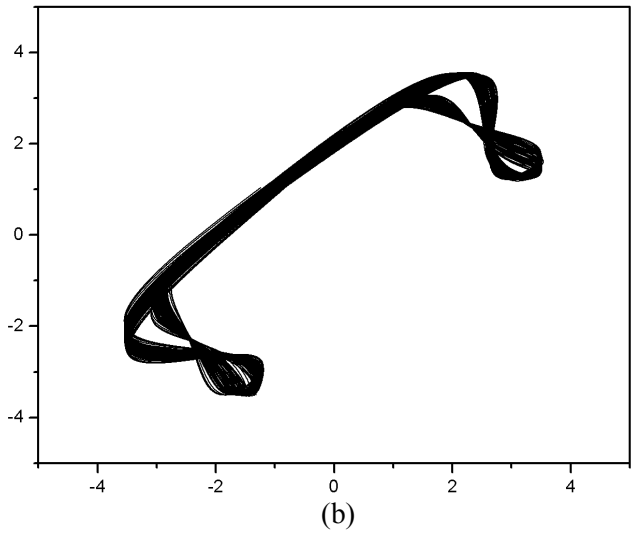


Fig.10. Experimental phase portrait x_1' versus x_1 for R_C=5kΩ (ξ=0.23).
(Horiz.V_{C2}: 1V/div., Vert. V_{C2}': 1V/div.).
The system is in a chaotic state.

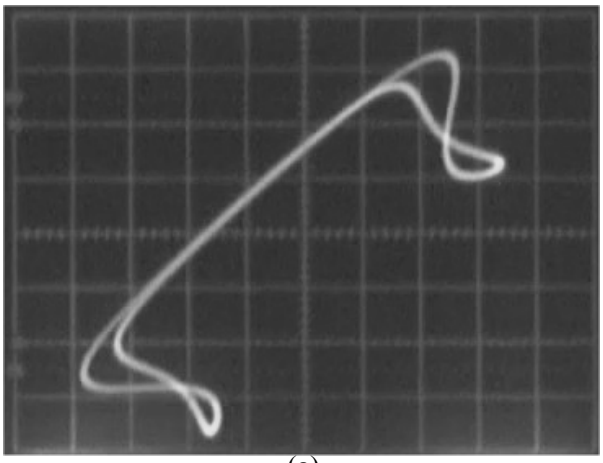


(a)

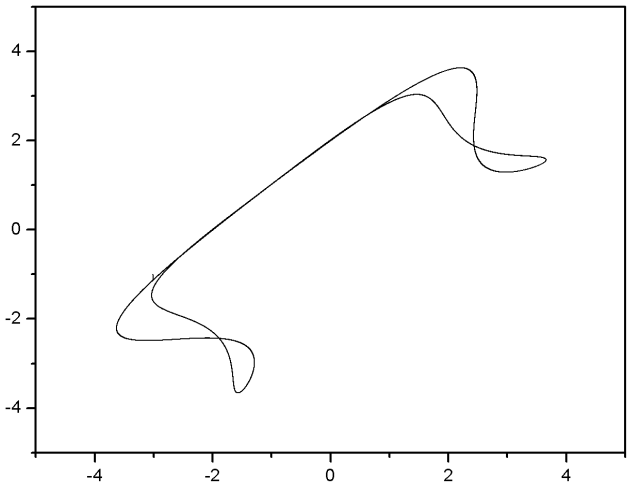


(b)

Fig.11. (a) Experimental phase portrait x_1' versus x_1 (Horiz.V_{C2}: 1V/div., Vert. V_{C2}': 1V/div.),
(b) Theoretical phase portrait x_1' versus x_1, for R_C=1.3kΩ (ξ=0.38) The system is in a chaotic state.



(a)



(b)

Fig.12. (a) Experimental phase portrait x_1' versus x_1 (Horiz.V_{C2}: 1V/div., Vert. V_{C2}': 1V/div.),
(b) Theoretical phase portrait x_1' versus x_1, for R_C=1kΩ (ξ=0.4).
The system is in period-1 state.

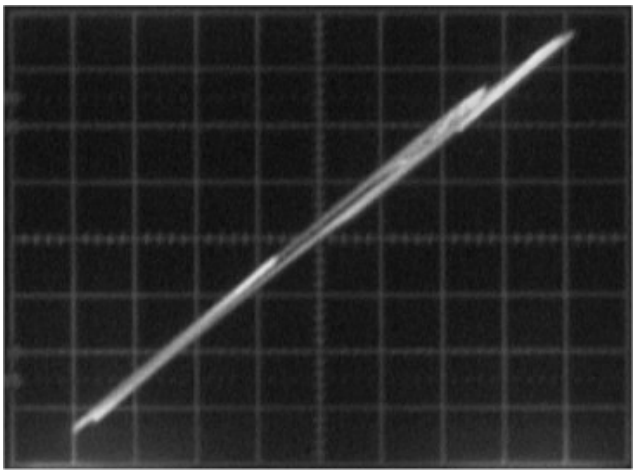
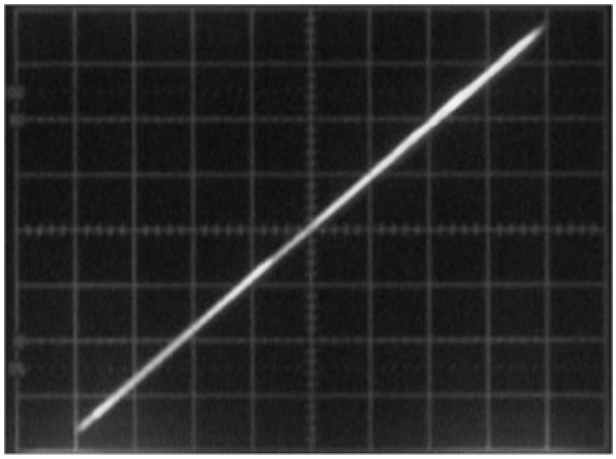
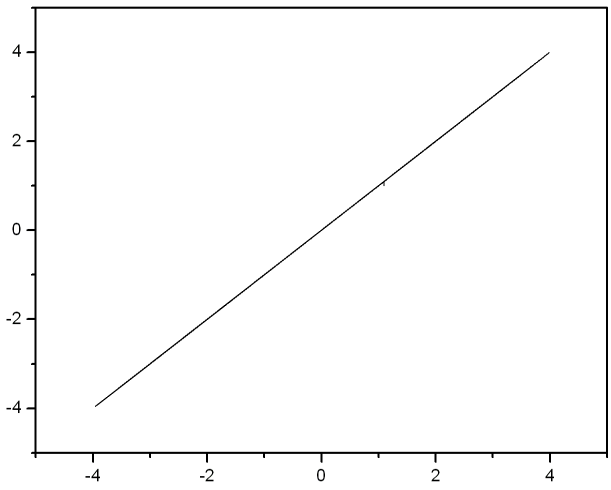


Fig.13. Experimental phase portrait x_1' versus x_1 for R_C=260Ω (ξ=0.47). (Horiz.V_{C2}: 1V/div., Vert. V_{C2}': 1V/div.). The system is in a chaotic state.



(a)



(b)

Fig.14. (a) Experimental phase portrait x_1' versus x_1 (Horiz.V_{C2}: 1V/div., Vert. V_{C2}': 1V/div.).,
(b) Theoretical phase portrait x_1' versus x_1, for R_C=170Ω (ξ=0.48).
The system is in a chaotic synchronization.

5 Conclusion

In this paper we have studied the dynamics of two resistively coupled nonlinear Duffing-type electrical oscillators. The two circuits are identical, having chaotic dynamical behavior, as we have found out from both theoretical and experimental results. We experimentally confirmed the expected behavior of the system for various values of the coupling resistor R_C. We have shown periodic and chaotic states, in different ranges of values of the coupling factor ξ. Finally, we observed a chaotic synchronization when the coupling factor $\xi > 0.48$.

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