A Visual Cryptography based system for sharing multiple secret images

Shang-Kuan Chen
Department of Computer Science and Information Engineering
Yuanpei University
Hsinchu, 300
Taiwan, R. O. C.

Abstract: - A Visual Cryptography (VC)-based system for sharing multiple secret images is proposed. Several weighted transparencies are generated so that people can reveal multiple secret images by stacking qualified subsets of some transparencies together. The transparency with relatively larger weight decides which secret image will be revealed. The proposed method has the following characteristics: decoding without computation, multiple secret images recovery, max-weight dominance, and quality-control design.

Key-Words: visual cryptography, image sharing, secret sharing, multi-secret sharing.

1 Introduction

Visual cryptography (VC) [1-4] is a kind of secret image sharing scheme that decodes a given secret image by human visual system without any computation. Naor and Shamir [1] introduced the so-called (r, n)-threshold visual cryptography scheme. In their scheme, n transparencies, called shares, are generated. Stacking at least r of the n generated shares can decode the secret, but stacking less than r shares gives no information about the secret. In their encoding phase of VC, each pixel of the secret image is extended to a block for each of the n generated shares. Let $p \times q$ be the block size. (Therefore, each share will be $p \times q$ times bigger than the secret image in size.) In the stacking result, if at least d_b black elements exist in each block representing black, and if at most d_w black elements exist in each block representing white, then the contrast of the stacking result was defined in Ref. [2] as $\frac{d_b - d_w}{p \times q}$.

Multi-secret images sharing system is an interesting research area for protecting many secret images. Wu and Chang [4] embedded two secret images into two circle shares. If two shares are stacked together, the content of the first secret image can be revealed. Moreover, rotating one of the two shares by a given angle degree, the other secret image can be revealed then. Their method is more flexibility than known VC methods. However, so far, only two secret images can be applied by their method.

Tsai et al. [5] and Feng et al. [6] proposed sharing methods for multiple secret images. In [5], they adopted XOR computing for embedding and extracting the secret images, and in [6], they adopted Largrange's interpolation that is applied from Thien and Lin [7] for generating several shares in order to recover the secret images. The method of [5] and [6] should use computer to extract the secret images. Although their methods are very convenient in network environment, but when in war, a computer may be not easy for use; the multi-secret image sharing must have other approaches, like VC.

Visual Cryptography for general access structure was introduced in [8-10]. Stacking each qualified subset of shares together can reveal the secret image, but stacking other, forbidden, sets of shares together has no information on the secret image. In [8-10], only single secret image is applied. Notably, in [6], their method can be applied on general access structure; however, as mentioned above, their method is not for VC, thus, can not reveal secret images by stacking the shares together.

In this paper, we propose a new VC scheme for progressive viewing, and the scheme shares k multi-secret images among k+1 shares. The method is a kind of access structure based on VC. The forbidden set contains only subsets of which each contains single share. All other subsets, containing at least two shares, can reveal at least one of the secret images after stacking them together. The answer to the question "which secret image will be revealed?" is totally up to the share whose weight is relatively larger among the received shares.

The remaining parts of this paper are organized as follows: the proposed method is stated in Sec. 2; the experimental results are shown in Sec. 3; the concluding remarks are described in Sec. 4.

2 The proposed method

The proposed scheme shares k binary secret images $S_1, S_2, \dots, S_k$ to generate k weighted shares and a basic weighted share. The k shares $T_1, \dots, T_k$ are with weights $w_1, w_2, \dots, w_k$, respectively, in increasing order: $w_1 < w_2 < \dots < w_k$. The basic share T_0 is with weight w_0, where $w_0 \le w_1$; however, the best contrast will be achieved if $w_0 = w_1$. The reason will be stated in Sec. 2.1. Let S_i consists of the binary pixels $\{b_{iz} \mid 1 <= z <= \text{size}(S_i)\}$. For each binary pixels $(b_{1z}, b_{2z}, \dots, b_{kz})$ of the secret images $(S_1, S_2, \dots, S_k)$, respectively, they are encoded into binary blocks $t_{0z}, t_{1z}, \dots, t_{kz}$ that belongs to $T_0, T_1, \dots, T_k$, respectively. Let $p\times q$ be the number of binary terms in each block. The block t_{iz} is defined to have w_i black terms and $p\times q - w_i$ white terms for each corresponding position z. In other words, each share will be $p ´ q$ times bigger than the secret image in size. In this multi-secret VC system, stacking T_i and some of $\{T_j \mid j < i\}$ together, the secret image S_i is revealed. To reach the goal, the shares are designed in the following steps that will be stated in Sec. 2.2.

Before stating the encoding algorithm, the stacking result to represent the secret images will be defined in Sec. 2.1, as the number of black terms in each block of stacking results that represent each binary pixels $(b_{1z}, b_{2z}, \dots, b_{kz})$ of the secret images $(S_1, S_2, \dots, S_k)$.

2.1 The definition of stacking result

Let the stacking result to represent S_i be X_i. For each block of X_i, the number of black terms corresponding to each original pixel b_i of S_i is defined as follows.

1. **"White"** pixel of S_i corresponds to w_i black terms in the block X_i. Notably, the number of black terms defined to correspond to a "white" pixel of S_i is the same as the one of t_{iz}. The w_i is the least number of black terms including T_i and some of $\{T_j \mid j < i\}$.

2. **"Black"** pixel of S_i corresponds to the following two cases:
($i < k$): more than w_i black terms and no more than w_{i+1} black terms in the block X_i.
($i = k$): more than w_i black terms and no more than $p\times q$ black terms in the block X_i.

For example, $\{S_1, S_2\}$ is the set of secret images, and T_0, T_1, T_2 are generated with weight w_0, w_1, and w_2. Therefore, for each position z, the number of black terms of blocks t_{0z}, t_{1z}, t_{2z} are w_0, w_1, and w_2, respectively. If the block of X_1 (the stacking result of T_0 and T_1) has w_1 black terms, the same as the number of black terms of t_{1z}, then it corresponds to a "white" pixel of S_1. If the block of X_1 (the stacking result of T_0 and T_1) has more than w_1 but no more than $\min\{w_0+w_1, w_2\}$ black terms of t_{1z}, then it corresponds to a "black" pixel of S_1. Notably, if w_0 equals to w_1, the difference between the "white" and the "black" of X_1 will be maximized. Similarly, if the block of X_2 (the stacking result of T_2 and one or both of T_0 and T_1) has w_2 black terms, then it corresponds to a "white" pixel of S_2. That is the reason why the block of X_1 (the stacking result of T_0 and T_1) has no more than w_2 black terms. If so, the stacking result of T_0, T_1 and T_2 can not represent the "white" pixel of S_2. If the block of X_2 (the stacking result of T_2 and one or both of T_0 and T_1) has more than w_2 black terms, then it corresponds to a "black" pixel of S_2.

2.2 The encoding algorithm

Now, we state the encoding process. For every position z in the corresponding block $t_{0z}, t_{1z}, \dots, t_{kz}$ of $T_0, T_1, \dots, T_k$, respectively, the block $t_{0z}, t_{1z}, \dots, t_{kz}$, are generated by turns: the block t_{0z} of basic share T_0 is generated first, the corresponding block t_{1z} of T_1 is generated then, … , and finally the corresponding block t_{kz} of T_k is generated. Let u_z be an array to record the accumulative terms during the work of generating the corresponding blocks $t_{0z}, t_{1z}, \dots, t_{kz}$ of the shares $T_0, T_1, \dots, T_k$.

Algorithm

For each position z of the secret image, the steps are descripted as follows:

Step 1: (Generate basic share T_0)
1. For generating t_{0z} of basic share T_0, w_0 terms are randomly assigned to black and others are assigned to white.
2. After generating t_{0z}, array u_0 records the black terms of t_{0z}.
3. Set i to 1.

Step 2: (Generate basic share $T_1 \sim T_k$)
1. For generating t_{iz} of share T_i, there are two cases for concern:
   Case 1: the corresponding pixel of secret image S_i is "white".
      1. Let g be a number of nonzero terms in current u_z. For these g corresponding terms in t_{iz}, they are assigned to black, and other terms in t_{iz} are assigned to white.
      2. The $w_i - g$ terms are randomly selected from zero-terms in u_z, and

for these corresponding terms in t_{iz}, they are assigned to black. The rest unassigned terms in t_{iz} are assigned to white.

3. After assigning t_{iz}, the corresponding terms of u increase according to the w_i black terms in t_{iz}.

4. If $i < k$, then increase i and return to Step2.
   If $i = k$, then the work for this position is completed and go through next position.

Case 2: the corresponding pixel of secret image S_i is "black"

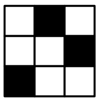
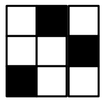
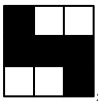
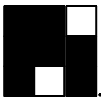
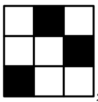
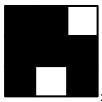
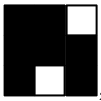
Case 2.1 : ($i < k$)

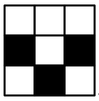
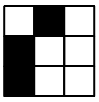
1. Select the minimal w_i terms from the maximal w_{i+1} terms in u_z.
2. For all corresponding terms of t_{iz}, they are assigned to black.
3. After generating t_{iz}, the corresponding terms of u_z increase according to the black terms of t_{iz}.
4. Increase i and return to Step 2.

Case 2.2: ($i = k$)

1. Assign the minimal w_i terms from u_z.
2. For all corresponding terms of t_{iz}, they are assigned to black.
3. The work for this position is completed and go through next position.

The following example shows how to share three binary secret images S_1, S_2, and S_3. For each position z and for each binary pixels (b_{1z}, b_{2z}, b_{3z}) taken from (S_1, S_2, S_3), respectively, the binary blocks t_{0z}, t_{1z}, t_{2z}, and t_{3z} are generated by turns. Let the weights $w_0 = w_1 = 3$, $w_2 = 5$, and $w_3 = 7$ for these shares and the size expansion of each share be 3×3. The binary pixels (b_{1z}, b_{2z}, b_{3z}) must be in the following 8 cases: (W, W, W), (W, W, B), (W, B, W), (W, B, B), (B, W, W), (B, W, B), (B, B, W), and (B, B, B), where W means "white" and B means "black". Table 1 shows the generated blocks t_{0z}, t_{1z}, t_{2z}, and t_{3z} for (b_{1z}, b_{2z}, b_{3z}) in these eight cases. Without the loss of generality, we show the process of generating the blocks t_{0z}, t_{1z}, t_{2z}, and t_{3z} as follow, where (b_{1z}, b_{2z}, b_{3z}) are in the two cases (W, B, W) and (B, W, B).

In the case (b_{1z}, b_{2z}, b_{3z}) = (W, B, W), t_{0z} is first generated by randomly selecting $w_0 = 3$ black terms, as  and u_z records accumulative terms as {0, 1, 0, 0, 0, 1, 1, 0, 0}. Since b_{1z} =W, to generate t_{1z} (with $w_1 = 3$), the black terms of t_{0z} is followed. Therefore t_{1z} is generated as . After t_{1z} generated, u_z records accumulative terms as {0, 2, 0, 0, 0, 2, 2, 0, 0}. Since b_{2z} =B, to generate t_{2z}, the minimal $w_2 = 5$ terms are selected from the maximal $w_3 = 7$ terms of u_z. The 7 terms include three value-2-terms and four zero-terms. Therefore, only four zero-terms can be selected and additional one term should be randomly selected from value-2-terms of u_z, such that, t_{2z} is generated as , and u_z records accumulative terms as {1, 2, 0, 1, 1, 3, 2, 0, 1}. Finally, to generate t_{3z}, since b_{3z} =W, there should be $w_3 = 7$ black terms, and the number of non-zero-terms of u_z is equal to 7. Therefore, t_{3z} is generated as . Now, the stacking results are shown below: the stacking result of "t_{0z} and t_{1z} " is , only $w_1 = 3$ black terms, thus it represents "white" in S_1; the stacking result of "t_{1z} and t_{2z} " and "t_{0z}, t_{1z} and t_{2z} " is , 7 black terms, representing "black" in S_2 (more than $w_2 = 5$ black terms); the stacking result of t_{3z} and subset (t_{0z}, t_{1z}, t_{2z}) is , $w_3 = 7$ black terms, thus it represents "white" in S_3.

In the case (b_{1z}, b_{2z}, b_{3z}) = (B, W, B), t_{0z} is also generated by randomly selecting $w_0 = 1$ black term, as . and u_z records accumulative term as {0, 0, 0, 1, 0, 1, 0, 1, 0} . Since b_{1z} =B, to generate t_{1z}, the minimal $w_1 = 3$ terms are selected from the maximal $w_2 = 5$ terms of u_z; thus, $w_2 - w_1 = 2$ terms are selected from zero-terms of u_z and remaining one term are selected from value-1-terms of u_z. Therefore, t_{1z} is generated as , and u_z records accumulative terms as {0, 1, 0, 2, 0, 1, 1, 1, 0}. Since b_{2z} =W, to generate t_{2z}, there should be $w_2 = 5$ black terms, and the number of non-zero terms of u_z is equal to 5.

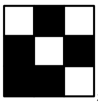
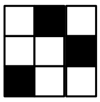
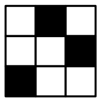
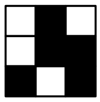
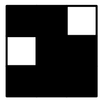
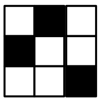
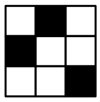
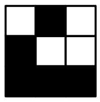
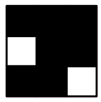
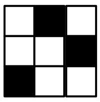
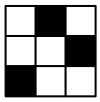
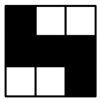
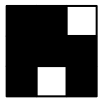
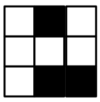
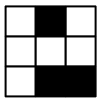
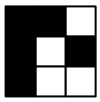
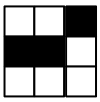
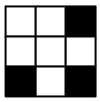
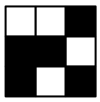
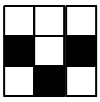
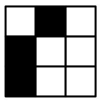
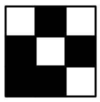
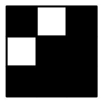
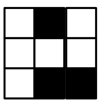
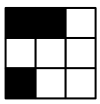
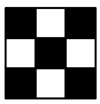
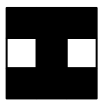
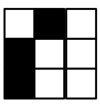
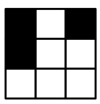
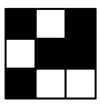
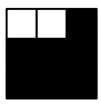
Therefore t_{2z} is generated as , and u_z records accumulative terms as {0, 2, 0, 3, 0, 2, 2, 2, 0}. Finally, to generate t_{3z}, since b_{3z} =B, the minimal w_3 = 7 terms are selected from u_z. Therefore, four zero-terms of u_z are selected, and then three terms are randomly selected from value-2-terms of u_z. The t_{3z} is generated as .

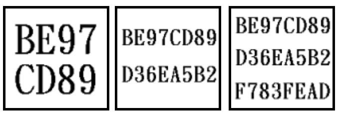
Table 1. Eight cases of encoding for each position z

b_{1z}	b_{2z}	b_{3z}	t_{0z}	t_{1z}	t_{2z}	t_{3z}
W	W	W				
W	W	B				
W	B	W				
W	B	B				
B	W	W				
B	W	B				
B	B	W				
B	B	B				

If we adequately assign the weights, quality-control design is also available. Obviously, when $T_0, T_1, \ldots, T_i$ are stacked together, the contrast is $\frac{w_{i+1} - w_i}{p \times q}$ (if $i = k$, then $w_{i+1} = p \times q$). Therefore, if the value of $w_{i+1} - w_i$ is larger, then quality of stacking result is better. For example, there are two secret images S_1 and S_2 and the size expansion $p \times q = 3 \times 3 = 9$. If the stacking result representing S_1 (Lena) is not easier to be distinguish than that representing S_2 (YPU-logo), the weights can be assigned as $w_0 = w_1 = 4$, and $w_2 = 8$. When stacking T_0 and T_1 together, the contrast is $\frac{w_2 - w_1}{p \times q} = \frac{8-4}{9} = \frac{4}{9}$. However, when stacking T_0, T_1 and T_2 together, the contrast is only $\frac{p \times q - w_2}{p \times q} = \frac{9-8}{9} = \frac{1}{9}$. If the stacking result representing S_2 (Lena) is not easier to be distinguished than that representing S_1 (YPU-logo), the weights can be assigned as $w_0 = w_1 = 4$, and $w_2 = 5$. When stacking T_0 and T_1 together, the contrast is only $\frac{w_2 - w_1}{p \times q} = \frac{5-4}{9} = \frac{1}{9}$. However, when stacking T_0, T_1 and T_2 together, the contrast is $\frac{p \times q - w_2}{p \times q} = \frac{9-5}{9} = \frac{4}{9}$. Moreover, for the example of Table 1, the assignment of weights $w_0 = w_1 = 3$, $w_2 = 5$, and $w_3 = 7$ is an average contrast assignment for all secret images.

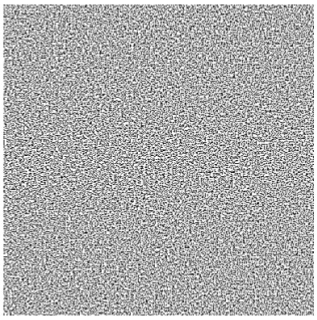
3 Experimental results

In the experiment, there are three secret images, shown in Fig. 1(a)-(c), are shared. The shares with weights $w_0 = w_1 = 3$, $w_2 = 5$, and $w_3 = 7$, shown in Fig. 2(a)-(d), respectively, are generated. Stacking Fig. 2(a) and (b) together, the secret message in Fig. 1(a) is revealed (see Fig. 3); stacking Fig. 2(b) and (c) together, the secret message in Fig. 1(b) is revealed (see Fig. 4); stacking Fig. 2(d) and at least one images from Fig. 2(a)-(c), the secret message in Fig. 1(c) is revealed. Fig. 5 shows the result of stacking all generated shares in Fig. 2.

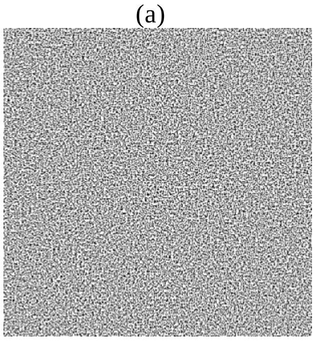


(a) (b) (c)

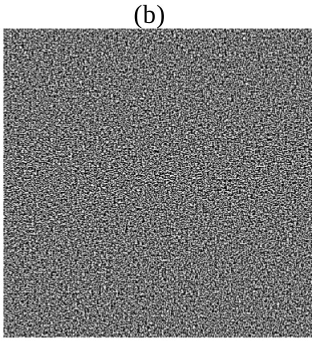
Figure 1. The secret images.



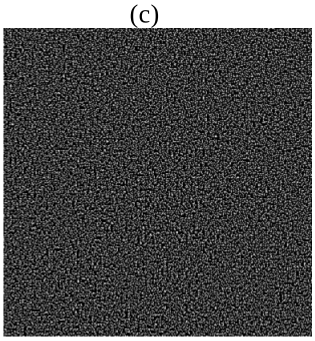
(a)



(b)



(c)



(d)

Figure 2. The generated shares ((a) basic share, (b) the share with weight = 3, (c) the share with weight = 5, (d) the share with weight = 7.).

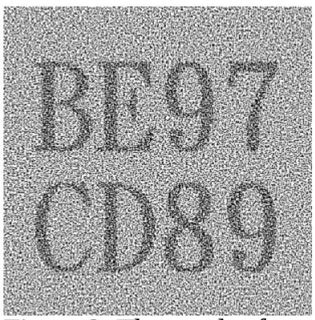


Figure 3. The result of stacking Fig. 2(a) and 2(b).

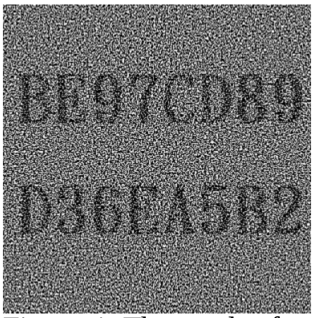


Figure 4. The result of stacking Fig. 2(b) and Fig. 2(c).

Figure 5. The result of stacking all shares in Fig. 2.

4 Concluding remarks

The proposed method is to share multiple secret images simultaneously. Several weighted shares are generated, and each is a binary noisy share. The revealing of multi-secret images is by stacking some shares together. Among the stacked shares, the one whose weight is relatively maximal will decide which secret image is revealed. The characteristics of the proposed method are (1) decoding without computation; (2) multiple secret image revealing; (3) the relatively maximal weight share will decide which secret image is revealed. Therefore, these properties make the tool applicable to the management of multiple secret images in a company. Another application is the so-called "game cards". When a player begins to play the game, he can get game card T_0 initially. When he passes the test at stage i ($i > 0$), he is granted the game card T_i. The player can view the secret image S_i by stacking T_i and T_j together (j is the number corresponding to an earlier stage).

References:

[1] M. Naor and A. Shamir, Visual Cryptography, *In Advances in Cryptology – Eurocrypt 94*, Springer, Berlin, 1995, pp. 1-12.

[2] T. Hofmeister, M. Krause, and H. U. Simon, Contrast-optimal k out of n secret sharing schemes in visual cryptography, *Theoretical Computer Science*, Vol. 240, No. 2, 2000, pp. 471-485.

[3] C. N. Yang and T. S. Chen, Aspect ratio invariant visual secret sharing schemes with minimum pixel expansion, *Pattern Recognition Letters*, Vol. 26, No. 2, 2005, pp. 193-206.

[4] H. C. Wu and C. C. Chang, Sharing visual multi-secrets using circle shares, *Computer Standards & Interfaces*, Vol. 28, 2005, pp. 123-135.

[5] C. S. Tsai, C. C. Chang, and T. S. Chen, Sharing multiple secrets in digital images, *The Journal of Systems and Software*, Vol. 64, No. 2, 2002, pp. 163-170.

[6] J. B. Feng, H. C. Wu, C. S. Tsai, and Y. P. Chu, A new multi-secret images sharing scheme using Largrange's interpolation, *The Journal of Systems and Software*, Vol. 76, No. 3, 2005, pp. 327-339.

[7] C.C. Thien and J.C. Lin, Secret image sharing, *Computers & Graphics*, Vol 26, 2002, pp. 765-770.

[8] G. Ateniese, C. Blundo, A. De Santis, and D. R. Stinson, Visual Cryptography for General Access Structures, *Information and Computation*, Vol. 129, 1996, pp. 86-106.

[9] G. Ateniese, C. Blundo, A. De Santis, and D. R. Stinson, Extended capabilities for visual cryptography, *Theoretical Computer Science*, Vol. 250, 2001, pp. 143-161.

[10] A. De Bonis and A. De Santis, Randomness in secret sharing and visual cryptography schemes, *Theoretical Computer Science*, Vol. 314, 2001, pp. 351-374.